The Impact of CDS on Firm Financing and Investment: Borrowing Costs, Spillovers, and Default Risk[1]

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Abstract

This paper studies the effects credit default swaps (CDS) have on firm financing in a general equilibrium production economy with heterogeneous firms. CDS induce borrowing cost spillovers in both covered and naked CDS economies. Covered CDS lower borrowing costs for all firms even when CDS contracts are traded on only one firm type. This leads to higher investment levels in the economy, but raises the likelihood of default when productivity shocks are bad. Conversely, naked CDS raise borrowing costs for all firms even when CDS contracts trade on a single firm type. This lowers investment levels in the economy, and lowers the likelihood of default.

JEL Classification: D52, D53, E44, G10, G12

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1 Introduction

The Global Financial Crisis that began in 2007 underscores the need to better understand how financial market participants price and take risk. Credit default swaps (CDS) are particular types of financial instruments that market participants use with increasing regularity to not only price different types of risk, but also manage balance sheet risk and regulatory capital requirements. According to the Bank for International Settlements (BIS) the notional size of the CDS market (value of all outstanding contracts) at its peak before the market crash in 2007 was \$57 trillion. While that number has abated, its size at the end of 2011 was \$29 trillion. Clearly the CDS market remains large and active.

CDS contracts are traded on various types of debt. For example, CDS contracts trade on corporate debt, sovereign debt, market indexes or a basket of credit entities to name a few. According to the BIS, in 2011 60% of the gross notional value–or roughly \$17.5 trillion–of the CDS market are contracts on single-name entities. Of this \$17.5 trillion, CDS contracts on corporate debt make up roughly 80% (\$14 trillion) of the single-name CDS market value. Almost 60% of the single-name corporate CDS market is traded on non-financial firms, making it the single-largest reference entity category in the CDS market with outstanding contracts valued at approximately \$8.4 trillion. Hence the focus of our analysis will be on the effects CDS contracts have on corporate debt.

There is currently very little theoretical work detailing how credit derivatives affect borrowing cost and investment in a production environment. This paper aims to fill this gap. We investigate the role of credit derivative markets on corporate borrowing costs, endogenous investment, and default risk. We find that CDS alters the collateral value of cash in the economy. This changes the demand for bonds, which impacts firm borrowing costs, investment levels and alters economy-wide default risk despite there being no changes to fundamentals in the economy.

We first consider a static general equilibrium model with two states characterized by an aggregate productivity shock. There are two firm types endowed with different production technologies. Each firm endogenously issues non-contingent, collateralized debt in order to produce. Debt financing comes from investors with heterogeneous beliefs over the probability of future states. We solve the baseline model without CDS for equilibrium bond prices, firm borrowing costs and firm investment demand.

Second, we allow investors to issue covered CDS contracts on firm debt. A covered CDS means that the buyers also own the underlying bond. Thus, in covered CDS economies, the number of CDS contracts sellers can issue is limited to the number of underlying bonds. Optimistic investors demand consumption in good states *i.e* "Arrow-Up" securities, which can be achieved through selling CDS. This tends to raise bond prices and lower borrowing costs.[4] Firms endogenously respond to lower borrowing costs by increasing investment. Additionally, firms are more likely to default for certain parameters as they increase their investment decisions.

Third, we allow investors to purchase naked CDS. Naked CDS means that CDS contract owners do not own the underlying bond. The demand for naked CDS increases the overall demand for credit derivatives. This results in an increase in firm borrowing costs and lower subsequent firm investment levels leading to decreased firm default risk over certain parameter regions. This suggests that there are costs and benefits to both types of CDS contracts when investment is endogenous; covered CDS lower firm borrowing costs and increase default risk, while naked CDS increase firm borrowing costs and lower default risk.

Fourth, the introduction of a single CDS type affects *both* firm types' borrowing costs and investment decisions. In other words, CDS generate what we call "initial" spillovers. The spillovers are positive in covered CDS economies. The reason is that covered CDS allow a smaller set of the most optimistic investors to price and hold all credit risk because the number of CDS contracts that can be sold is limited to the number of underlying bonds. Consequently, the investors who purchase the other firm type's debt are more optimistic relative to an economy in which CDS do not exist. This drives up bond prices and lowers firm borrowing costs. Conversely, the spillovers are negative in naked CDS economies. Pessimistic investors demand naked CDS and drive up CDS prices. This induces more investors to sell credit derivatives instead of buying the other firm type's bonds, causing its bond price to fall.

Last, introducing CDS contracts on the second firm type generates "additional" spillovers. Additional CDS types allow multiple but equivalent ways to buy Arrow securities. This reduces the need to use CDS on any *specific* firm type. In other words, with aggregate shocks investors do not care what type of firm debt CDS contracts insure. In covered CDS economies, this *raises* borrowing cost for firm types for whom CDS *previously existed.* Conversely, in naked CDS economies, additional CDS types *lower* borrowing costs for firms with pre-existing CDS contracts.

The key difference in how additional CDS influence borrowing costs is whether or not investors can purchase naked CDS. For example, in covered CDS economies, introducing additional CDS types reduces the competitive pressure on the *sell-side* of the credit insurance market. Optimistic investors who choose to sell CDS can do so using either firm types' bond as the underlying asset which reduces the competition to sell credit derivatives on the initial firm type's debt. This tends to increase the CDS price and raise borrowing costs. Conversely, in naked CDS economies, introducing additional CDS types reduces the competitive pressure on the *buy-side* of the initial

[4] As we show later in the paper, Arrow-Up securities and bonds are complimentary assets in covered CDS economies. Optimistic investors implicitly raise the demand for bonds since the only way for them to purchase synthetic Arrow-Up securities is through insuring credit risk.

CDS market. Pessimistic investors who choose to buy naked credit derivatives can do so on either firm types' bond. This reduces the price of credit protection insuring the initial firm type's bond, which lowers its borrowing costs.

Our model's borrowing cost spillovers in bond markets is novel. CDS can either free or tie up capital to buy other assets depending whether investors are required to hold the underlying bond. Ashcraft and Santos (2009) evaluate how CDS affect bond prices and borrowing costs, but only test for how CDS affect the borrowing costs for the underlying reference entity, and do not consider any spillover effects. Norden et. al (2014) find evidence of interest rate spillovers in syndicated bank lending markets, but not bond markets. The authors attribute interest rate spillovers in bank lending markets from banks' use of CDS to more effective portfolio risk management.

Our work adds to a growing theoretical literature on the economic impact of credit derivative markets. Our work is most closely related to a class of heterogeneous agent models developed by Fostel and Geanakoplos ((2012) & (2013)). Fostel and Geanakoplos (2012) show in an endowment economy that financial innovation in credit derivatives markets alters asset collateral capacities, and asset prices. Fostel and Geanakoplos (2013) study the effect credit derivatives have on endogenous investment outcomes and show that credit derivatives can lead to investment beneath the first best level obtained in an Arrow-Debreu Economy. Our model is distinguishable from their models because we treat production and financial investment separately. This allows us to incorporate heterogeneous production and endogenous default in a very simple way. Che and Sethi (2012) study how CDS affect borrowing costs for a representative firm with a random output draw that raises an exogenous amount of capital. Our model adds several relevant features by explicitly modeling a production environment with different firm types so that CDS need not have a uniform effect on potential borrowers. Moreover, the flexible specification we utilize allows one to generate comparative static results for changes in productivity, economy wide technology shocks, as well as firm optimism. Second, our model gives rise to a more in-depth discussion of investment and default decisions because investment needs are endogenous. Oehmke and Zawadowski (2013) study the effects CDS have on bond market pricing when investors not only have heterogeneous beliefs, but also heterogeneous trading frequencies. The authors do not consider investment in production or default. Bolton and Oehmke (2011) show how CDS lead to an empty creditor problem, whereby lenders' incentives to rollover loans are reduced, leading to increased bankruptcy and default risk. Our model has implications for endogenous default without the need for debt renegotiations.

The organization of the paper is as follows: In Section 2, we first describe a very general model of firms, debt contracts and investors. We solve the baseline economy with no CDS contracts, and describe the relevant comparative statics. In Section 3, we introduce covered CDS, first on one firm type then on both. In section 4 we introduce naked CDS. We discuss in detail the comparisons across all economies in Section 5. We close with concluding remarks.

2 Non-CDS Economy

2.1 Model

2.1.1 Time and Uncertainty

The model is a two-period general equilibrium model, with time $t = \{0, 1\}$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature $s = \{U, D\}$ at time 1. There is one durable consumption good in this economy that is also the numeraire good. We will refer to this throughout the paper as cash.

2.1.2 Agents

Firms

There are two firms $i = \{G, B\}$ in the economy where firm G is the "good" type and firm B is the "bad" type. Each firm is owned and operated by a manager with access to a production technology. The managers run the firms and consume from firm profits. The only difference between the two firms is the productive technology at the respective managers' disposal. The firms use the durable consumption good as an input at time 0 and produce more of this good for consumption at time 1. The respective firms' have standard decreasing returns to scale production functions given by $A^s I_i^{\alpha_i}$ for $s = \{U, D\}$, $i = \{G, B\}$ with the following properties: $I_i^{'} > 0$, $I_i^{''} < 0$. Firm G is more productive than firm B, *i.e.* $I^{\alpha_g} > I^{\alpha_b}$, $\forall\, 0 < I < 1$. In other words, using the same input, firm G can always produce more than firm B.

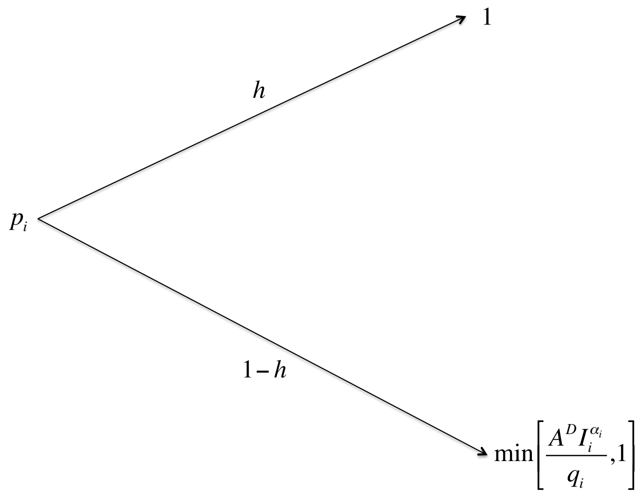
The technology shock A^s takes on binary values $s = \{U, D\}$ at time 1, with $A^U > A^D$. The technology parameter is identical for both firms. Consequently, the only type of uncertainty in our model is aggregate. For simplicity we normalize the technology shock A^U to 1. Both firms have identical expectations about the future, where each firm expects $s = U$ with probability γ and $s = D$ with probability $(1 - \gamma)$ at time 1. Lastly, these firms are competitive price takers in the market for the durable consumption good.

Investors

We consider a continuum of uniformly distributed risk neutral investors $h \in H \sim U(0, 1)$, who do not discount the future. Investors are characterized by linear utility for the single consumption good x_s for $s = \{U, D\}$ at time 1. Each investor $h \in H = (0, 1)$ is endowed with one unit of the consumption good, e^h, and assigns probability h to the up-state U and $(1 - h)$ to the down-state D. Thus, a higher h denotes more optimism. The von-Neumann-Morgenstern expected utility function for investor h is given by

$$U^h(x_U, x_D) = hx_U + (1 - h)x_D \tag{1}$$

Figure 1: Bond Payout



2.1.3 Firm Financing

Firms have no initial endowment and need to raise capital from the investors in order to produce. At time 0, firms issue debt contracts (bonds) using the firm's future output as collateral. The lender (investor) has the right to seize as much of the collateral up to the value of the promise, but no more. This enforcement mechanism ensures that the borrower (firm) will not simply default on its promise to repay at time 1.

Each bond, priced p_i at time 0, promises a face value of 1 upon maturity at time 1. The two firms issue bonds denoted by q_i at time 0, for $i = \{G, B\}$. In the up-state, each bond returns full face value, 1.[5] In the down-state each bond pays $\min\left[1, \frac{A^D I_i^{\alpha_i}}{q_i}\right]$, for $i = \{G, B\}$, which we call the bond's "recovery value." Firm borrowing costs r_i are given by the difference in what the firm owes on maturity and the amount of capital they receive at the time of issuance, $r_i = 1 - p_i$. Figure 1 depicts the bond payouts.

2.1.4 Firm Maximization Problem

Each firm chooses investment I_i, $i = \{G, B\}$ to maximize profits. Since firms have no initial endowment, all of their investment for production will have to be financed

[5] We assume the bonds repay in full at $s = U$, $\min\left[1, \frac{A^U I_i^{\alpha_i}}{q_i}\right] = 1$ to make the model interesting; otherwise, the firm does not invest in production.

by issuing debt.[6] Hence $I_i = p_i q_i$, for $i = \{G, B\}$. We denote firm profit, π_i^s for $s = \{U, D\}$, $i = \{G, B\}$.

Each firm for $i = \{G, B\}$ maximizes expected profits given its state contingent repayment decision:

$$\left\{ \begin{array}{c} \max_{q_i} E[\pi_i] = \left\{\gamma\left[A^U I_i^{\alpha_i} - q_i\right] + (1-\gamma)\left[A_i^D I_i^{\alpha_i} - \min\left[q_i, A^D I_i^{\alpha_i}\right]\right]\right\} \\ s.t.\ I_i = p_i q_i \end{array} \right\} \tag{2}$$

2.1.5 Investor Maximization Problem

We can now characterize each agents budget set. Given bond prices (p_g, p_b), each investor $h \in H$ chooses cash holdings $\{x_0^h\}$ and bond holdings $\{q_b^h, q_g^h\}$ at time 0 to maximize utility given by (1) subject to the budget set defined by:

$$\begin{aligned} B^h(p_g, p_b) \;=\; & \{ \left(x_0^h, q_g^h, q_b^h, x_U^h, x_D^h\right) \in R_+ \times R_+ \times R_+ \times R_+ \times R_+ : \\ & x_0^h + p_g q_g^h + p_b q_b^h = e^h, \\ & x_U^h = x_0^h + q_g^h + q_b^h, \\ & x_D^h = x_0^h + \min\left[1, \frac{A^D I_g^{\alpha_g}}{q_g}\right] q_g^h + \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right] q_b^h \} \end{aligned}$$

Each investor consumes from two potential sources in either state of nature: consumption based on risk-less cash holdings and consumption from their total bond portfolio. In the up-state, consumption from bond holdings is equal to the quantity of bonds an investor owns in his portfolio because each bond has face value of 1. In the down-state, firms may default on their debt, in which case investors take ownership of the firm and consume from the firms' recovery value on a per bond basis. Furthermore, note that we rule out short sales of bonds by assuming $q_i \in R_+$.

2.1.6 Equilibrium

An equilibrium in the *Non-CDS Economy* is a collection of bond prices, firm investment decisions, investor cash holdings, bond holdings and final consumption decisions

$$\begin{aligned} & (p_g, p_b), (p_{gc}, p_{bc}), (I_g, I_b), (x_0, q_g, q_b, q_{gc}, q_{bc}, x_U, x_D)_{h \in H} \\ \in\ & (R_+ \times R_+) \times (R_+ \times R_+) \times (R_+ \times R_+) \times (R_+ \times R_+ \times R_+ \times R \times R \times R_+ \times R_+) \end{aligned}$$

such that the following are satisfied:

[6] We abstract away from equity issuance and focus only on the impact credit derivatives have on debt financing.

1. $\int_0^1 x_0^h dh + \int_0^1 p_b q_b^h dh + \int_0^1 p_g q_g^h dh = \int_0^1 e^h dh$

2. $\sum_{i=G,B} \int_0^1 q_i^h min\left[1, \frac{A^s I_i^{\alpha_i}}{q_i^h}\right] dh + \sum_{i=G,B} \pi_i^s = \sum_{i=G,B} A^s I_i^{\alpha_i}$ for $s = \{U, D\}$

3. $I_i = \int_0^1 p_i q_i^h dh$ for $i = \{G, B\}$

4. $\pi_i (I_i) \geq \pi_i(\hat{I}_i), \forall \hat{I}_i \geq 0$ for $i = \{G, B\}$

5. $\left(x_0^h, q_g^h, q_b^h, x_U^h, x_D^h\right) \in B^h (p_g, p_b) \Rightarrow U^h (x) \leq U^h \left(x^h\right), \ \forall h$

Condition (1) says at time 0 all of the initial cash endowment is held by investors for consumption or used to purchase bonds from firms. Condition (2) says the goods market clears at time 1 such that all firm output is consumed either by firm managers via profits or goes to creditors via bond payments. The bond market clearing conditions correspond with (3). Condition (4) says that firms choose investment to maximize profits, and condition (5) states that investors choose portfolios that optimize their budget sets.

2.2 Results

We now characterize the equilibrium for the *Non-CDS Economy* for a given set of parameters. Let $A^D = 0.2$, $\gamma = 0.5$, $\alpha_g = 0.5$, and $\alpha_b = 0.75$. Our results are not particular to these parameter values. Parameters are chosen such that probability of default in the down-state is positive for both firms.[7]

In equilibrium, as a result of linear utilities and the continuity of utility in h, and the connectedness of the set of agents $H = (0, 1)$, at state $s = 0$ there will be *marginal buyers*, $h_1 > h_2$. Every agent $h > h_1$ will buy bonds issued by firm type B, every agent $h_2 < h < h_1$ will purchase bonds issued by firm type G, and every agent $h < h_2$ will remain in cash. This regime is shown in Figure 2.

Table 1 shows that firm G is more profitable than firm B. This is due to firm G being able to raise capital on better terms, *i.e* it's bond is priced higher. Moreover, firm G invests more and profits are increasing in production.

In the remainder of the section we explain how to solve for the equilibrium in the *Non-CDS Economy.* Readers not interested in the technical details can skip to the

[7]Firms could borrow at the risk-free interest rate if there was never a positive probability of default in some state of nature. In such a world, investors with rational expectations would not pay a positive price for a CDS contract and no CDS would be traded in equilibrium.

Figure 2: Non-CDS Economy

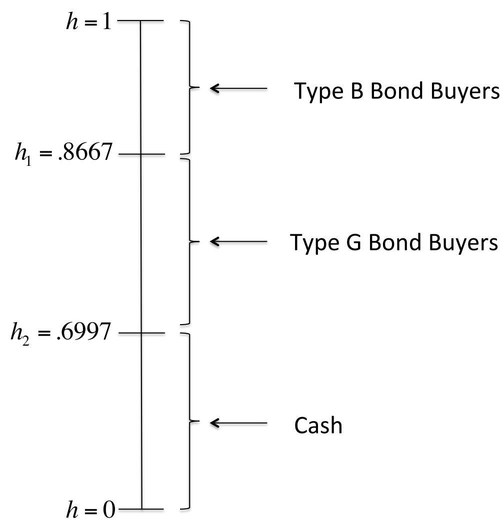


Table 1: Equilibrium Values: *Non-CDS Economy*

Non-CDS Economy		
	$i = G$	$i = B$
Price: p_i	.8198	.8041
Quantity: q_i	.2050	.1645
Investment: I_i	.1680	.1323
Output: Y_i^U	.4099	.2193
Exp.Profit: $E[\pi_i]$	.1025	.0274

next section. We first guess a particular regime (default at $s = D$) and check to see if it is indeed optimal.[8]

The eight endogenous variables are $(p_i, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$. The system

[8]Firms can choose investment such that they either default or fully repay in the downstate. We only present the equilibrium when firms default on their debt obligations in the main body of the text because CDS will not exist if firms fully honor their debt obligations in both states of nature. The results for other possible regimes are presented in Appendix A.

Table 2: Comparative Static Results: *Non-CDS Economy*

	p_i	I_i	q_i	$E[\pi_i]$
$\frac{\partial(\cdot)}{\partial A^D}$	$\frac{\partial p_i}{\partial A^D} > 0, i = G, B$	$\frac{\partial I_i}{\partial A^D} > 0, i = G, B$	$\frac{\partial q_i}{\partial A^D} > 0, i = G, B$	$\frac{\partial E[\pi_i]}{\partial A^D} > 0, i = G, B$
$\frac{\partial(\cdot)}{\partial \gamma}$	$\frac{\partial p_i}{\partial \gamma} = 0, i = G, B$	$\frac{\partial I_i}{\partial \gamma} = 0, i = G, B$	$\frac{\partial q_i}{\partial \gamma} = 0, i = G, B$	$\frac{\partial E[\pi_i]}{\partial \gamma} > 0, i = G, B$

of equations is:

$$\frac{h_1 + (1 - h_1)\min\left\{1, \left[\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right\}}{p_g} = \frac{h_1 + (1 - h_1)\min\left\{1, \left[\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right\}}{p_b} \tag{3}$$

$$\frac{h_2 + (1 - h_2)\min\left\{1, \left[\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right\}}{p_g} = 1 \tag{4}$$

$$A^U \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i} \text{ for } i = \{G, B\} \tag{5}$$

$$I_i = p_i q_i \text{ for } i = \{G, B\} \tag{6}$$

$$1 - h_1 = p_b q_b \tag{7}$$

$$h_1 - h_2 = p_g q_g \tag{8}$$

Equation (3) says that the marginal investor who is indifferent between purchasing type G and type B debt must believe that the expected returns of the two assets are equal. Similarly, equation (4) says the marginal investor who is indifferent between buying type G debt and holding cash must believe that the returns to the two assets are equal. Equation (5) is each firm's marginal investment decision that equates the marginal product of each additional unit of capital with its marginal cost. Next, equation (6) says that each firms' bond issuance q_i will be in accordance with the desired investment level given the market price for bonds. Equation (7) is the market clearing condition for type B firm debt, which says the supply of cash used to purchase type B bonds must equal the amount of capital the firm raises for production. Likewise, equation (8) states the supply of cash investors use to purchase type G bonds must equal the amount of capital firm G raises for production.

2.3 Comparative Statics

In this section we discuss a few of the model's comparative static results for different values of γ and A^D. The results of our model are in line with the classic theory of the firm as shown in Table 2.

Bond Prices

Both firms' bond prices are increasing in A^D for $p_i < 1$, $i = \{G, B\}$. This is because an increase in A^D raises the recovery value of the firm in the down-state. This raises the expected value of the bond, hence its price.

For $p_i < 1$, $i = \{G, B\}$, bond prices are constant in γ. Firms only care about the up-state because they default in the down-state. Thus, their expectations do not factor into their investment decision.

Investment

Investment levels are increasing for both firms as A^D increases when $p_i < 1$, $i = \{G, B\}$. Bond recovery values increase as A^D increases. This lowers borrowing costs and increases investment. Investment is constant in firm optimism, γ, for both firms when $p_i < 1$, $i = \{G, B\}$. Firms only care about the up-state since they expect to default at $s = D$. Thus, no matter how optimistic firms become they will not alter their investment level as γ increases.

Profits

Expected profits are monotonically increasing for both firms in A^D and γ. Profits are increasing in the value of down-state shock A^D because the recovery value of the firm in default increases. Investors are thus willing to pay a higher price for firm debt because it is less risky. *Expected* profits are increasing in firm optimism γ because the probability they assign to the up-state, where they make profits, is rising.

Marginal Buyers

The marginal buyer indifferent between type B and type G bonds is always more optimistic than the buyer indifferent between type G bonds and cash. This is intuitive since type B's marginal bond buyer must be willing to take on more down-side risk. Hence he must believe the down-state is less likely to occur than the marginal buyer who purchases type G bond. Marginal bond buyers do not change in γ for $p_i < 1$, $i = \{G, B\}$. As explained earlier, firms only care about investment in the up-state and do not change their investment position based on their expectations. Thus, investment does not change and neither do the marginal bond buyers.

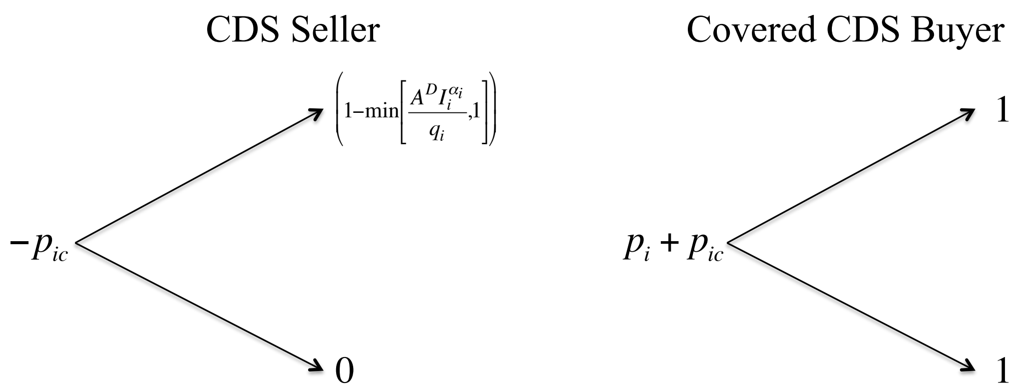
3 Covered CDS Economy

3.1 Covered CDS

In this section, we incorporate credit default swaps (CDS) to the baseline model. A CDS is a financial contract in which a CDS seller compensates the buyer for losses to the value of an underlying asset for a pre-specified credit event or default. The underlying assets in this economy are firm bonds. CDS contracts will compensate buyers the difference between a bond's face value at maturity and its market value at the time of the credit event. Thus, CDS allow investors to hedge against idiosyncratic default risk.[9]

[9]CDS do not allow investors to insure away aggregate risk.

Figure 3: Covered CDS Payout



We first consider covered CDS, where CDS buyers are required to also hold the underlying asset *i.e.* the firm's bond for whom the CDS was written. Figure 3 shows the payout to the CDS seller and buyer. Note that for the CDS seller, writing CDS is equivalent to holding an Arrow-Up security since it pays out only when $s = U$ and pays nothing when $s = D$.

We assume the CDS seller must post enough collateral to cover the payment in the worst case scenario to rule out any counter-party risk.[10] Let q_{ic}^h for $i = \{G, B\}$ be the number of CDS investor h can sell, and let p_{ic} for $i = \{G, B\}$ be the CDS price. Therefore the following relationship must hold:

$$1 + p_{ic}\, q_{ic} = q_{ic}^h \left(1 - \min\left[1, \frac{A^D I_i^{\alpha_i}}{q_i}\right]\right) \tag{9}$$

At time 0, given our assumption of risk neutral investors, a CDS seller will post his endowment and total revenue (received from selling CDS) as collateral. The collateral posted by the CDS seller must equal the total exposure of his CDS position. This represents the maximum CDS payout. Solving for the total number of CDS contracts gives:

$$q_{ic}^h = \frac{1}{1 - \min\left[1, \frac{A^D I_i^{\alpha_i}}{q_i}\right] - p_{ic}}. \tag{10}$$

[10]Geanakoplos (2010) shows that when an array of contracts with different margin requirements are present, the only one that will be traded is the Value-At-Risk equal to zero contract that we assume.

3.1.1 Investor Maximization Problem

Given bond and CDS prices $(p_g, p_b, p_{gc}, p_{bc})$, each investor h decides on their cash, bond and CDS holdings $\left\{x_0^h, q_b^h, q_g^h, q_{bc}^h, q_{gc}^h\right\}$, to maximize utility (1) subject to the following budget set:

$$
\begin{aligned}
B^h\left(p_g, p_{b,} p_{gc}, p_{bc}\right) \;=\; & \{ \left(x_0^h, q_g^h, q_b^h, q_{gc}^h, q_{bc}^h, x_U^h, x_D^h\right) \in R_+ \times R_+ \times R_+ \times R \times R \times R_+ \times R_+\} : \\
& x_0^h + p_g q_g^h + p_b q_b^h + p_{gc} q_{gc}^h + p_{bc} q_{bc}^h = e^h, \\
& x_U^h = x_0^h + q_g^h + q_b^h - p_{gc} q_{gc}^h - p_{bc} q_{bc}^h, \\
& x_D^h = x_0^h + \min\left[1, \frac{A^D I_g^{\alpha_g}}{q_g}\right] q_g^h + \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right] q_b^h + \\
& \left(1 - \min\left[1, \frac{A^D I_g^{\alpha_g}}{q_g}\right]\right) q_{gc}^h + \left(1 - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_g}\right]\right) q_{bc}^h, \\
& max\left\{0, q_{ic}^h\right\} \leq q_i^h \; for \; i = \{G, B\} \}
\end{aligned}
$$

The first three equations are analogous to the investor budget set in the *Non-CDS Economy*. The fourth equation states that since CDS buyers are required to hold the underlying asset, the maximum number of CDS contracts that can be purchased cannot be greater than the number of bonds owned. Notice that there is no sign restriction on q_{ic}^h. Selling CDS implies that $q_{ic}^h < 0$, while $q_{ic}^h > 0$ implies purchasing CDS. Short selling of bonds is still ruled out by the restriction $q_i \in R_+$ as in the *Non-CDS Economy*.

3.1.2 Equilibrium

An equilibrium in the *Covered-CDS Economy* is a collection of bond prices, CDS prices, firm investment decisions, investor cash holdings, bond holdings, CDS holdings and final consumption decisions:

$$
\begin{aligned}
& \left(p_g, p_b\right), \left(p_{gc}, p_{bc}\right), \left(I_g, I_b\right), \left(x_0, q_g, q_b, q_{gc}, q_{bc}, x_U, x_D\right)_{h \in H} \\
\in & \left(R_+ \times R_+\right) \times \left(R_+ \times R_+\right) \times \left(R_+ \times R_+\right) \times \left(R_+ \times R_+ \times R_+ \times R \times R \times R_+ \times R_+\right)
\end{aligned}
$$

such that the following are satisfied

1. $\int_0^1 x_0^h dh + \int_0^1 p_b q_b^h dh + \int_0^1 p_g q_g^h dh = \int_0^1 e^h dh$
2. $\sum_{i=G,B} \int_0^1 q_i^h min\left[1, \frac{A^s I_i^{\alpha_i}}{q_i^h}\right] dh + \sum_{i=G,B} \pi_i^s = \sum_{i=G,B} A^s I_i^{\alpha_i}$ for $s = \{U, D\}$
3. $\int_0^1 q_{ic}^h = 0$
4. $I_i = \int_0^1 p_i q_i^h dh$ for $i = \{G, B\}$
5. $\pi_i (I_i) \geq \pi_i(\hat{I}_i),\ \forall \hat{I}_i \geq 0$for $i = \{G, B\}$
6. $\left(x_0^h, q_g^h, q_b^h, q_{gc}^h, q_{bc}^h, x_U^h, x_D^h\right) \in B^h (p_g, p_b, p_{gc}, p_{bc}) \Longrightarrow U^h (x) \leq U^h \left(x^h\right),\ \forall h$

Condition (1) states that all of the initial endowment is held by investors or used to purchase bonds. Condition (2) says the goods markets clear such that total firm output is consumed by firm managers in the form of profits and by bond holders. Condition (3) says that the CDS market is in zero net supply, while (4) states that the bond markets clear. Condition (5) says firms choose investment to maximize profits. Lastly, (6) states that investors choose a portfolio that maximizes their utility given their budget set.

We make use of the following lemma to characterize equilibrium in the covered CDS economy.

Lemma 1 *If* $0 < \frac{A^D I_i^{\alpha_i}}{q_i} < 1$*, for* $i = \{G, B\}$ *then no bonds for which CDS is sold will be purchased without CDS.*

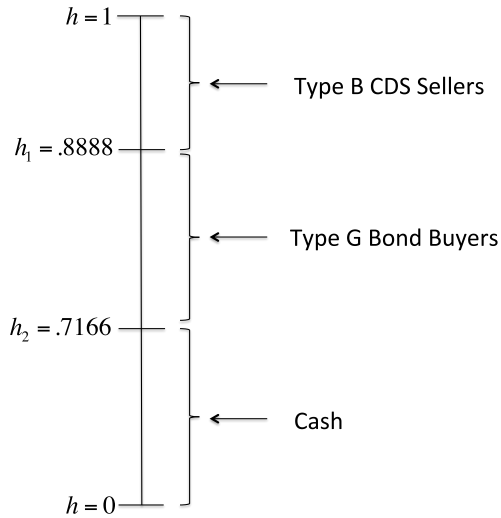
Proof. See appendix. ■

The intuition behind Lemma 1 is that any investor optimistic enough to buy a bond without a CDS will be better off selling CDS on that bond. Additionally, if the recovery value of the bond is zero, then CDS and bonds pay the same amount in both states, thus making CDS redundant assets. Finally, if the recovery value of the bond is 1, then bonds are risk free and no CDS will trade in equilibrium.

3.2 Type B Covered CDS Economy

We first introduce CDS only on type B debt. Introducing CDS in this sequential manner allows us to examine how CDS affect firm financing for whom CDS contracts are written, but also any additional benefits or cost CDS impose on other firms

Figure 4: Type B Covered CDS Economy



seeking financing. We now characterize the equilibrium for this economy for the following parameter values $A^D = 0.2$, $\gamma = 0.5$, $\alpha_g = 0.5$, and $\alpha_b = 0.75$.[11]

As before, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDS on type B debt, every agent $h_2 < h < h_1$ will purchase type G debt, and every agent $h < h_2$ is indifferent to holding covered CDS and cash. They hold a portfolio of covered positions on type B and cash.[12] This regime is shown in Figure 4.

Table 3: Equilibrium Values: *Type B Covered CDS Economy*

Type B Covered CDS Economy		
	$i = G$	$i = B$
Price: p_i	.8300	.8463
Quantity: q_i	.2075	.1918
Investment: I_i	.1722	.1623
Output: Y_i^U	.4150	.2557
Exp.Profit: $E[\pi_i]$	.1037	.0320

[11] The characterization of the equilibrium when either one or both firms does not default can be found in Appendix A.2.2.

[12] In equilibrium, every agent $h < h_2$ will be indifferent between cash and a covered position and each agent will hold a portfolio consisting of 26.73% in covered positions and the rest in cash. To compute this portfolio allocation we simply divide the number of type B bonds investors hold by their total cash endowment: $\frac{q_b}{h_2}$

Table 3 shows equilibrium values of the endogenous variables in the *Type B Covered CDS Economy.* Introducing covered CDS produces a number of interesting results. Compared to the *Non-CDS Economy*, covered CDS lower borrowing costs for the firm on which CDS are traded (firm B) due to separation of credit risk from firm financing needs. More interestingly, firm G is also able to borrow on better terms despite there being no fundamental changes to the firm. Thus, we find a positive spillover to firm G when investors are able to sell CDS on firm B debt. As both firms are able to borrow on better terms, they raise more capital and thus are more profitable compared to the *Non-CDS Economy.* A detailed analysis of these results is presented in the Section 5.

In the remainder of this section we explain how to solve for the equilibrium in the *Type B Covered CDS Economy.* We first guess a particular regime (default at $s = D$) and check to see if it is optimal.

The nine endogenous variables in this economy are $(p_i, p_{bc}, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$. The system of equations is:

$$\frac{h_1\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}} = \frac{h_1+(1-h_1)\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]}{p_g} \tag{11}$$

$$\frac{h_2+(1-h_2)\left\{\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right\}}{p_g} = 1 \tag{12}$$

$$A^U \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i} \text{ for } i = \{G, B\} \tag{13}$$

$$I_i = p_i q_i \text{ for } i = \{G, B\} \tag{14}$$

$$\frac{(1-h_1)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}} = q_b \tag{15}$$

$$h_1 - h_2 = p_g q_g \tag{16}$$

$$p_b + p_{bc} = 1 \tag{17}$$

Equation (11) states that marginal investor h_1 is indifferent between selling CDS on type B debt and purchasing type G bonds. Similarly, equation (12) says that marginal investor h_2 is indifferent between purchasing type G bonds and holding a risk free portfolio consisting of cash and covered CDS positions. Equation (13) is each firm's investment level that maximizes profits. Equation (14)says that each firms' bond issuance q_i will be in accordance with the desired investment level given the market price for bonds. Next, equation (15) is the CDS market clearing condition for CDS contracts on type B. In any covered CDS equilibrium, the total number of CDS contracts investors sell equals the number of bonds purchased, due to Lemma 1. Equation (16) is the bond market clearing condition for type G debt. Lastly, equation (17) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly.

Note that there are four markets that must clear; the bond market for each firm type, the CDS for type B debt, and the market for cash. By Walras' Law, three market clearing conditions is sufficient to ensure all four markets clear. However, the restriction that in any covered equilibrium all CDS must also be purchased with the underlying bond implies that the bond market and CDS market for type B debt will be linked. In fact, one cannot clear without the other, and a single market clearing condition for the two markets is all that is needed. That is why there are only two market clearing conditions, (15) and (16).

3.3 Covered Two CDS Economy

In this section, we allow CDS to be traded on both firm's debt. The characterization of equilibrium in this economy is for the same set of parameters used to solve all previous economies.[13]

As before, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDS on type B debt, every agent $h_2 < h < h_1$ will sell CDS on type G debt, and every agent $h < h_2$ is indifferent between holding either covered CDS and cash. They hold a portfolio of covered positions on firm type B, type G, and cash.[14] This regime is shown in Figure 5.

Table 4 shows the equilibrium values of the endogenous variables in the *Covered Two CDS Economy*.

Table 4: Equilibrium Values: *Two Covered CDS Economy*

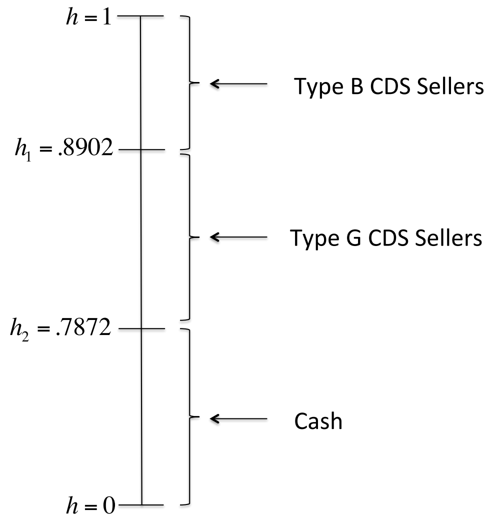
Two Covered-CDS Economy		
	$i = G$	$i = B$
Price: p_i	.8723	.8439
Quantity: q_i	.2181	.1902
Investment: I_i	.1902	.1605
Output: Y_i	.4362	.2536
Profit: π_i	.1090	.0317

Recall that the initial introduction of CDS lowered borrowing costs for both firm types leading to increased investment levels. In this economy, competition in the CDS market results in different bond pricing and investment decisions. Additional

[13]The characterization of the equilibrium for the *No Risk Regime* and *Partial Risk Regime* can be found in Appendix A.2.2.

[14]In equilibrium, every agent $h < h_2$ will be indifferent between cash and a covered position and each agent will hold a portfolio consisting of 51.87% in covered positions and the rest in cash. To compute this portfolio allocation we simply divide the number of type B and G bonds investors hold by their total cash endowment: $\frac{q_b+q_c}{h_2}$.

Figure 5: Two Covered CDS Economy



CDS creation can actually *increase* borrowing costs and *decrease* investment for firms for whom CDS previously traded, even when CDS are restricted to covered positions.

Introducing covered CDS on type G (the second firm) lowers its borrowing costs relative to the *Non-CDS Economy* and *Type B Covered CDS Economy* due to the separation of credit risk from firm financing. However, type B now borrows on worse terms compared to the *Type B Covered CDS Economy* despite there being no fundamental changes to the firm itself. Thus, we find a negative spillover to firm B as a result of firm G also having CDS traded on its debt. This leads to higher investment for type G and lower investment for type B. Not surprisingly, profits rise for G and fall for B when compared to the *Type B Covered CDS Economy.* A detailed analysis of these results is presented in Section 5.

In the remainder of this section we explain how to solve for the equilibrium in the *Covered Two CDS Economy.* We first guess a particular regime (default at $s = D$) and check to see if it is optimal.

The ten endogenous variables in this economy are $(p_i, p_{ic}, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$.

The system of equations is:

$$\frac{h_1\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}} = \frac{h_1\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]-p_{gc}} \tag{18}$$

$$\frac{h_2\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{p_g-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]} = 1 \tag{19}$$

$$A^U \alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i} \text{ for } i = \{G, B\} \tag{20}$$

$$I_i = p_i q_i \text{ for } i = \{G, B\} \tag{21}$$

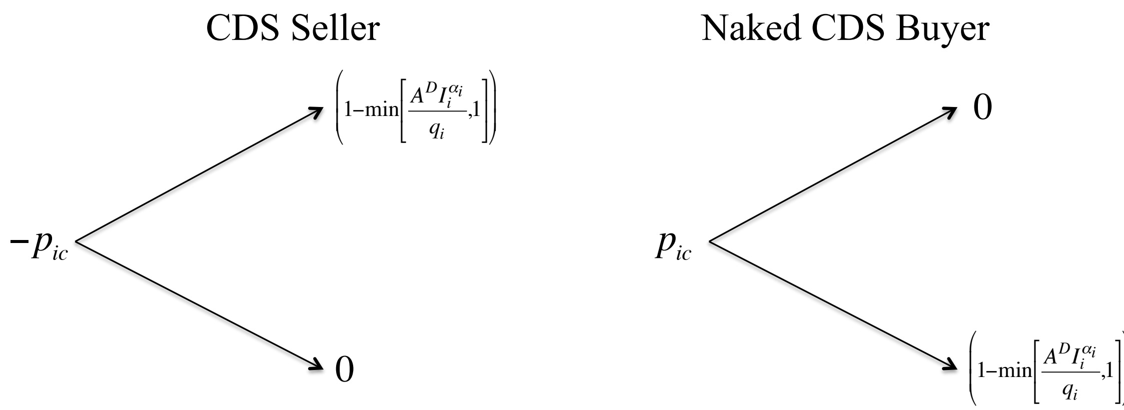
$$\frac{(h_1 - h_2)}{1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]-p_{gc}} = q_g \tag{22}$$

$$\frac{(1 - h_1)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}} = q_b \tag{23}$$

$$p_i + p_{ic} = 1 \text{ for } i = \{G, B\} \tag{24}$$

Equation (18) says that marginal investor h_1 will be indifferent to writing a CDS using either firm type as the underlying reference entity. In equilibrium, CDS prices are purely a function of fundamentals and not investor expectations. Moreover, the equilibrium is unchanged if we re-rank investors by having the most optimistic set selling CDS on type G debt instead. The relative mass of investors selling different CDS types remains unchanged when investors are re-ranked. This confirms that optimistic investors are indifferent between selling either CDS type because, in equilibrium, the returns are identical. Equation (19) states that marginal investor h_2 will be indifferent between selling a CDS on type G debt and holding cash. As in previous economies, equation (20) is each firm's profit maximizing investment level, and equation (21) shows the bond market clearing conditions. Equations (22) and (23) are CDS market clearing for type G and type B debt, respectively. Recall that in covered CDS economies CDS can only be purchased with the underlying asset. Thus, the respective bond and CDS markets jointly clear. By Walras' Law the market for cash will automatically clear. Lastly, (24) is a non-arbitrage pricing implication of assuming the return to holding either covered CDS position is equivalent to holding cash and will be priced accordingly.

Figure 6: Naked CDS Payout



4 Naked CDS Economy

4.1 Naked CDS

In this section, we extend the model by allowing investors to hold *naked* CDS positions: investors do not need to hold the underlying asset to purchase a CDS.

A naked CDS buyer expects to receive the difference between the face value of the bond and its value at the time of default. The naked CDS payout structure is given in Figure 6. Furthermore, notice that buying a naked CDS is equivalent to buying the Arrow-Down security since it pays out only when $s = D$ and pays nothing when $s = U$.

At time 0 an investor can purchase a naked CDS by paying p_{ic}. The buyer believes with probability h that the up-state will occur at time 1, the firm will not default, and the CDS will not payout. The buyer believes with probability $(1 - h)$ that the downstate will occur at time 1. In this case he expects to receive the difference between the face value of the bond and its recovery value at time 1, $\left(1 - \min\left[1, \frac{A^D I_i^{\alpha_i}}{q_i}\right]\right)$.

We continue to assume that CDS sellers post enough collateral to cover payments in the worst-case scenario. Therefore the maximum CDS payout carries over from previous economies and is given by Equation (10). Moreover, the implications of Lemma 1 still hold.

4.1.1 Investor Maximization Problem

Given bond and CDS prices $(p_g, p_b, p_{gc}, p_{bc})$, each investor chooses cash, bond and CDS holdings $\left\{x_0^h, q_b^h, q_g^h, q_{bc}^h, q_{gc}^h\right\}$ to maximize utility (1) subject to the budget set:

$$
\begin{aligned}
B^h\left(p_g, p_{gc,} p_b, p_{bc,}\right) \quad = \quad & \{\left(x_0^h, q_g^h, q_{gc}^h, q_b^h, q_{bc}^h, x_U^h, x_D^h\right) \in \\
& R_+ \times R_+ \times R \times R_+ \times R \times R_+ \times R_+\} : \\
& x_0^h + p_g q_g^h + p_{gc} q_{gc}^h + p_b q_b^h + p_{bc} q_{bc}^h = e^h, \\
& x_U^h = x_0^h + q_g^h + q_b^h - p_{gc} q_{gc}^h - p_{bc} q_{bc}^h, \\
& x_D^h = x_0^h + \min\left[1, \frac{A^D I_g^{\alpha_g}}{q_g}\right] q_g^h + \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right] q_b^h + \\
& \left(1 - \min\left[1, \frac{A^D I_g^{\alpha_g}}{q_g}\right]\right) q_{gc}^h + \left(1 - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]\right) q_{bc}^h \}.
\end{aligned}
$$

The investor's budget set is exactly the same as the one described in the *Covered Two CDS Economy* except that investors can now buy CDS without holding the underlying asset. Hence there is no restriction that ties the maximum number of CDS contracts bought to the number of bonds held.

Equilibrium Existence and Retail Investor

Fostel and Geanakoplos (2013) show that there are robust parameter regions in which equilibrium fails to exist when naked CDS are introduced. The reason is that investors will prefer to buy or sell CDS, but no investor strictly prefers to buy bonds. CDS are derivative instruments. Hence, they cannot exist if there is no underlying bond. However, as shown in the *Non-CDS Economy*, as soon as CDS cease to exist, investors choose to buy bonds. In other words, there are parameter regions for which there is no *fixed-point*.

To circumvent this problem, we assume there exists an investor, M, outside the continuum who is required to hold no risk on his balance sheet. We assume that this investor has a large enough endowment to satisfy all potential firm funding needs and associated cost of insurance in equilibrium. Let the maximum amount of capital both firms can raise be given by $\bar{M}$. His endowment must therefore be such that $e^M \geq \bar{M}$.[15] Furthermore, we assume this investor simply takes utility in holding covered bonds of either type since both covered positions have the same payout in both states. Let his utility be given by $U^M(c_i) = \sum_i c_i \epsilon$, for $i = \{G, B\}$, $\epsilon > 0$, where c_i is a covered CDS package made up of a bond and CDS: $c_i = q_i + q_{ic}$ for $i = \{G, B\}$.[16] Since he holds no risk, a CDS contract will accompany every bond

[15] Note that since the investor will have no risk on his balance sheet, he needs also to be able to purchase covered CDS in addition to purchasing bonds. $\bar{M}$ will therefore be sufficient to purchase the requisite amount of credit protection as well.

[16] We abstract away from expected utility maximization since the investor's portfolio pays the same in both states of nature.

purchased. Furthermore, let $U^{M'}(c_i) > 0$ so that the investor always prefers to invest when the opportunity exists.

We can now characterize the *retail investor's* budget set:

$$\begin{aligned} B^M(p_g, p_{gc}, p_b, p_{bc}) &= \left\{ \left(x_0^M, q_g^M, q_{gc}^M, q_b^M, q_{bc}^M, x_U^M, x_D^M \right) \in R_+ \times R_+ \times R_+ \times R_+ \times R_+ \times R_+ \times R_+ \right\} : \\ & x_0^M + (p_b + p_{bc})\, c_b^M + (p_g + p_{gc})\, c_g^M = e^M, \\ & x_U^M = x_0^M + c_b^M + c_g^M, \\ & x_D^M = x_0^M + c_b^M + c_g^M. \end{aligned}$$

The investor uses his endowment to either purchase bonds with covered CDS or for consumption. The retail investor consumes the same amount in either state since covered CDS positions have identical payouts in both states.

4.1.2 Equilibrium

An equilibrium in the *Naked-CDS Economy* is a collection of bond prices, CDS prices, firm investment decisions, and investor consumption decisions

$$\begin{aligned} & (p_g, p_b), (p_{gc}, p_{bc}), (I_g, I_b), (x_0, q_g, q_{gc}, q_b, q_{bc}, x_U, x_D)_{h\in H} \\ & \in (R_+ \times R_+) \times (R_+ \times R_+) \times (R_+ \times R_+) \times (R_+ \times R_+ \times R \times R_+ \times R \times R_+ \times R_+) \end{aligned}$$

such that the following are satisfied:

1. $\int_0^1 x_0^h dh + x_0^M + \sum_{i=G,B} p_i q_i^M + \sum_{i=G,B} p_{ic} q_{ic}^M = \int_0^1 e^h dh + e^M$
2. $\sum_{i=G,B} q_i^M + \sum_{i=G,B} \pi_i^s = \sum_{i=G,B} A^s I_i^{\alpha_i}$ for $s = \{U, D\}$
3. $\int_0^1 q_{ic}^h + q_i^M = 0$
4. $I_i = p_i q_i^M$ for $i = \{G, B\}$
5. $\pi_i(I_i) \geq \pi_i(\hat{I}_i)$, $\forall \hat{I}_i \geq 0$, for $i = \{G, B\}$
6. $\left(x_0^h, q_g^h, q_{gc}^h, q_b^h, q_{bc}^h, x_U^h, x_D^h\right) \in B^h(p_g, p_b, p_{gc}, p_{bc}) \Rightarrow U^h(x) \leq U^h\left(x^h\right)$, $\forall h$
7. $\left(c_i^M\right) \in B^M(p_i, p_{ic}) \Rightarrow U^M(c) \leq U^h\left(c^M\right)$, for $i = \{G, B\}$

Condition (1) states that all endowment including the *retail investor's* endowment goes to one of three uses: (a) held as collateral to issue CDS, (b) held by the *retail investor* for consumption, or (c) used by the *retail investor* to purchase bonds and covered CDS. Condition (2) says the goods market clears such that total firm output is consumed by firm managers in the form of profits and used to repay bond holders. Condition (3) says that the CDS market is in zero net supply because all

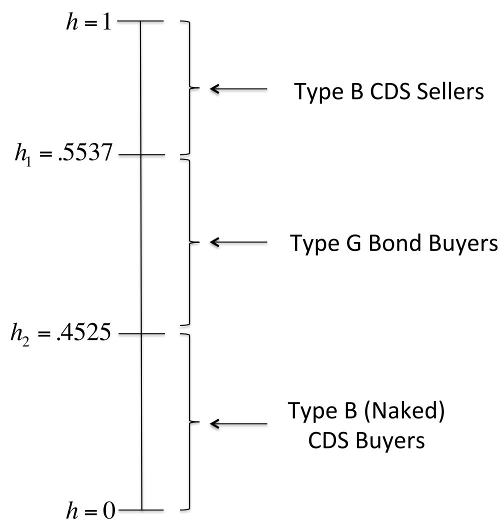
CDS purchased as naked investments and all CDS purchased by the *retail investor* as covered investments will be equal to all of the CDS issued. Condition (4) states that the bond markets clear. Condition (5) says firms choose investment to maximize expected profits. Condition (6) states that investors choose a portfolio that maximizes their utility given their budget sets, while condition (7) says that the *retail investor* holds a portfolio that maximizes his utility given his budget set.

4.2 Type B Naked CDS Economy

In this section investors are allowed to trade CDS only on type B debt. Introducing CDS on one firm's debt at a time allows us to capture spillovers in naked CDS economies.

The equilibrium for this economy is characterized for the same parameters used in previous economies.[17]

Figure 7: Type B Naked CDS Economy



As before, there will be *marginal buyers*, $h_1 > h_2$. In equilibrium, every agent $h > h_1$ will sell CDS on type B debt, every agent $h_2 < h < h_1$ will purchase bonds issued by type G, and every agent $h < h_2$ will buy naked CDS on type B debt. This regime is shown in Figure 7 and the equilibrium values corresponding to this economy are shown in Table 5.

[17]The equilibrium characterization for the *No Risk Regime* and *Partial Risk Regime* can be found in Appendix A.2.3.

Naked CDS raise type B's borrowing costs because pessimists are able to purchase Arrow-Down securities. Additionally, type G firm borrowing costs rise because the marginal buyer pricing type G bonds is significantly more pessimistic than in previous economies. Thus, both firms' financing costs are higher than in the *Non-CDS Economy.* Consequently, investment and profits are the lowest when compared to all the other economies. A detailed analysis of these results is presented in Section 5.

Table 5: Equilibrium Values: *Type B Naked CDS Economy*

Type B Naked CDS Economy		
	$i = G$	$i = B$
Price: p_i	.6363	.6195
Quantity: q_i	.1591	.0752
Investment: I_i	.1012	.0466
Output: Y_i^U	.3182	.1003
Exp.Profit: $E[\pi_i]$	.0795	.0125

Lemma 1 continues to hold implying that no bonds on which CDS are traded will be purchased without protection. Furthermore, we derive the following lemma to solve for equilibrium in the *Type B Naked CDS Economy.*

Lemma 2 *No investors hold cash or covered assets in equilibrium.*

Proof. See Appendix. ■

The intuition is that any investor pessimistic enough to remain in cash will be better off buying a naked CDS.

In the remainder of this section we explain how to solve for the equilibrium in the *Type B Naked CDS Economy.* We first guess a particular regime (default at $s = D$) and check to see if it is optimal.

The nine endogenous variables in this economy are $(p_i, p_{bc}, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$.

The system of equations that solve the set of variables is:

$$\frac{h_1\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{p_b-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]}=\frac{h_1+(1-h_1)\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]}{p_g} \tag{25}$$

$$\frac{(1-h_2)\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-p_b}=\frac{h_2+(1-h_2)\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]}{p_g} \tag{26}$$

$$A^U\alpha_i I_i^{\alpha_i-1}=\frac{1}{p_i} \text{ for } i=\{G,B\} \tag{27}$$

$$I_i=p_i q_i \text{ for } i=\{G,B\} \tag{28}$$

$$(h_1-h_2)=p_g q_g \tag{29}$$

$$\frac{(1-h_1)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}}=\left(q_b+\frac{h_2}{p_{bc}}\right) \tag{30}$$

$$p_b+p_{bc}=1 \tag{31}$$

Equation (25) says that marginal investor h_1 will be indifferent between issuing CDS on type B debt or buying type G bonds. Equation (26) says that marginal investor h_2 is indifferent between buying type G bonds and buying naked CDS on type B debt. Next, equations (27)-(28), as in previous sections, are firm profit maximizing investment decisions and funding conditions. Equation (29) is the type G bond market clearing condition. Equation (30) is the type B CDS market clearing condition, which states the supply of CDS investors issue will equal the demand for CDS from both the outside invetor and investors who purchase naked CDS. Lastly, (31) is a non-arbitrage pricing implication of assuming the return to holding either covered CDS position is equivalent to holding cash and will be priced accordingly.

4.3 Naked Two CDS Economy

In this section naked CDS are traded on both firms' debt. The equilibrium for this economy is characterized for the same set of parameters used to analyze previous economies.[18] Table 6 and Figure 8 show the equilibrium values for this economy.

There is 1 *marginal buyer* h_1 who is indifferent between buying and selling either CDS. In equilibrium, every agent $h > h_1$ will sell a CDS while every agent $h < h_1$ will buy a naked CDS. Notice that just as in Fostel and Geanakoplos (2012), this corresponds to the Arrow-Debreu economy. The ability to buy and sell naked CDS on all firms in the economy allows investors to convert their endowment into assets that deliver state-contingent consumption paths. Investors who sell CDS use their endowment as collateral to consume only in the up-state. Investors purchasing naked

[18]The characterization of the equilibrium for the *No Risk Regime* and *Partial Risk Regime* can be found in Appendix A.2.3.

Figure 8: Two Naked CDS Economy

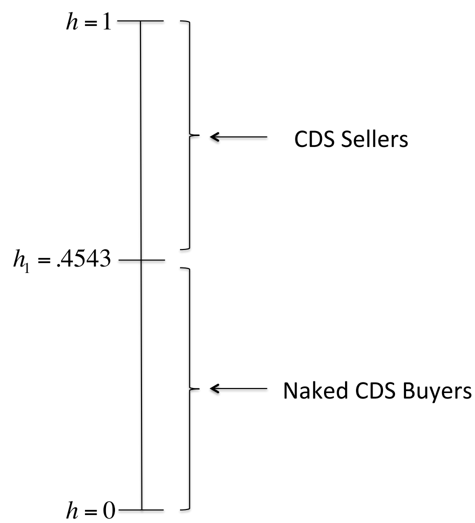


Table 6: Equilibrium Values: *Two Naked CDS Economy*

Two Naked-CDS Economy		
	$i = G$	$i = B$
Price: p_i	.7026	.6365
Quantity: q_i	.1757	.0816
Investment: I_i	.1234	.0519
Output: Y_i^U	.3513	.1088
Exp.Profit: $E[\pi_i]$	.0878	.0136

CDS use their endowment to buy assets that pay out only in the down-state. In other words, at time 0 all investors buy and sell state-continent consumption assets. Additionally, investors have no future endowments at time 1. However, one could replicate the Arrow-Debreu economy with future endowments so long as they were pledgeable at time 0.

The previous section showed that introducing naked CDS on one firm negatively impacts both firms' borrowing costs. However, introducing naked CDS on an additional firm allows both firms to borrow on better terms compared to the *Type B Naked CDS Economy.* The additional CDS on firm G debt reduces the demand for naked CDS issued on type B debt, lowering the price of type B CDS, which in turn lowers the firm's borrowing cost. However, financing costs for both firms are higher relative to the *Non-CDS Economy*, while investment and profits are lower. A detailed analysis these results is presented in the Section 5. The set of equations used to solve the *Two Naked CDS Economy* can be found in Appendix C.1.

5 Economy Comparisons

In this section we review the main findings of the paper by comparing bond prices and borrowing costs, investment demand, and default decisions across economies. Table 7 succinctly shows the effects of CDS on all variables for both firms.

Table 7: Economy Comparisons

	Non-CDS	Cov 1	Cov 2	Naked 1	Naked 2
Price: p_g	.8198	.8300	.8723	.6363	.7026
Quantity: q_g	.2050	.2075	.2181	.1591	.1757
Investment: I_g	.1680	.1722	.1902	.1012	.1234
Output: Y_g^U	.4099	.4150	.4362	.3182	.3513
Exp. Profit: $E[\pi_g]$	.1025	.1037	.1090	.0795	.0878
Price: p_b	.8041	.8463	.8439	.6195	.6365
Quantity: q_b	.1645	.1918	.1902	.0752	.0816
Investment: I_b	.1333	.1623	.1605	.0466	.0519
Output: Y_b^U	.2193	.2557	.2536	.1003	.1088
Exp. Profit: $E[\pi_b]$	.0274	.0320	.0317	.0125	.0136
Marginal Buyer: h_1	.8677	.8888	.8902	.5537	.4543
Marginal Buyer: h_2	.6997	.7166	.7872	.4525	.4543

5.1 Borrowing Costs

Borrowing costs are lower in covered CDS economies relative to economies without CDS and economies with naked CDS. Issuing CDS allows investors to separate firm funding needs from firm credit risk, and subsequently, allows the most optimistic investors to hold all of the credit risk. CDS sellers are only required to hold enough collateral to cover firm default in the down-state, so that a single investor can now insure multiple bonds. Thus, every dollar of collateral used to write CDS insures more than a dollar's worth of bonds. In other words, the ability to write CDS allows investors to pseudo-leverage their cash positions in a way that all firm funding needs can be insured by a smaller set of more optimistic investors. Moreover, competition among CDS issuers to write a limited number of CDS contracts drives down CDS prices. Hence, borrowing costs fall and bond prices rise.

Naked CDS raise borrowing costs. Naked CDS allow pessimistic investors to price and buy Arrow-Down securities. Pessimistic investors believe with relatively high probability that $s = D$ at time 1. Their preferred portfolio, therefore, is one that pays when $s = D$, which is what the Arrow-Down security achieves. The demand for CDS increases in naked CDS economies when investors are allowed to purchase CDS without also having to own the underlying bond. This drives up CDS prices and subsequent borrowing costs. In other words, pessimists implicitly price bonds through their demand for naked CDS. Recall that firm borrowing costs in the

Non-CDS Economy are determined by bond buyers, and are implicitly determined by CDS sellers in covered CDS economies. Pessimistic investors in these economies are limited to holding cash or purchasing bonds with protection; their beliefs do not affect bond prices. Subsequently, bond prices are higher than in naked CDS economies.

Fostel and Geanakoplos (2012) point out that derivative securities raise the collateral value of cash relative to the collateral value of the underlying asset. This reduces demand for the asset and lowers its prices. The notion that derivative securities raise the collateral value of cash is present in our model as well, but with a slight distinction. Bond prices can rise or fall in our model depending on whether or not naked CDS are permitted, yet either type of CDS always increases cash's collateral value. The reason is that CDS and bonds are asset *compliments* in covered CDS economies. Thus, CDS increase the demand for cash as collateral, which also increases the demand for the bond since it is only through issued bonds that covered CDS can be created. This raises bond prices. Conversely, CDS and bonds are asset *substitutes* in naked CDS economies. Thus, CDS raise the collateral value of cash and investors substitute away from debt markets and into the derivative security market. This lowers bond prices.

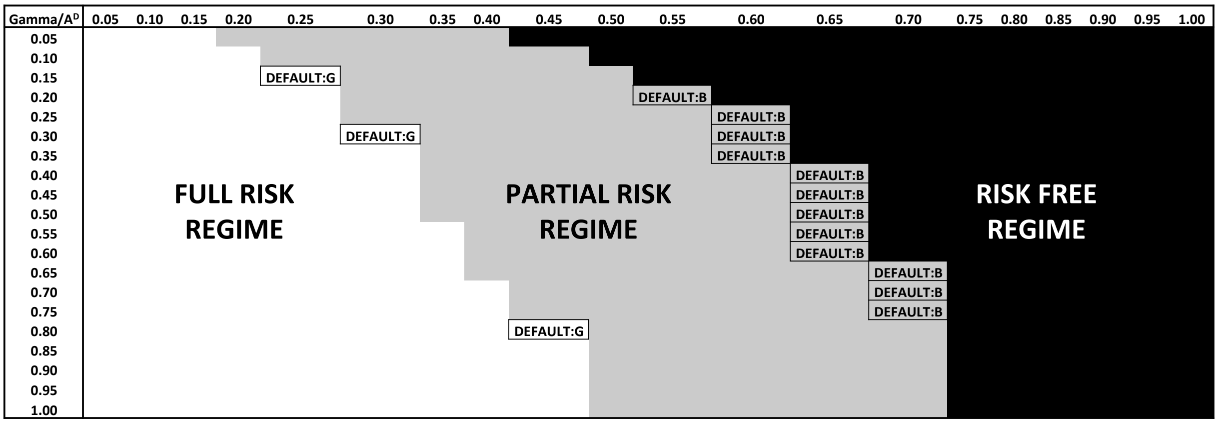
Lastly, notice that once any type of CDS is traded, subsequent CDS introduction *always* lowers borrowing costs for that specific firm. The reason borrowing costs are lower in covered CDS economies is because credit risk can be separated from funding needs. Hence, fewer and more optimistic investors insure all credit risk. This holds true even for the *Naked Two CDS Economy*. Almost all investors who want to hold naked CDS can do so once *any* naked CDS is allowed to trade. The creation of additional CDS does nothing in terms of creating new ways for investors to transfer state-contingent consumption paths, and the resulting impact on total market demand for naked CDS is minimal. The CDS effect of separating funding needs from credit risk has a greater impact on bond prices than the increase in demand for naked CDS. In other words, additional CDS will not raise firm borrowing cost as long as Arrow-Down securities are already present in the market.

5.2 Investment and Default

Both firms' investment demands are highest in covered CDS economies. The marginal decision to invest weights the expected increase in profits from higher investment against the marginal cost of issuing debt, *i.e.* the promise to repay each bond. Covered CDS lower borrowing costs, which allows firms to increase investment.

Conversely, each firms' investment falls in naked CDS economies. Investor demand for naked CDS drives up borrowing costs causing firms to scale back their demand for investment capital. The marginal borrowing cost increase outweighs the marginal gains in production, causing firms to reduce investment and production.

Figure 9: Increased Default Risk: *Non-CDS vs Covered CDS*

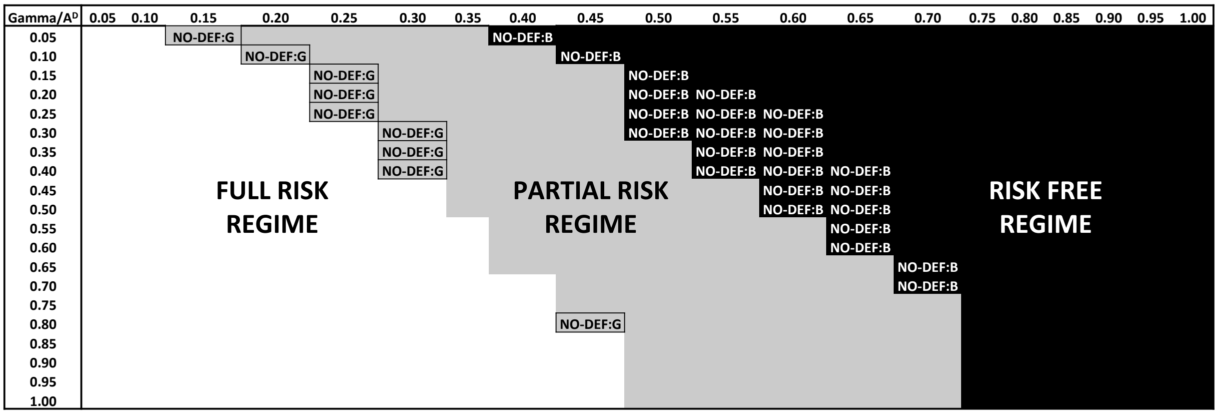


The joint effect of CDS on borrowing costs and investment has interesting implications regarding economy-wide firm default risk. Default risk is the likelihood a firm cannot repay its debt obligations when $s = D$. Firms choose investment to maximize expected profits. Because of limited liability, firms are more likely to default on their debt obligations when A^D is relatively low. This drives investors to charge positive interest rates on the capital they lend to firms, or a default premium. Covered CDS lower the default premium investors seek because they allow the most optimist investors to price and hold more credit risk. The corresponding investment levels firms choose given the lower default premiums are more likely to result in default than the investment levels chosen in an economy without covered CDS. The set of parameters $\left(\gamma,\, A^D\right)$ in covered CDS economies for which each firm defaults *expands* relative to the set of parameters in the *Non-CDS Economy*.

Figure 9 illustrates the effect covered CDS have on default relative to the *Non-CDS Economy*. The diagram shows the different possible default regimes. The area labeled "Full Risk Regime" is the set of parameter values for which both firms default when $s = D$. Similarly, "Partial Risk Regime" corresponds to the region where only type B defaults, and neither firm defaults in the "Risk Free Regime." The individual cells outlined and labeled "Default G" and "Default B" highlight the impact of introducing covered CDS. For example, relative to the *Non-CDS Economy*, all of the Default B cells indicate firm B defaulting on its debt obligations once covered CDS are introduced. Likewise, relative to the *Non-CDS Economy*, all of the Default G cells indicate firm G defaulting on its debt obligations once covered CDS are introduced.

CDS prices are higher in naked CDS economies. Higher CDS prices correspond

Figure 10: Decreased Default Risk: *Covered CDS vs Naked CDS*



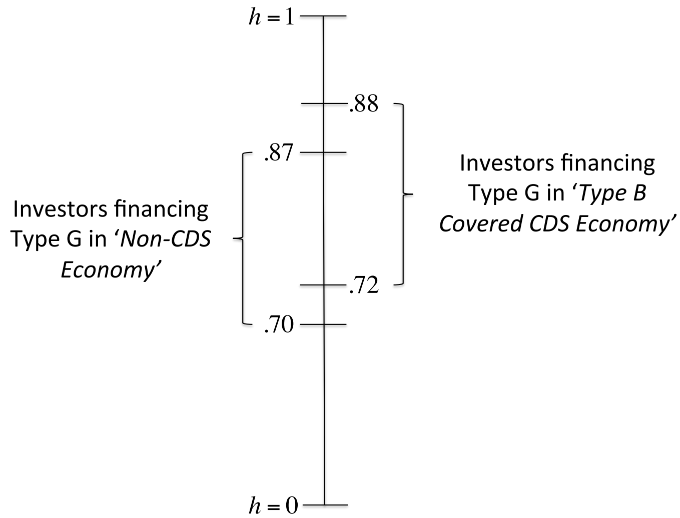
to higher firm borrowing costs. Firms respond to higher borrowing costs by choosing investment that is less likely to result in default. The set of parameters for which both firms default when naked CDS are introduced shrinks relative to the *Non-CDS Economy.*

Figure (10) shows the equilibrium comparing the *Covered Two CDS Economy* and the *Naked Two CDS Economy.* It is the analogue to Figure 9 in that it shows the impact of introducing naked CDS conditional on covered CDS already existing. We make this comparison to show the impact on default of banning naked CDS; presumably, covered CDS would still be allowed. The cells labeled "No Def G" indicate that firm type G switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. Similarly, the cells labeled "No Def B" indicate that firm type B switches from defaulting on debt obligations in covered CDS economies to fully repaying when naked CDS are introduced. Banning naked CDS in our model results in increased firm default risk as shown by the highlighted cells in Figure (10). These results highlight a tradeoff between borrowing costs and default risk not identified in the existing CDS literature.

5.3 Spillovers

Spillovers occur whenever the introduction of CDS on a specific firm type affects borrowing costs for the other unrelated firm type. Covered CDS lower borrowing cost even for the firm with no CDS being traded. Consider the *Type B Covered CDS Economy* from Section 3.2 where the borrowing costs for firm type G are lower due to CDS trading on type B. The reason is that the derivative asset allows investors to

Figure 11: Covered CDS Economy: *Type G Positive Spillover*



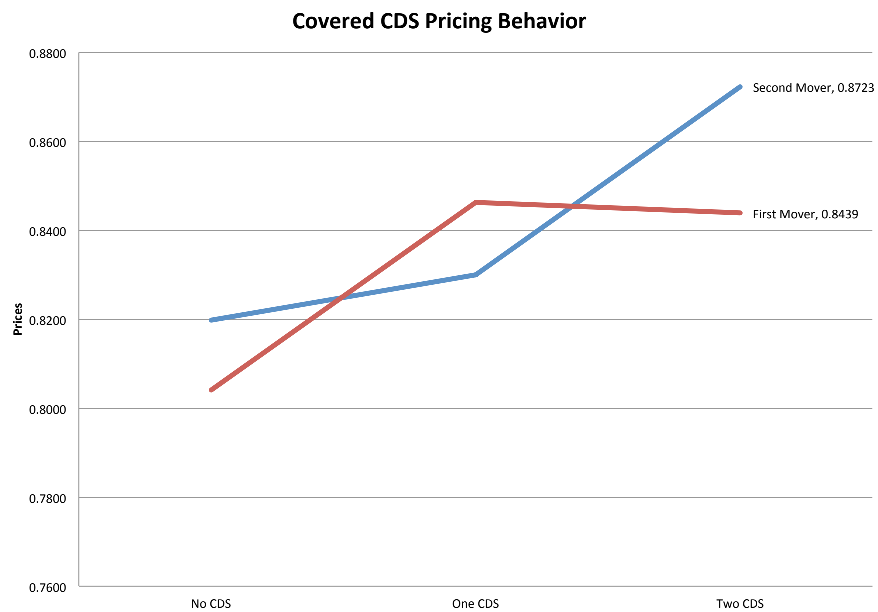
leverage their cash in a way that buying the underlying bond does not. This results in fewer, more optimistic investors holding all of the credit risk associated with the firm's debt issuance. More capital is then free to purchase other assets in the economy relative to an economy without covered CDS. The resulting marginal investor buying type G bonds is more optimistic than the counterpart in the *Non-CDS Economy*, raising type G bond prices. This is depicted in Figure 11.

Subsequent CDS introduction leads to negative borrowing cost spillovers for the firm with CDS already trading. For example, introducing covered CDS on Type G in the *Type B Covered CDS Economy* raises firm B's borrowing costs. The most intuitive way to understand the negative "additional" borrowing cost spillover is through the Arrow security market.

Selling a CDS is equivalent to buying the Arrow-Up security since it provides the issuer a state contingent consumption path. Investors acquire Arrow-Up securities by using their cash as collateral to sell CDS, which allows them to consume the proceeds only when the underlying bond does not default *i.e.* in the up state. In other words, the price an investor pays for an Arrow-Up security is the amount of his *own* collateral he must post to issue a CDS contract.

Investors realize that selling a CDS on either firm type's debt achieves the same state contingent consumption path. Therefore, investors are indifferent between issuing either CDS as long as they are priced proportionally. Broadening the CDS market to include both firms effectively increases the supply of Arrow-Up securities, which drives down their price. The fall in the price of the Arrow-Up security is equivalent to a fall in the amount of his *own* collateral an investor must post to issue a CDS. Insurance sellers raise CDS prices relative to the *Type B Covered CDS Economy*

Figure 12: Result 1



to ensure that all contracts remain fully collateralized. As a result, investors who hold covered positions pay less for the bond since the price of its insurance goes up. This leads to the following implication:

Result 1 *In any covered CDS economy, there is a first mover advantage in bond pricing that is invariant to the order in which CDS types are introduced.*

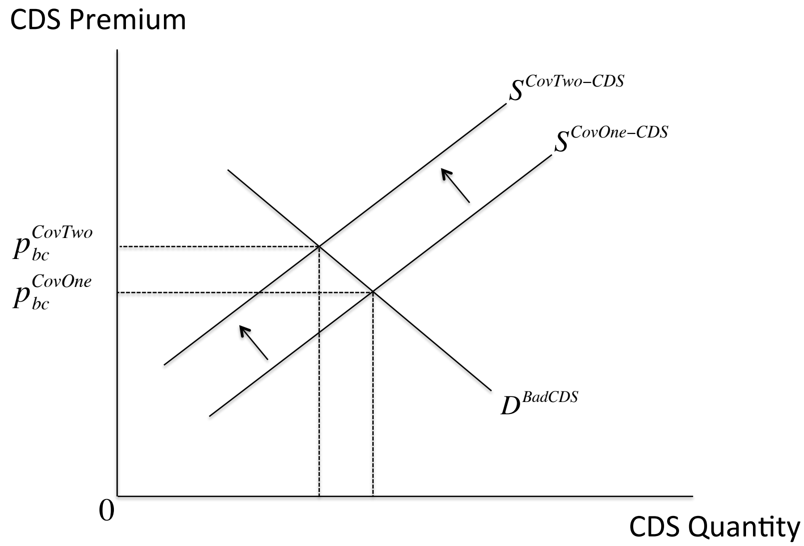
Let p^i, $i = \{G, B\}$ be each firms' bond price in the *Non-CDS Economy.* Let p^i_{jk}, $i = \{G, B\}$, $j = \{1, 2\}$, $k = \{c, n\}$ denote bond prices for each firm in either a one or two firm covered or naked CDS economy, where j denotes the number of firm types on which CDS trade, and k denotes the type of CDS trading, *i.e.* covered or naked. For example p^g_{1c} denotes firm G's bond price in a *Type G Covered CDS Economy* while p^b_{2n} denotes firm B's bond price in the *Naked Two CDS Economy.*

Result 1 says that in a *Non-CDS economy* where p^i is the price of the first firm for which CDS are introduced and $p^{\bar{i}}$ is not p^i, then the following pricing implications hold:

$$p^i < p^i_{2c} < p^i_{1c}, \; i = \{G, B\}$$
$$\text{and}$$
$$p^{\bar{i}} < p^{\bar{i}}_{1c} < p^{\bar{i}}_{2c}, \; i = \{G, B\}$$

Result 1 says that in an economy where multiple covered CDS may eventually be written, borrowing costs for the first mover will increase as subsequent CDS are

Figure 13: Covered CDS Economy: *Bad Firm Negative Spillover*

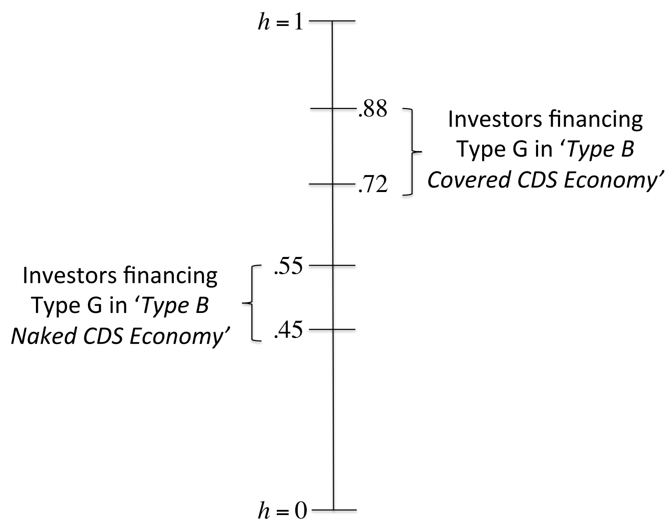


introduced. All of the borrowing costs benefits that accrue to firms from an active CDS market *i.e.* separating credit risk from funding needs, go initially to the first mover. The borrowing cost benefits then diminish as CDS are issued on the second mover's debt. This is illustrated in Figure 13.

Naked CDS raise borrowing costs even for the firm with no CDS trading. For example, firm G borrowing costs rise when naked CDS against firm B are introduced. Buying a naked CDS is equivalent to buying the Arrow-Down security, which pessimists demand, since it pays out only if the firm defaults in the down-state. The demand for the derivative assets pulls natural bond buyers out of the bond market and into the derivative security market. This leads to a fall in firm G's bond price. This is illustrated in Figure 14.

Unlike in covered CDS economies, introducing additional naked CDS leads to positive borrowing cost spillovers for the first mover. For example, introducing naked CDS on type G debt in the *Type B Naked CDS Economy* lowers type B's borrowing costs. Pessimists can buy Arrow-Down securities by purchasing naked CDS on either of the two firms' debt. Therefore, the two naked CDS will be priced in proportion to their respective pay-outs in the down state. Thus, naked CDS buyers no longer have to buy Arrow-Down securities using only type B naked CDS. This lowers the CDS price on type B debt, and leads to an increase in its bond price.

Figure 14: Naked CDS Economy: *Good Firm Negative Spillover*



Result 2 *In any naked CDS Economy, there is a first mover dis-advantage in bond pricing that is invariant to the order in which CDS are introduced. Subsequent CDS introduction raises all bond prices.*

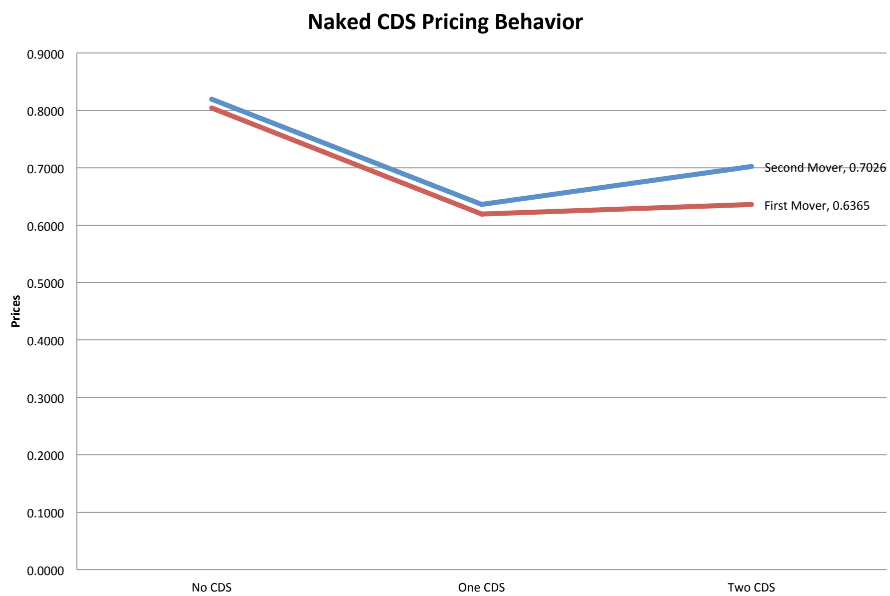
$$p^i_{1n} < p^i_{2n} < p^i,\ i = G, B.$$

Here, both bonds exhibit the same price movement. The first naked CDS that is introduced lowers bond prices for both firms because the CDS raises the collateral value of cash, which induces natural bond buyers to exit debt markets and enter derivative markets. Subsequent naked CDS raise both firms' bond prices. The additional CDS alleviate the need to use a particular firm's debt as the underlying reference entity on which to issue credit derivatives. This lowers the price of the CDS and leads to higher bond prices.

6 Conclusion

This paper highlights the fact that derivative securities can impact real outcomes even when economy fundamentals remain constant. Credit derivatives in particular can induce a tradeoff between firm borrowing costs and default probability. Access to cheaper capital can induce firms to invest in a manner that renders them unable or unwilling to repay creditors in the presence of negative productivity shocks. Alternatively, more expensive capital leads to debt financing that firms are more likely to repay for given productivity shocks.

Figure 15: Result 2



The introduction of credit derivatives alters the economy-wide value of cash as collateral. The subsequent impact on debt markets depends on whether credit derivatives and the underlying bonds are asset compliments or substitutes. Bond prices rise when the only way to purchase a credit derivative is to also own the underlying bond (covered economies) because the assets are compliments. Credit derivatives raise the value of cash as collateral and induce optimistic investors to enter derivative markets. Moreover, since in covered economies, CDS and bonds are compliments, optimistic investors implicitly increase the demand for bonds through their demand to issue CDS. This raises bond prices. Bond prices fall when investors can purchase CDS without having to own the underlying bond (naked economies). The CDS and bonds become asset substitutes. CDS raise the value of cash as collateral, which diverts funding away from bond markets and into derivative markets because investor demand to issue derivative assests is not directly tied to the underlying bond market. This decreases the demand for bonds and lowers their prices.

7 References

Ashcraft, Adam and João Santos, (2009) "Has the CDS Market Lowered the Cost of Corporate Debt?", *Journal of Monetary Economics,* 56, 514-523

Bernanke, Ben, (2008), "The Future of Mortgage Finance in the United States.", *Address at* UC Berkeley/UCLA Symposium: The Mortgage Meltdown, the Economy and Public Policy, Berkeley, California.

Bolton, Patrick and Martin Oehmke (2011), " Credit Default Swaps and the Empty Creditor Problem.", *Review of Financial Studies*, 24 (8), February 2011.

Che, Yeon-Koo and Rajiv Sethi, (2012) "Credit Derivatives and the Cost of Capital," *American Economic Journal: Microeconomics*, 2012.

Financial Times, (2011), "Bankers Fear Political Motives Will Kill Off CDS" David Oakley and Tracy Alloway. Capital Markets, October, 25, 2011

Fostel, Ana and John Geanakoplos, (2013), "Financial Innovation, Collateral and Investment." Revise and Resubmit, *American Economic Journal: Macroeconomics*.

Fostel, Ana and John Geanakoplos, (2012), "Tranching, CDS, and Asset Prices: How Financial Innovation Can Cause Bubbles and Crashes." *American Economic Journal: Macroeconomics*, Vol. 4, no 1, 190-225.

Geanakoplos, John, (2010), "The Leverage Cyle," In *NBER Macroeconomics Annual 2009.* Vol 24 edited by Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, 1-65. Chicago: University of Chicago Press.

Norden, Lars, Consuelo Silva Buston and Wolf Wagner (2014). "Financial Innovation and Bank Behavior: Evidence from Credit Markets." Forthcoming: *Joural of Economic Dynamics and Control*.

Sambalaibat, Batchimeg, (2012), "Credit Default Swaps and Sovereign Debt with Moral Hazard and Debt Renegotiation", *Working Paper No. 1093*, Society for Economic Dynamics, 2012.

A Appendix

A.1 Risk-Free Equilibrium

In section 2.2 we informed the reader to the fact that equilibrium investment behavior is endogenous to the set of parameter values chosen. The analysis in the main body of the paper focuses on an equilibrium in which both firms default, so that CDS will eventually be traded against both firms.

The system of equations that characterizes the equilibrium when neither defaults on its debt when $s = D$ includes eight endogenous variables $(p_i, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$ and is as follows:

$$1.\frac{h_j \times 1 + (1-h_j) \times 1}{p_i} = 1, \text{ for } i = \{G, B\}, j = \{1, 2\}$$

$$2.\alpha_i I_i^{\alpha_i - 1} = \frac{1}{p_i \left[\gamma A^U + (1-\gamma) A^D\right]}, \text{ for } i = \{G, B\}$$

$$3.I_i = p_i q_i \text{ for } i = \{G, B\}$$

$$4.1 - h_1 = p_b q_b$$

$$5.h_1 - h_2 = p_g q_g$$

Equation (1) says that bonds are essentially the same investment instrument as cash. This implies that the role investors play will simply be to fund the firms' investment needs and not set relative bond prices. Equation (2) is the firm optimizing decision that takes into account its profitability in the down-state when it chooses an investment level that ensures it will not default. Equation (3) says that each firms' bond issuance q_i for $i = \{G, B\}$ will be in accordance with the desired level given market price for bonds. Finally, Equations (4) & (5) correspond to the bond market clearing conditions for the respective firms.

A.2 Partial-Risk

A.2.1 Non CDS Equilibrium

This equilibrium is characterized by the fact that the good firm is always able to fully repay its debt while the bad firm defaults when $s = D$. This equilibrium will be characterized as a combination of the *Risk-Free Regime* presented in the preceding section and the *Full-Risk Regime* described in Section 2.2.

The system of equations that characterizes equilibrium in the *Non-CDS Economy* includes eight endogenous variables $(p_i, q_i, I_i, h_1, h_2)$ for $i = \{G, B\}$ and is as follows:

$$1.h_1 + (1-h_1) \min\left\{1, \frac{A^D I_b^{\alpha_b}}{q_b}\right\} = p_b$$

$$2.h_2 + (1-h_2) = p_g$$

$$3.\alpha_g I_g^{\alpha_g - 1} = \frac{1}{p_g \left[\gamma A^U + (1-\gamma) A^D\right]}$$

$$4.\alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b}$$

$$5.I_i = p_i q_i \text{ for } i = \{G, B\}$$

$$6.1 - h_1 = p_b q_b$$

$$7.h_1 - h_2 = p_g q_g$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) corresponds to type G's optimizing decision that takes into account firm profitability in the down-state while (4) corresponds to type B's optimizing decision by considering profits exclusively in the up-state. Equation (5) says that each firms' bond issuance q_i for $i = \{G, B\}$ will be in accordance with the desired level given market price for bonds. Finally, Equations (6) & (7) correspond to the bond market clearing conditions for the respective firms.

A.2.2 Covered CDS Equilibrium

CDS allow investors to sell insurance on firm default. Note that in the *Covered-CDS Economy,* CDS buyers are required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime.*

The system of equations that characterizes equilibrium in the *Covered-CDS Economy* includes ten endogenous variables $(p_i, p_{bc,} q_i, q_{bc,} I_i, h_1, h_2)$ for $i = \{G, B\}$ and is as follows:

$$1. \frac{h_1 \left(1 - \min 1, \frac{A^D I_b^{\alpha_b}}{q_b}\right)}{p_b - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]} = 1$$

$$2. h_2 + (1 - h_2) = p_g$$

$$3. \alpha_g I_g^{\alpha_g - 1} = \frac{1}{p_g\left[\gamma A^U + (1 - \gamma) A^D\right]}$$

$$4. \alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b}$$

$$5. I_i = p_i q_i \text{ for } i = \{G, B\}$$

$$6. 1 - h_1 = p_b q_b - \min\left[q_b, A^D I_b^{\alpha_b}\right]$$

$$7. h_1 - h_2 = p_g q_g$$

$$8. p_b + p_{bc} = 1$$

$$9. q_{bc} = \frac{1}{1 - p_{bc} - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]}$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i for $i = \{G, B\}$ will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

A.2.3 Naked CDS Equilibrium

In the *Naked-CDS Economy* CDS buyers are not required to hold the underlying asset. In equilibrium, CDS will only be traded on type B debt since type G does not default in the *Partial-Risk Regime*.

The system of equations that characterizes equilibrium in the *Naked-CDS Economy* includes ten endogenous variables $(p_i, p_{bc,} q_i, q_{bc,} I_i, h_1, h_2)$ for $i = \{G, B\}$ and is as follows.

$$1.\frac{h_1\left(1-\min 1, \frac{A^D I_b^{\alpha_b}}{q_b}\right)}{p_b - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]} = \frac{(1-h_1)\left(1-\min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-p_b}$$

$$2.h_2 + (1-h_2) = p_g$$

$$3.\alpha_g I_g^{\alpha_g - 1} = \frac{1}{p_g\left[\gamma A^U + (1-\gamma) A^D\right]}$$

$$4.\alpha_b I_b^{\alpha_b - 1} = \frac{1}{p_b}$$

$$5.I_i = p_i q_i \text{ for } i = \{G, B\}$$

$$6.(1-h_1) + p_{bc} q_b + h_2 = \left(q_b + \frac{h_2}{p_{bc}}\right)\left(1-\min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)$$

$$7.h_1 - h_2 = p_g q_g$$

$$8.p_b + p_{bc} = 1$$

$$9.q_{bc} = \frac{1}{1 - p_{bc} - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]}$$

Equations (1) & (2) are used to determine bond prices in equilibrium. Equations (3) & (4) corresponds to the firms' optimizing decisions. Equation (5) says that each firms' bond issuance q_i for $i = \{G, B\}$ will be in accordance with the desired level given market price for bonds. Equations (6) & (7) correspond to the CDS market and bond market clearing conditions respectively. Equation (8) is a non-arbitrage pricing implication of assuming the return to holding a covered CDS position is equivalent to holding cash and will be priced accordingly. Equation (9) solves for the number of CDS contracts an individual investor can sell given their initial endowment.

B Appendix

The investors maximization problem is to maximize (1) subject to the budget constraint $p^e q^e + p^a q^a = 1$, where p^e is the price of cash normalized to 1, q^e is the quantity of cash held, p^a is the price of the asset, and q^a is the quantity of the asset held.

Consumption when $s = U$ is given by $q^a + q^e$. Consumption when $s = D$ is given by $q^e + \min\left[1, \frac{A^D I^\alpha}{q^f}\right] q^a$, where q^f is the quantity of bonds the firm issues to raise capital. In the down state the investors consume the quantity of cash held plus the minimum between the value of the firm on a per bond basis, or one in the case of full repayment. In the case when the firm defaults the investor maximizes

$$\begin{aligned} U_D &= \max_{q^a} \left\{ h\left[q^a + q^e\right] + (1-h)\left[q^c + \frac{A^D I^\alpha}{q^f} q^a\right] \right\} \\ &= \max_{q^a} \left\{ hq^a + \left[1 - p^a q^a\right] + (1-h)\left[\frac{A^D I^\alpha}{q^f} q^a\right] \right\}. \end{aligned}$$

The first order condition for a maximum solves

$$U_D^{'} = \frac{h + (1-h)\left[\frac{A^D I^\alpha}{q^f}\right]}{p^a} = 1.$$

This relationship says that an investor will be indifferent between the expected value of the return on purchasing an asset and holding cash. In the case when the firm does not default the investor maximizes. The first order condition for a maximum solves

$$U_{ND}^{'} = p^a = 1.$$

Proof. *Lemma 1*: Suppose to the contrary that bonds are purchased unprotected. Then it must be the case that the utility of the agent who buys the unprotected bond is given by $u^b(h_1) = \frac{h_1 + (1-h_1)\left\{\min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]\right\}}{p_b} > 1$, which can be written as $\frac{h_1\left(1 - \min\left[1, \frac{A^D I_b^{\alpha_b}}{q_b}\right]\right) + \frac{A^D I_b^{\alpha_b}}{q_b}}{p_b} > 1$. Note that the utility of the CDS seller is given by $u^s(h_{cds}) = \frac{h_{cds}\left(1 - \min\left[1, \frac{A_d I_b^{\alpha_b}}{q_b}\right]\right)}{p_b - \min\left[1, \frac{A_d I_b^{\alpha_b}}{q_b}\right]}$. Now suppose that the investor h_1 who purchases the bad bond unprotected instead writes the CDS. His utility would be given by $u^s(h_1) = \frac{h_1\left(1 - \min\left[1, \frac{A_d I_b^{\alpha_b}}{q_b}\right]\right)}{p_b - \min\left[1, \frac{A_d I_b^{\alpha_b}}{q_b}\right]}$. To finish the proof it suffices to show that h_1 prefers to write CDS over buying unprotected bonds. Let $h_1\left(1 - \min\left[\frac{A^D I_b^{\alpha_b}}{q_b}, 1\right]\right) = X$, $p_b = Y$, and $\frac{A^D I_b^{\alpha_b}}{q_b} = \Lambda$. We can then rewrite the utilities in the following way: $u^b(h_1) = \frac{X+\Lambda}{Y}$ and $u^s(h_1) = \frac{X}{Y+\Lambda}$. If $u^s(h_1) > u^b(h_1)$ then,

$$\begin{aligned} &\implies \frac{X+\Lambda}{Y} < \frac{X}{Y-\Lambda} \\ &\implies (X+\Lambda)(Y-\Lambda) < XY. \\ &\implies -X\Lambda + \Lambda Y - \Lambda^2 < 0 \\ &\implies (X - Y + \Lambda) > 0. \end{aligned}$$

Substituting back in for X, Y, and Λ we see that $h_1\left(1-\min\left[\frac{A^D I_b^{\alpha_b}}{q_b},1\right]\right)-p_b+\frac{A^D I_b^{\alpha_b}}{q_b}>0$, which is the same as $\frac{h_1+(1-h_1)\left\{\min\left[1,\frac{A^D I_b^{\alpha}}{q_b}\right]\right\}}{p_b}>1$. Thus, any agent who would buy unprotected bonds would be better off selling CDS.

■

Proof. *Lemma 2:* Suppose to the contrary that h_1 holds cash. It must be the case then that the investor prefers holding cash to any other instrument int the economy. Thus we can say

$$h_1+(1-h_1)\min\left[A^D I_b^{\alpha_b}\right]<p_b, \tag{32}$$

and

$$1>\frac{(1-h_1)\left(1-\min\left[1,\frac{A_D I_b^{\alpha_b}}{q_b}\right]\right)}{1-p_b}. \tag{33}$$

Inserting (32) into the denominator of the *r.h.s* of (33) we do not perturb the inequality

$$1>\frac{(1-h_1)\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-\left[h_1+(1-h_1)\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right]}.$$

Rearranging and regrouping we get

$$(1-h_1)\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)>(1-h_1)\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)\ \otimes$$

a contradiction. ■

C Appendix

C.1 Naked Two Results

In this section we explain how to solve for the equilibrium in the *Naked CDS Two Firm Economy.* We first guess a particular regime (default at $s=D$) and check to see if it is optimal.

The thirteen endogenous variables in this economy are $(p_i, p_{ic}, q_i, q_{ic}, I_i, h_1, h_2, h_3)$ for $i=\{G,B\}$.

The system of equations is:

$$\frac{h_1\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{p_b-\min\left[1,\frac{A^D I_b^{\alpha}}{q_b}\right]}=\frac{h_1\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{p_g-\min\left[1,\frac{A^D I_g^{\alpha}}{q_g}\right]} \tag{34}$$

$$\frac{(1-h_3)\left(1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]\right)}{1-p_b}=\frac{(1-h_3)\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{1-p_g} \tag{35}$$

$$\frac{h_2\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{p_g-\min\left[1,\frac{A^D I_g^{\alpha}}{q_g}\right]}=\frac{(1-h_3)\left(1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]\right)}{1-p_g} \tag{36}$$

$$A^U\alpha_i I_i^{\alpha_i-1}=\frac{1}{p_i} \text{ for } i=\{G,B\} \tag{37}$$

$$I_i=p_i q_i \text{ for } i=\{G,B\} \tag{38}$$

$$\frac{(h_1-h_2)}{1-\min\left[1,\frac{A^D I_g^{\alpha_g}}{q_g}\right]-p_{gc}}=\left(q_g+\frac{h_2-h_3}{p_{gc}}\right) \tag{39}$$

$$\frac{(1-h_1)}{1-\min\left[1,\frac{A^D I_b^{\alpha_b}}{q_b}\right]-p_{bc}}=\left(q_b+\frac{h_3}{p_{bc}}\right) \tag{40}$$

$$p_i+p_{ic}=1 \tag{41}$$