Trade with Endogenous Market Power under Asymmetric and Incomplete Information

By

Manitra A. Rakotoarisoa
Economist
Trade and Markets Division
Economic and Social Development Department
Food and Agriculture Organization of the United Nations
D-835 Viale delle Terme di Caracalla 00153, Rome, Italy
+390657053809 Manitra.rakotoarisoa@fao.org

Paper selected for presentation at the Fourteenth Annual Missouri Economics Conference of the
US Federal Reserve Bank of Saint Louis,
US Federal Reserve Bank of Kansas City,
and
Department of Economics, University of Missouri-Columbia
Columbia, Missouri, March 21-22, 2014

Copyright © Rakotoarisoa 2014. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies. Views expressed here are the author’s own and do not represent the views of any Institutions.
Abstract

This paper addresses whether or not trade under both asymmetric information and endogenous market power fits standard assumptions and outcomes of mechanism design of imperfect competition. I analyse the outcomes of a bilateral trade in which a manufacturer (the principal) purchases $n$ inputs from a seller (the agent). Each input has a continuum of types, but the principal has no information on these input types, excepting their distributions. The model allows input types to shift input supply curves and flexibly accounts for any endogenous monopsony power (i.e. determined by the mechanism). Focusing on an optimal Bayesian mechanism, I find that the monotonicity assumption may not be enough to ensure price discrimination based on type. Truthful implementation implies that when allocation is increasing and weakly convex (curvature zero or positive) in the input type, the principal’s monopsony power decreases as the input type increases (i.e. is higher near competitive price for higher input type). However, under increasing but concave allocation, ambiguity remains as it is no longer guaranteed that high types would receive high prices. I also examine some extensions of the analysis in the cases of a benevolent principal and a mechanism with multiple agents.

Keywords: Mechanism design, Asymmetric information, Endogenous market power, Bilateral trade
JEL classification: D82, C72, L13, F12
1. Introduction

Buyers and sellers rarely have the same level of bargaining power or access to information. This reality renders the standard outcomes from the literature of mechanism design for auction and bilateral trade impractical. Additional framework and flexible functional forms are needed. The objective of this paper is to examine the outcome allocation of bilateral trade between a manufacturer (the principal) that may have some degrees of monopsonistic power over its purchases and a seller (the agent) of n inputs. The principal has no information about the agent’s input types, which are related to input cost, except the probability distribution of the input types, i.e., the distribution of the attributes that define the input cost. The aim are to develop a mechanism that leads to the equilibrium allocation, the amount and price of the inputs traded, and to determine endogenously the principal’s market power to such allocation.¹

A rational agent may announce a false type for each input hoping that the input will be purchased at a higher price. Under the ‘revelation principle’, the optimal allocation is assimilated to a mechanism for which the agent, when presented with an optimal payoff, has the incentive to reveal his private information about the type of each input. Moreover, the agent would nevertheless sell if the mechanism pays an amount greater than his minimum needs. I borrow these key assumptions but present a different model. I assign multidimensional types as in Severinov (2008) and model the degree of market power of the monosponistic firm endogenous (i.e. determined by the mechanism). Moreover, I assume that the types shift the input supply curve outward and that the principal has a production function allowing inputs to be complementary. Focusing solely on optimal Bayesian mechanism implementation, I examine the equilibrium allocation under profit maximization for the principal and how the mechanism determines the levels and changes in market power. I also examine the cases of a benevolent principal maximizing total surplus and of the ex post efficient allocations. Some extension and implications of the analysis for non-competitive markets are also discussed.

2. The Model

2.1 Basic Setting

The principal is a manufacturing firm employing n inputs of amount $x_i$, $i = (1, ..., n)$, to produce a single final output $Y = \Gamma(x_1, x_2, ...)$, where $\Gamma(.)$ is the production function, before selling the output at a price $P$. The principal buys all his $n$ inputs from a single agent (the seller) but has no information on the seller’s input types $\theta_i$ which determine the input cost $c_i(\theta_i)$. For each input $i$, $i = (1, ..., n)$ the input type $\theta_i$ is continuum within a known interval $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$. For instance, if the inputs are $i$ (e.g. capital) and $-i$ (e.g. labour), the type $\theta_i$ and $\theta_i$ can be the degrees of input productivity of capita and labour. Each type $\theta_i$ is assumed to be distributed with a positive probability density function $g_i(\theta_i)$ and a cumulative density function $G_i(\theta_i)$. The distributions of the input types are public knowledge. The principal’s problem is to choose a mechanism that

¹ See Veretka (2010) on endogenous market power for the bilateral oligopoly case.
determines the allocation \( x = x(\theta), \omega = \omega(\theta) \), namely the quantity \( x \) and price \( \omega \) of each input to be traded.

The optimal mechanism is based on the principal maximizing profit to determine the allocation. It is customary to begin with the agent’s decision as he holds private information on input types and hence on cost. A rational agent’s profit is

\[
\Pi_i(\hat{\theta}_1, \ldots, \hat{\theta}_n) = \sum_i \{x_i(\omega_i(\hat{\theta}_i) - c_i(\hat{\theta}_i))\} - \bar{c}_i,
\]

where \( x_i \) is the amount of input of the announced input type \( \hat{\theta}_i \) and \( \omega_i(\cdot) \) is the unit price at which the principal pays the input seller (the agent) per unit of input \( x_i \). The variable \( c_i(\theta_i) \) is the unit cost that depends on the input types, and for simplicity, I assume that the cost increases with the values of the input types: \( \frac{dc_i}{d\theta_i} > 0 \). The fixed cost \( \bar{c}_i \) takes into account fixed expenses by the agent while collecting the inputs.

The principal on the other hand has no information on the true input type and his expected product is

\[
E\Pi_\sigma = \{ \sum_i \int g(\theta) \left[ P_\sigma(\Gamma(x_1, \ldots, x_n) - x_i\omega(x_i, \hat{\theta}_i) - \bar{c}_0, \hat{\sigma}, \hat{\theta}_i) \right] d\hat{\theta}_i \},
\]

where \( P \) is the output price, \( \Gamma(\cdot) \) is the principal’s production function and the inputs are both technically and economically pairwise complementary. 1 The principal buys \( x_i \) amount of the input \( i \) with attributes \( \theta_i \) at price \( \omega_i \) from the agent and also operates with a fixed cost \( \bar{c}_0 \). The function \( g(\theta) \) is the probability density function of the distribution of the input type \( \theta_i \) for each input \( i \). The principal knows the probability densities \( g_i > 0 \) and \( G_i \) of the seller’s input types and some priors of their cumulative distribution such that \( G(\theta_i = 0) = 0 \) and \( G(\theta_i) = 1 \).

### 2.2 Key Assumptions

In focusing on optimal Bayesian mechanism, some key assumptions are required.

(i) The Individual Rationality (IR) constraint: This assumption ensures that the agent is willing to sell, i.e. the equilibrium allocation is \( x > 0, \omega > 0 \) such that

\[
\sum_i \{x_i(\omega_i(\hat{\theta}_i) - c_i(\hat{\theta}_i)) - \bar{c}_i\} \geq 0.
\]

This assumption can easily be justified because the seller needs a minimum revenue to cover all or at least a portion of the fixed costs (e.g. mortgage on the trucks, pension of permanent

---

1 The inputs are pairwise economically complementary in the sense that an increase in price of one input will reduce the demand for the other input; they are also technically complementary as the increase in the amount of one input increases the marginal product of the other input. These properties arise from the nature of the production function.
employees). Therefore, one way to ensure that IR holds is that the agent supplies at least a minimum amount $x_i$ for each input such that

$$
0 \leq E_x \left( \sum_i x_i \left( \omega_i - c_i \right) \right) \leq \bar{c}_i.
$$

Another particular, perhaps more practical, solution is to impose that the fixed cost is equally shared by all input revenues. In other words, the fixed cost is spread uniformly across all inputs and the revenue from each input sale must at least cover the share of the fixed cost for that input:

$$
x_i \omega_i \geq \frac{1}{n} \bar{c}_i \text{ for all } i.
$$

(ii) The Bayesian Incentive Compatibility (BIC) constraint: This assumption ensures that on average the agent (seller) will always benefit (via higher profit or higher utility) from claiming the true types rather than claiming other different types.

$$
E_{\theta_i} \left[ \sum_i \{ x_i \left( \omega_i (x_i, \theta_i) - c_i (\theta_i) \right) \} - \bar{c}_i \right] \geq 0
$$

For all $\theta_i \neq \theta_i'$

Following McAfee and McMillan (1988), Myerson (1989) and Mas-Colell et al. (1995), incentive compatibility implies monotonicity of the allocation of the input type $\theta_i$:

$$
\frac{dx_i}{d\theta_i} \geq 0.
$$

The monotonicity has been written and interpreted in various forms. One intuitive interpretation useful for this paper’s purposes is that the derivative of the marginal MRS with respect to the type has to increase with the type for the agent selling the input, as described in Laffont, Maskin and Rochet (1987) and detailed eloquently in Fudenberg and Tirole (2002):

$$
d\omega_i / d\theta_i = K_i (dx_i / d\theta_i), \text{ where } K_i \text{ is nonzero constant that may depend on the type.}
$$

Another equivalent interpretation is the Generalized Single Crossing Property (GSCP) summarized in McAfee and McMillan (1987):

$$
\omega_i (x_i, \theta_i) - \omega_i (x_i', \theta_i') = \rho \frac{d\omega_i (x_i, \theta_i)}{d\theta_i} (\theta_i - \theta_i') \text{ for } i \neq i'.
$$

---

3 This is also the ex-post IC assumptions in the terminology of Garg et al (2008).
4 If $x$ is labor and $\theta$ is number of years of professional experience, the amount of labor supplied by an educated worker will be larger than that of uneducated worker. Similarly, if $x$ is management skill and $\theta$ is level of education, education will shift the supply of management skill outward.
5 $K_i$ can be positive or negative depending on whether the agent is the main buyer or the main seller. Relation (6) is a direct consequence of quasi-linearity (Mirrlees 1971) and signifies that the marginal rate of substitution between what the agent supplies and what he receives as price is positive, although McAfee and McMillan (1987) affirms that this constant may depend on the cost types.
Equation (9) ensures that, with $\rho$ being constant, if the input price increases (or decreases) locally with the types, it will also globally increase (or decrease) with the types. In other words, the difference in the prices based on the true type and the price based on any other (false) claimed type is proportional to the difference between the ‘values’ of the true and false type.

(iii) Monopsony power

I assume that for each input $i$, the principal has a varying degree of monopsony power that can be measured by the non-negative parameter $\lambda_i$. I further assume that the supply curve is shifted by the agent’s input types by a known-valued function $\phi_i(.)$ that depends on the types. Formally, the price that the manufacturer pays to the agent is defined as

\[
\omega_i = \phi_i^{-1}(\theta_i)x_i^{\lambda_i},
\]

where $\lambda_i \geq 0$ indicates the level of monopsony power; perfect competition on the input purchase means $\lambda_i = 0$. Additionally, $\phi_i(.)$ has the property that

\[
(11) \quad \frac{d\phi_i}{d\theta_i} \geq 0 \quad \text{and} \quad \frac{dx_i}{d\phi_i} \geq 0
\]

These two properties imply that

\[
(13) \quad \frac{dx_i}{d\theta_i} \geq 0, \quad \text{which is also in line with the monotonicity implied by the IC in (6) and (7).}
\]

Equation (13) ensures that for each input $i$, higher types will be traded at higher volume than lower types. For instance, if $i$ is labour and $\theta_i$ is the level of education or number of years of experience of a worker, it is reasonable to assume that the principal would hire more of highly educated or more experienced worker. Overall, equations (10) and (13) mean that at a given labour price, a more educated labour or a more experienced worker will provide more labour (hence more productivity). This model’s ability to take into account endogenous monopsony power shall become clear in the next sections.
Figure 1. Price discrimination under monopsony
Note: For the same input, the monopsonist faces two types $\theta'$ and $\theta''$ with different equilibrium allocation. Focusing solely on the right half of the graph (type $\theta'$) helps explain how monopsony price and quantities are set. The supply curve facing the monopsonist is the average expenditure $\text{AE}'$ of his purchase, and it slopes upward because the marginal expenditure $\text{ME}'$ increases with the unit purchased. In other words, the average expenditure (the supply curve) increases because when an additional unit is purchased, the unit expenditure for all the units already purchased increases as well, and therefore the average expenditure increases. To maximize profit, the monopsonist purchases units up until his willingness to pay (MVP) intersects his marginal expenditures $\text{ME}'$. But because of his market power, the monopsonist offers a price that is at most equal to average expenditure of the last unit purchased. This price is lower than marginal expenditures. The pure rent per unit is the difference $\omega'_m - \omega'_c$. Note that the monopsonist’s purchased quantity is smaller than that purchased under perfect competition. Note also that, in the end, there is no demand schedule as the monopsonist’s demand is reduced to a single point $M'$ $(x'_m, \omega'_m)$. Analogously, $M''$ indicates the equilibrium monopsony price and quantity for type $\theta''$. The aim is to develop a mechanism that determines how the principal (buyer) and the agent (supplier) settle on either $M'$ or $M''$.

3. The Optimal Bayesian Mechanism for a Profit Maximizing Monopsonist

3.1 Equilibrium Outcomes
The principal’s problem is to choose the allocation that maximizes expected profit. Maximizing expected profit is equivalent to maximizing total surplus minus the virtual profit of the agent
(Fudenberg and Tirole, 2002). The virtual profit of the agent can be obtained by the quasi-linearity approach (Mirrlees, 1971). Using the assumptions of IC and IR above, the agent’s profit is:

\[(14) \Pi_1 = \max_\theta \sum_i \{ x_i \omega(x_i, \theta_i) - c_i(\theta_i) \} - \bar{c}_{1,}. \]

Using a linear form of the (14), the maximized profit is

\[(15) \Pi_1 = \pi(\theta) + \sum_i \int \frac{\partial}{\partial \theta_i} \{ x_i \omega(x_i, \theta_i) - c_i(\theta_i) \} d\theta_i. \]

where \(\pi(\theta)\) is the minimum profit (or minimum revenue), which can be set to zero for zero profit firm (or equal to the fix cost as minimum revenue).

Integration by parts leads to the agent’s virtual profit:

\[(16) \Pi_1 = \pi(\theta) + \sum_i \frac{1-G_i(\theta_i)}{g_i(\theta_i)} \frac{\partial}{\partial \theta_i} \{ x_i \omega(x_i, \theta_i) - c_i(\theta_i) \}. \]

The manufacturer maximizes the expected social surplus minus the agent’s virtual profit. Using equations (1), (2) and (16), the maximization problem is to solve

\[
\max_{\theta} \sum_{i=1}^n \int_{g_i(\theta_i)}^{\theta_i} \left\{ [P \Gamma(x_1, \ldots, x_i, \ldots, x_n) - x_i \omega_i(x_i, \theta_i) - \bar{c}_{1,}] ight\}
\]

\[+ [x_i \omega_i(x_i, \theta_i) - c_i(\theta_i)]

\[- [\pi(\theta) + \sum_i \frac{1-G_i(\theta_i)}{g_i(\theta_i)} \frac{\partial}{\partial \theta_i} (x_i \omega_i(x_i, \theta_i) - c_i(\theta_i))] g_i d\theta_i, \]

subject to a series of constraints stated earlier in section 2.2 Instead of using the Lagrangian method directly to include these constraints, I introduce them in sequences in solving the problem. Substituting the monopsonistic constraint in (9) directly into the objective function, the first order condition is

\[(18) \int_{g_i(\theta_i)}^{\theta_i} \left[ P \frac{d\Gamma}{dx_i} \right] - \left\{ [c_i(\theta_i)] + \frac{(1-G_i)}{g_i} \left[ x_i \phi_i(\theta_i)(1 + \lambda_i) \frac{\phi_i(\theta_i)}{\phi_i'(\theta_i)} - c_i(\theta_i) \right] \right\} d\theta_i = 0. \]

A condition to ensure that (18) respects that marginal product has to be positive is described in Appendix 1. More important, assuming that \(c_i''(\theta_i) = 0\), it can be shown (see Appendix 2) that a sufficient condition for the integrand to be increasing in \(\theta\) (a condition to ensure the existence of a solution) is that

\[(19) \phi_i'(\theta_i) = \frac{d^2 \phi_i}{d\theta_i^2} \leq 0. \]

This condition (19) ensures that the first order condition (f.o.c.) in (18) is solvable in \(x_i\) and also allows one to elicit the manufacturer’s strategy as a monopsonist, specifically to show how the monopsonist chooses prices.
The relaxed program solving the integration in (18) leads to equilibrium solutions that must satisfy the following f.o.c. for all \( \theta_i \):

\[
(20) \quad \frac{d\Gamma}{dx_i} = \left[ c_i(\theta_i) - J_i \left[ x_i^\frac{\lambda_i}{\lambda_i + 1} \left( 1 + \lambda_i \right) \frac{\phi_i'(\theta_i)}{\phi_i^2(\theta_i)} + c_i'(\theta_i) \right] \right] \quad \text{for all } i = 1, \ldots, n \quad \text{and all } \theta_i
\]

with \( J_i = \frac{1 - G_i(\theta_i)}{g_i(\theta_i)} \) is the inverse of the hazard rate.

The f.o.c. in (20) looks familiar, with the value of marginal product of the input on left side and a function involving input price and parameter of market power \( \lambda \) on the other side. Using all \( n \) equations of the form of (17) and solving for each \( x_i \) leads to a polynomial function in \( x \). There could be multiple solutions, but the generic expression of the allocation satisfying the individual rationality constraint in (5) is

\[
(21) \quad \left\{ \begin{array}{l}
  x_i = f(\theta_i, \theta_{-i}, \lambda_i, \lambda_{-i}, ; P, J_i, J_{-i}) , \\
  x_i \geq \left( \frac{c_1}{n} \phi(\theta_i) \right)^{\frac{1}{\lambda_i + 1}}
\end{array} \right.
\]

The allocation \( x_i \) depends on output price, the distribution of the types, and all parameters such as the degrees of market power. The allocation also depends on all the types especially because of the assumption that the principal’s production function features complementarity among inputs. The purchasing price is obtained by substituting (10) into (21):

\[
(22) \quad \omega_i = \phi_i^{-1}(\theta_i) \left( f(\theta_i, \theta_{-i}, \lambda_i, \lambda_{-i}, ; P, J_i, J_{-i}) \right)^{\frac{1}{\lambda_i}}.
\]

### 3.2 Endogenous Market Power in conjunction with Bayesian Incentive Compatible Equilibrium

In light of (21) and (22), when the agent (the seller) announces his type \( \theta_i \), the principal remains unable to set either the amount to be purchased or the input price unless he has information on \( \lambda \). The principal may set an arbitrary \( \lambda \) using the tatônnement process, but it is more reasonable to assume that he takes into account the types (false or true) announced by the seller. Because of the monotonicity constraint in (7), the amount to be purchased must increase with the type, and it is only after the announcement of the type and after knowing how the quantity purchased varies with the type (as expressed in equation 21) that the monopsonist would be able to set price using (22). In other words, price setting, i.e. the market power, has to be somewhat determined by the mechanism. The principal has to process first the information announced by the agent and to identify the function linking the type with the quantity \( x \) before finding out whether or not he has the power to set price. Two questions stand: What is the price level (how near to or far from the

---

6 Note that one can directly substitute in (20) the expression of the monopsony assumption \( \omega_i = \phi_i^{-1}(\theta_i) x_i^{\frac{1}{\lambda_i}} \) to find an implicit function containing the input price.
competitive price)? And more precisely, does the principal’s market power soften (small $\lambda$) or harden (large $\lambda$) as input type $\theta$ varies?

3.2.1 The Level of Market Power

To determine more precisely the parameter $\lambda$ that sets monosponist’s rent, it is important to examine the monosponist’s price setting rules in equation (10), namely the expression $\omega_i = \phi_i^{-1}(\theta_i) x_i^{\lambda_i}$. To start with, equation (10) implies that the degree of market power is

$$ (23) \lambda_i = \left( \frac{d\omega_i}{\omega_i} + \frac{d\phi_i}{\phi_i} \right) \left/ \frac{dx_i}{x_i} \right. . $$

Define the supply elasticity with respect to the shift function $\phi(.)$ as

$$ (24) \sigma_{x,\phi} = \frac{dx_i / x_i}{d\phi_i / \phi_i} , $$

which is positive and as implied by the definition in (12). (The more productive the inputs are, the more educated and more experienced the hired labour, the more the supply curve shifts outward.)

Moreover, the supply elasticity with respect to input price is also non-negative by definition:

$$ (25) \sigma_{x,\omega}(\theta_i) = \left( \frac{dx_i}{x_i} / \frac{d\omega_i}{\omega_i} \right) . $$

More important, the IC assumption in (8) hints that (25) shall be non-negative and also that, because all the economic variables and outcomes of the mechanism under study, $x$ and $\omega$ are positive.

Eq. (21) can now be rewritten as

$$ (26) \lambda_i = (\sigma_{x,\omega}^{-1} + \sigma_{x,\phi}^{-1}) \geq 0 . $$

In the absence of market power, $\lambda_i = 0$, which may arise only when both components of the right-hand side of (26) are zero, i.e. when the quantity responses to both small changes in price and small changes in the shifter are infinitely high. For instance, without market power, the buyer will lose significant input quantities by lowering the purchasing price even a small amount; in other words, the price is almost fixed. Similarly, at a given input price, a slight inward shift of the supply curve would result in a huge reduction in the quantity traded.

But in the case where $\lambda_i > 0$, the market power is simultaneously determined by the outcomes of the mechanism, represented by $\sigma_{x,\omega}^{-1}$. The IC assumption in (8) ensures that $\sigma_{x,\omega}^{-1}$ has to be non-negative; a strictly positive $\sigma_{x,\omega}^{-1}$ is sufficient to ensure that (26) is positive, which shows that the market power may be endogenously determined by the outcomes of the mechanism. Because computing the values of elasticity requires knowledge of the slope and the ratio of the
equilibrium price and quantity, the Individual Rationality (or participation constraint) in (3) ensures that at least one value of $\sigma^{-1}_{x,\theta}$ can be computed at the break-even point (zero profit). Moreover, since $\phi_i(.)$ is a known positive-valued function, the value of $\sigma^{-1}_{x,\phi}$ can be computed. As a result, a value of the market power in (26) can be computed at least for a zero-profit agent, the break-even point. Such uses of the mechanism design assumptions help to determine market power endogenously.

3.2.2 Comparative Statics: How Market Power Changes with the Input Type

The elasticities $\sigma^{-1}_{x,\theta}$ and $\sigma^{-1}_{x,\phi}$ that define the market power may depend on the types (see McAfee and McMillan (1987), Fudenberg and Tirole (2002) for some illustrations). It is not enough that the level of market power is determined, at least partly, endogenously. To ensure that price discrimination based on the types for each input exists, the key question becomes, How does this market power change with the input type? In other words, is it the case that the true endogeneity lies not in how the level is determined (since a method such as the tatônnement process can be used to determine the level of market power) but in how the mechanism affects the market power? The challenge is to determine the sign of the derivative

$$d\lambda_i \frac{d}{d\theta_i} = \frac{d}{d\theta_i} \left(\sigma^{-1}_{x,\theta} + \sigma^{-1}_{x,\phi}\right).$$

A positive sign for (27) means that the monopsonist prefers to assert his market power (lowering price and amount purchased) to increase his rent as the input type rises. Conversely, a negative sign for (27) implies that as input type rises, the monopsonist uses less market power (keeping price and quantity purchased near their competitive levels) and reduces rent.

To probe the sign of (27), I recall that the weak monotonicity and single crossing property assumptions in (9) imply that the supply elasticity must increase (or at least remain constant) with respect to the input type (see Fudenberg and Tirole (2002) or Maskin (1984)), which equivalently means that

$$\frac{d}{d\theta_i} \sigma^{-1}_{x,\theta} \leq 0.$$

In other words, the inverse of the supply elasticity with respect to price decreases with or is independent of the type. But what about the sign of $\frac{d}{d\theta_i} \sigma^{-1}_{x,\phi}$?

Using chain rules, one can write:

$$\frac{\partial^2 x}{\partial \theta^2} = \frac{\partial}{\partial \theta} \left(\frac{\partial x}{\partial \phi}\right) + \frac{d^2 \phi}{d\theta^2}.$$

Therefore,

---

7 Indeed, it maybe there is a reversal of market power; in some cases it is the agent who has the market power, not the principal depending on the types and the function $x()$. 

10
(30) \[ \text{sign} \left[ \frac{d}{d\theta} \sigma_{x,\theta}^{-1} \right] = -\text{sign} \left[ \frac{d^2 x(\theta)}{d\theta^2} - \frac{d^2 \phi}{d\theta^2} \right]. \]

Equation (30) conveys an important information: because (19) imposes that \( \frac{d^2 \phi}{d\theta^2} \leq 0 \), the sign of the change in degree of market power with respect to the cost type depends entirely on the curvature of \( x(\theta) \), namely the absolute value and sign of \( \frac{d^2 x(\theta)}{d\theta^2} \). This shows that the market power is endogenously determined and dictated by the outcomes of the game-theoretic mechanism. Taking into account the complex shape of the curve \( x(\theta) \) (which arises mostly from (21) and by substituting the price levels), several possible cases have to be examined (see figure 2).

Case 1: \( \frac{d^2 x(\theta)}{d\theta^2} < 0 \), i.e. the change in the slope of the curve \( x(\theta) \) is negative, that is, the supply increases with the type at a decreasing rate. In this case, the sign of \( \frac{d}{d\theta} \sigma_{x,\theta}^{-1} \) in (30) is undetermined without further assumption. There are two possibilities:

Case 1.1

if \( \left| \frac{d^2 \phi}{d\theta^2} \right| > \left| \frac{d^2 x(\theta)}{d\theta^2} \right| \), then \( \frac{d}{d\theta} \sigma_{x,\theta}^{-1} < 0 \) and \( \frac{d\lambda_i}{d\theta_i} < 0 \). In this case, the monopsonist will indeed soften his market power (buy more at a higher price, nearer the competitive price) to acquire high input types but will increase his market power (purchases less amount and at lower price) for lower input types;

Case 1.2

if \( \left| \frac{d^2 \phi}{d\theta^2} \right| < \left| \frac{d^2 x(\theta)}{d\theta^2} \right| \), then \( \frac{d}{d\theta} \sigma_{x,\theta}^{-1} > 0 \) and the sign of \( \frac{d\lambda_i}{d\theta_i} \) is undetermined. In other words, the effect of type on market power is ambiguous and depends on which effect is larger, the effect on the inverse supply elasticity w.r.t. price or that on the inverse supply elasticity w.r.t. the shifter function \( \phi \).

Case 1.2.1

If \( \left| \frac{d}{d\theta_i} \sigma_{x,\theta}^{-1} \right| > \left| \frac{d}{d\theta_i} \sigma_{x,\theta}^{-1} \right| \), i.e. if the effect on the inverse supply elasticity with respect to the shift is greater than the effect on the inverse supply elasticity with
respect to price, then \( \frac{d\lambda_i}{d\theta_i} > 0 \), i.e. the monopsonist exerts more market power (depressing price and quantity) as the type increases.

**Case 1.2.2**

Conversely, if \( \left| \frac{d}{d\theta_i} \sigma^{-1}_{x,\phi} \right| > \left| \frac{d}{d\theta_i} \sigma^{-1}_{x,\omega} \right| \), i.e. if the effect on the inverse supply elasticity with respect to the shift is smaller than the effect on the inverse supply elasticity with respect to price, then \( \frac{d\lambda_i}{d\theta_i} < 0 \) and the monopsonist will soften his market power as the input type increases.

**Case 1.2.3**

If \( \left| \frac{d}{d\theta_i} \sigma^{-1}_{x,\phi} \right| = \left| \frac{d}{d\theta_i} \sigma^{-1}_{x,\omega} \right| \), i.e. if the two effects cancel each other out, then the market power does not depend on the type at all.

**Case 2:** \( \frac{d^2 x(\theta)}{d\theta^2} \geq 0 \), i.e. the slope of the curve \( x(\theta) \) is increasing or constant, that is, the supply increase caused by the cost type increases with the type at a non-decreasing rate. In this case, \( \frac{d}{d\theta} \sigma^{-1}_{x,\phi} < 0 \) and \( \frac{d\lambda_i}{d\theta_i} < 0 \). This case joins the conclusion in Case 1.2.2 above. In this particular case, the monopsonist will indeed soften his market power (offering higher prices nearer the competitive price) to acquire high input types. Conversely, the monopsonist will increase his market power (purchase less and at lower prices) for lower types.
Figure 2: Monotonic allocation function and endogenous market power

Figure 2 summarizes how the mechanism determines market power. If the allocation curve includes concavities, assertion of market power is undetermined or ambiguous in some areas (this is Case 1). The direct implication is that a high type may not necessarily be offered a higher price (i.e. near competitive price) when the allocations for these types fall under the concavity area. Similarly, a lower type may be paid at higher by the monopsonist if the type falls under the area where the curvature is non-negative (weakly convex). In sum, there remains a great deal of uncertainty of how the market power is exercised. The way out is either ‘ironing’ the curve, which is Myerson’s approach, straightening the concavity or concentrating only on the weak convexity (analyzing only regions where the curvature is nonnegative). These adjustments obviously entail other difficulties, especially that the allocation becomes non-differentiable at some points.

**Proposition:** In a mechanism design with weakly monotonic and convex allocation curve (i.e., the allocation \( x \) increases with the type \( \theta \) at a constant or increasing rate), if the principal has some degree of market power, then the principal’s market power is more (less) assertive when type is low (high).

More formally, consider a buyer facing an agent who has two types of the same product, \( \hat{\theta} \) and \( \hat{\theta} \), with \( \theta < \hat{\theta} \), and, the buyer has some degree of market power levels \( \lambda \) and \( \lambda \) to discriminate
price and quantity allocation between these two types of the same product. Consider also that the supply elasticity with respect to the shift caused by these types is increasing in the type, i.e. \( \sigma(\theta') > \sigma(\theta) \). The above proposition says that under an incentive compatible mechanism, monotonicty \( (\frac{d}{d\theta} x(\theta)) > 0 \) and weak convexity \( (\frac{d^2}{d\theta^2} x(\theta)) \geq 0 \), the buyer’s rule is \( \lambda' > \lambda'' \).

**Proof**

The proof is straightforward. Ignore monotonicity for the moment and assume that the opposite holds, that is, \( \lambda'' > \lambda' \) (i.e. market power is exerted more at the higher type \( \theta' \) than at lower type \( \theta' \)). This implies that for an equal purchasing price \( \omega' = \omega'' \) it must be that \( x' < x'' \).

Let’s confirm that this is true. The price equalization condition \( \omega'' = \omega' \) can be written as

\[
(31) \phi^{-1}(\theta').(x')^x = \phi^{-1}(\theta').(x')^x. \quad \text{Convexity of the allocation (as the graph shows) means}
\]

\[
\frac{dx}{d\theta}_{\theta'} > \frac{dx}{d\theta}_{\theta''} \quad \text{for} \quad \theta' > \theta \quad \text{which implies} \quad \frac{dx}{d\theta} \frac{d\phi}{d\theta'} > \frac{dx}{d\theta} \frac{d\phi}{d\theta''} > 0,
\]

which ensures that \( \frac{d\phi}{d\theta} > 0 \) in the vicinity of each type. This implies that \( \phi^{-1}(\theta') > \phi^{-1}(\theta'') \).

Therefore, the ratio between the shifters is

\[
(32) a = \frac{\phi^{-1}(\theta')}{\phi^{-1}(\theta'')} > 1.
\]

The price equalization can be re-written as

\[
(33) a.(x')^x = (x')^x. \quad \text{If} \quad \lambda'' > \lambda' \quad \text{then} \quad (x')^x > (x')^x \quad \text{and subsequently by transitivity}
\]

\[
a.(x')^x = (x')^x > (x')^x,
\]

which finally yields

\[
(34) \quad \frac{x'}{x''} > \left( \frac{1}{a} \right)^{\frac{1}{\lambda}}.
\]

It is easy to verify that for any \( a > 1 \) and \( \lambda' > 0 \), the term \( \left( \frac{1}{a} \right)^{\frac{1}{\lambda}} \) will always be less than one (only approaching 1 for very large \( \lambda' \)), implying a possibility that \( x' > x'' \), which contradicts the prior assertion that \( x' > x'' \) always, i.e. that market power is exerted more on \( x' \) than on \( x'' \).

This proposition fits economic intuition in that inputs with higher attributes convince the monopsonist to loosen his market power and forgo some rent. Equivalently, high-attribute inputs give some bargaining power to the input owner. The only requirement is that it happens under

---

8 Note that under monotonicity (GSP), \( \sigma = \frac{dx(x)}{d\phi(x)} / \frac{d\phi(x)}{d\phi(\phi)} \) has to be non-negative positive because by definition \( \lambda \) has to be non-negative. In other words, \( (dx(x)) / (d\phi(x)) \) is not compatible with (\( \lambda \) being positive) monotonicity (i.e. GSP).
weak convexity of the supply as function of the type. In other words, for the monopsonist to offer both higher input prices (higher than the pure monopsony price but still lower than the competitive price) and larger input purchases (larger than the monopsony quantity but possibly still lower than quantity under perfect competition), the input type (i.e. attribute) has to cause supply to increase and more important, the input type has to prevent the supply increase from decelerating. This ‘non-deceleration’ condition reflects how input is viewed in production and trade. For instance, the amount of labor (or capital) in the production function matters, but the labor or capital attributes matter most. Within the category of ‘skilled labor’, for instance, each worker still has different attributes. What this finding implies is that these attributes (which are sometimes hidden or overlooked) affect the non-deceleration (increasing at a constant or an increasing rate) of the input effects on the output, i.e. on productivity. And it is the non-deceleration effects of the type that leads to the monopsonist’s prices and quantity discrimination.

4. Some Comparisons and Extensions of the Models

4.1 Profit Maximization under Complete Information (the ex-post efficient solution)

It is to be noted that the first order condition in (20) leads to the interim efficient allocation, since the agent knows the input types without revealing them. In a more implicit form, (20) can be rewritten as

\[ P \frac{d\Gamma}{dx_i} = \left[ (c_i'')(\theta_i) - J_i \left[ \phi_i'\phi_i' (\theta_i) + c_i''(\theta_i) \right] \right] \]

What if the principal has complete information on the type for each input? The ex post efficient solution comes from the following profit-maximization problem:

\[ \max_{x_i} E \Pi_o = \sum_{\theta_i} \int [P \Gamma(x_i, ..., x_n) - x_i \omega(x_i, \theta_i) - \bar{c}_0] g_i(\theta_i) d\theta_i \]

Formally, the f.o.c. is

\[ \int [P \frac{d\Gamma}{dx_i} - (1 + \lambda_i) \omega(x_i, \theta_i)] g_i(\theta_i) d\theta_i = 0 \]

And the solution must satisfy

\[ P \frac{d\Gamma}{dx_i} = (1 + \lambda_i) \phi^{-1}(\theta_i) x_i^{\lambda_i}, \] which can implicitly be written as

\[ P \frac{d\Gamma}{dx_i} = (1 + \lambda_i) \omega_i. \]

Comparing (35) with (38a), it is as if the virtual monopsony price under incomplete information is
This virtual price set by the monopsonist under incomplete information takes into account the distribution of the types and especially the cost curve. It is noted, however, that the equilibrium ex post efficient allocation from (34) is of the form:

\[
    x_i = \begin{cases} \frac{f(\theta_i, \theta_{-i}, \lambda_i, \lambda_{-i}; P)}{\phi_i(\theta_i)} & \text{if } \phi_i(\theta_i) \neq 0 \\ \infty & \text{if } \phi_i(\theta_i) = 0 \end{cases}
\]

(40)

In theory as originally detailed in Coase, (1937, 1952) and Williamson, 1981, the solution under complete information in (40) shall yield higher welfare than the solution under incomplete information in (21) because transaction cost is reduced.

### 4.2 Maximization of Total Surplus: A Benevolent Principal or a Third-agent Intervention

One form of extensions worth pursuing is that of maximizing total surplus. Integration of input-collecting activity into the manufacturer’s activity can be seen through the eyes of a benevolent principal that maximizes total welfare. The focus is now on the outcomes of such an integration of the two activities. The surplus maximization problem is

\[
    \max_{x_0, x_1} \mathbb{E} \Pi = \sum_{i} \int_{\mathbb{I}_i} [P \Gamma(x_1, ..., x_n) - x_i c_i(\theta_i) - \bar{c}_i - \bar{c}_0] g_i d\theta_i.
\]

The f.o.c. is

\[
    (42) \int_{\mathbb{I}_i} P \frac{d\Gamma}{dx_i} - c_i(\theta_i), g_i(\theta_i), d\theta_i = 0, \text{ for all } i, \text{ or}
\]

Or more implicitly,

\[
    (43) P \frac{d\Gamma}{dx_i} = E_{\theta_i} [c_i(\theta_i)] \text{ for all } i.
\]

Solving \(x_i\) shows that each \(x_i\) is a function of output price and of the expected unit costs of procuring each input. The supply curve is therefore a straight vertical line, an inelastic supply for which all the buyer has no market power:

\[
    (44) x_i = f(P^{-1}, E_{\theta_i} [c_i(\theta_i)]) E_{\theta_i} [c_{-i}(\theta_{-i})].
\]

Note that this is an implementable outcome because the allocation \(x_i\) increases with its own input types (since the \(dc_i/d\theta > 0\)). Moreover, the reservation price the principal pays to the agent can
be derived by substituting (44) into a modified IR (participation). Since the modified IR constraint is \( x_i \omega_i \geq \frac{1}{n} (\bar{c}_0 + \bar{c}_1) \), the reservation price becomes

\[
(45) \quad \omega_i \geq \frac{(\bar{c}_0 + \bar{c}_1)}{n. f(P^{-1}, E_\alpha [c_i(\theta_i)], E_\alpha [c_{-i}(\theta_{-i})])}.
\]

Whether the equilibrium allocation under total surplus maximization as defined in (44) is higher than that under profit maximization in (21) deserves further analysis but is beyond the scope of this paper.

### 4.3 Further Extensions: Identically Independent Distribution of the Types, Multi-Agent Case

When the types in all inputs are identically distributed, i.e. \( g_i(\theta_i) = g(\theta_i) \) for all \( i \), this implies that all the hazard rate functions are also the same. This simplifies notations of the optimal mechanisms without changing the nature of the outcomes in the analysis.

A more interesting case is a principal facing multiple agents. The case of one agent selling all \( n \) inputs can be extrapolated to represent the case of \( n \) agents selling one input each. The outcomes of the Bayesian implementation may not change much because the assumptions and solutions are based on expected values of all the functions that depend on the input types. However, the participation (IR) constraints may differ when the firms’ fixed costs are not identical. This may affect the principal’s strategy and use of market power for each input seller. Moreover, the functional form of the principal’s production function matters, especially the degree of complementarity or substitutability of the \( n \) inputs.

There are many applications of the model developed in this paper. Some examples: airline companies aiming to offer services to passengers without having information on their income levels or preferences; insurance companies dealing with clients that have hidden credit histories; firms rushing to hire skill-diverse workers through a job placement agency; and, more generally, processing industries purchasing raw materials of unknown qualities.

### 5. Conclusion

It is often emphasized that with the monotonicity constraint, the optimal mechanisms of imperfect competition under asymmetric and incomplete information result in price and quantity discrimination (i.e. higher price and quantity allocation for higher type). Usually, the only worry has been about possible slope reversal as the type increases, but this was settled by the “no distortion at the top” assumption; the allocation may include concavity so long as the overall allocation is increasing in the type and the allocation does not decrease for higher type (Fudenberg and Tirole, 2002). This paper examines how these standard assumptions and results apply to the particular case of trade under asymmetric information and where buyer’s market power is endogenous, i.e. determined by the mechanism.

I focus on Bayesian implementation of bilateral trade between a principal purchasing \( n \) different inputs from a selling agent and for which each input has various types known only to the agent.
The analysis employs standard assumptions of Incentive Compatibility, which implies participation of the agent and especially monotonicity of the allocation. The main innovation in this paper is to allow flexible functional form so that the buyer may have and use market power over the \( n \) input purchase and so that such market power is not pre-determined but endogenous to the mechanism.

The results show that the equilibrium allocation depends on the input type and on level of market power that allow price discrimination. More important, I find that the monotonicity constraint is insufficient to ensure price discrimination. A much stronger assumption is needed: only weakly convex (i.e. with a positive curvature) and type-increasing allocation can ensure that a buyer attaches higher price and higher quantity to higher types. Under concavities, there is no guarantee of such price discrimination. I also briefly examine the allocation of the same mechanism under total surplus maximization and find that the equilibrium includes an inelastic input supply giving no market power to the principal. These findings enhance mechanism design analysis in explaining how price discriminations form in some markets under imperfect competition, especially when market power is not pre-determined. In addition to the possible extensions briefly discussed in the paper, two major improvements are worth pursuing. First is to introduce correlations among the input types, as in Severinov (2008), and address this issue for the monopsony or endogenous market power cases. Another refinement will be to compare the actual welfare gain or loss of moving from profit to total surplus maximization. These analyses will provide better understanding of the vertical merging of some companies and also the role that a third party (say the government or a social planner) can play to affect both individual and social outcomes of bilateral trade. This paper’s model and findings, especially on the need to review standard assumptions in mechanism design, offer some pathways leading to these improvements.
Appendix 1
Formally, the first order condition of the maximization problem in (18) can be written as:
$$\frac{d\Gamma}{dx_i} = E_\theta \left[ c_i(\theta_i) \right] + E_\theta \left[ J_i \omega_i(1 + \lambda_i)(-\frac{\phi_i'(\theta_i)}{\phi_i(\theta_i)}) - J_i c_i'(\theta_i) \right] , \text{ for all } i = 1, \ldots n. $$
Since the left-hand side is the value of marginal product and it has to be positive, and since the average cost $E_\theta [c_i(\theta_i)]$ and $E_\theta [c_i'(\theta_i)]$ are positive (since $c_i > 0$ and $\frac{dc_i}{d\theta_i} > 0$ within the domain considered in the study), it must be that
\begin{equation}
(A.1) \quad E_\theta \left[ c_i(\theta_i) - J_i \omega_i(1 + \lambda_i)(\frac{\phi_i'(\theta_i)}{\phi_i(\theta_i)}) \right] \geq E_\theta \left[ J_i c_i'(\theta_i) \right].
\end{equation}

Appendix 2
The f.o.c. of the maximization problem in (18) can also be written as
$$P. \frac{d\Gamma}{dx_i} = \int \frac{G_i}{g_i(\theta_i)} \omega_i(1 + \lambda_i)(-\frac{\phi_i'(\theta_i)}{\phi_i(\theta_i)}) - c_i'(\theta_i) \right] dG_i. $$
If the integrand has to be increasing in $\theta_i$, with the conditions that the hazard rate is monotonically increasing and that the unit cost rises at a constant rate with the type (i.e. $c'(\cdot) \geq 0$ and $c''(\cdot) = 0$), then it must be that
\begin{equation}
(A.2) \quad \frac{1-G_i(\theta_i)}{g_i(\theta_i)} \cdot \frac{d}{d\theta_i} \left\{ \omega_i(1 + \lambda_i)(-\frac{\phi_i'(\theta_i)}{\phi_i(\theta_i)}) \right\} \geq 0. \text{Substituting the price definition}
\end{equation}
$$\omega_i = \phi_i^{-1}(\theta_i)x_i^{\lambda_i}$$ into (A.2), the condition becomes
\begin{equation}
(A.3) \quad -\phi_i(\theta_i) + 2 \frac{\phi_i'(\theta_i)}{\phi_i(\theta_i)} \geq 0, \text{for which a sufficient condition is that } \phi_i(\theta_i) \leq 0.
\end{equation}

References


