Asymmetric Interest Rate Pass-Through from Monetary Policy:
The Role of Bank Regulation

Sebastian Roelands*

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Abstract

When the monetary policy rate increases, banks increase loan rates fairly quickly and by roughly the same amount. However, when the policy rate falls, bank loan rates adjust more slowly and not completely. I develop a model with which I show that this asymmetry in interest rate pass-through can be explained by the presence of capital and liquidity requirements imposed on banks by regulators. If the capital or liquidity constraints are binding, the shadow values of capital and liquidity are positive, which results in higher bank loan rates relative to the monetary policy rate. When the central bank lowers its policy rate, the critical values at which the constraints become binding are lowered, effectively tightening the regulatory requirements. This makes it more likely that banks become constrained, and hence reduce pass-through. Empirical evidence from United States bank holding companies over 2001Q1-2012Q1 corroborates the model predictions that (i) more banks are capital constrained during falling rate periods than rising rate periods; (ii) constrained banks adjust loan rates less relative to the federal funds rate, and (iii) constrained banks increase loan rates more after a drop in their capital ratios, relative to unconstrained banks.

JEL Classification: E43, E52, G21, G28

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*Ph.D. Candidate, Department of Economics, University of Notre Dame. Address: 434 Flanner Hall, Notre Dame, IN 46556. E-mail: roelands.1@nd.edu Website: http://nd.edu/~roelands/. I want to thank my advisors, Thomas Cosimano, Nelson Mark and Eric Sims, for their help and guidance, and all participants at the Macroeconomics Research Seminar and the Mathematical Finance Seminar at the University of Notre Dame, as well as the participants at the 2012 Midwest Macroeconomics Meetings and the 2012 International Finance and Banking Society Conference for helpful feedback. Any errors are my own.
1 Introduction

The asymmetric adjustment of bank loan rates relative to reference rates (such as the monetary policy rate) is well documented (Lim, 2001; Fuertes and Heffernan, 2006): banks tend to increase interest rates on loans at roughly the same speed as the reference rate, but lower their rates at a slower pace. In addition, pass-through often appears to be less complete during falling rate periods relative to rising rate periods. In this paper, I explain the asymmetry in interest rate pass-through by means of capital and liquidity requirements imposed on banks by regulators.

Previous work in this area has attempted to explain asymmetric pass-through by means of adjustment costs, borrowed from the non-bank literature (Hannan and Berger, 1991; Hoffman and Mizen, 2004). However, we do not observe a lack of adjustments of loan rates during falling rate periods, but rather very small adjustments, relative to reference rates. Moreover, Figure 1 shows that this asymmetry has worsened over time, but it would be hard to believe that adjustment costs have risen dramatically over the past twenty years. Since the banking sector is a relatively highly regulated industry, it is possible that some regulations create incentives for banks to limit pass-through under certain conditions. I develop a dynamic model of a single bank in the spirit of Chami and Cosimano (2010). This bank has market power in the market for bank loans and is subject to capital and liquidity constraints, and I show how these regulations affect loan rate decisions. I then test the predictions of the model using data on bank holding companies in the United States from 2001Q1 through 2012Q1.

In the model, the bank maximizes the present value of its profits by choosing the interest rate it charges on loans, as well as dividend payouts and how much (if any) new equity to issue. The shadow values of capital and liquidity are zero when the respective constraints are non-binding, but are positive when the bank has less capital or liquidity than required by the regulator. At a positive shadow value, the bank wants to obtain a higher capital-to-assets ratio or hold more liquid assets (depending on which constraint is binding). To achieve this, the bank will increase its loan rate, which is followed by a reduction in quantity of loans demanded. This can be used to either reduce total assets to increase the capital-to-assets ratio, or to free up funds to hold as liquid assets. In the case of a binding capital constraint, a higher loan rate will also generate higher returns, which can translate into higher retained earnings, and thus boost the amount of capital (which is made up of
The points at which the regulatory constraints become binding can be expressed in terms of exogenous shocks to loan demand, and are dependent on the return on liquid assets (which may be thought of as the monetary policy rate). An increase in the monetary policy rate will increase the range of loan demand shocks within which the bank is unconstrained, but a drop in the policy rate will cause this range to shrink, and effectively tighten the regulatory constraint. Therefore, ceteris paribus, initially unconstrained banks are more likely to become constrained after expansionary monetary policy than after a monetary tightening.

When banks are constrained, the pass-through from the policy rate is either incomplete or zero. In the case of a monetary tightening, banks are less likely to become constrained, but if they do, the models predicts zero pass-through into the bank’s loan rate. After the policy rate falls, on the other hand, only banks that were already constrained beforehand will leave their loan rates unchanged, whereas banks that were initially unconstrained but became constrained after the policy rate drop will adjust their loan rates in the direction of the policy rate, but not by the full amount.

Data on bank holding companies from 2001Q1 through 2012Q1 show that indeed more banks are capital constrained in falling rate environments than when rates are rising. The model predictions regarding loan rates that are empirically tested are the following: (i) capital constrained banks charge higher loan rates than unconstrained banks; (ii) the pass-through of the federal funds rate into bank loan rates is lower for capital constrained banks (in falling as well as rising rate periods); and (iii) the loan rates of capital constrained banks are more sensitive to fluctuations in the aggregate economy. Predictions (i) and (ii) are supported over the full sample period, while (iii) only appears to hold over the period 2007Q3-2012Q1.

Section 2 motivates this paper and discusses related literature. I introduce the model in Section 3 and analyze it in Section 4. Section 5 outlines the estimation strategy of the model predictions and shows the results. Section 6 concludes.
Figure 1: Interest Rates on Adjustable Rate Mortgages and Federal Funds, January 1992 - July 2012

2 Motivation

Why should we care about the pass-through of monetary policy into interest rates on bank loans? The credit channel of monetary policy states that expansionary monetary policy actions should increase the availability of bank credit through a drop in the cost of borrowing (Bernanke and Gertler, 1995). However, as became particularly clear during and shortly after the financial crisis of 2007-2009, bank credit did not increase, nor did interest rates on bank loans fall significantly, despite a lowering of the federal funds rate from 5.25% in July 2007 to almost 0% in December 2008. Aggregate lending by commercial banks in the United States fell 10.4% between October 2008 and February 2010. As of December 2011, aggregate lending was still down 5.5% from October 2008. Interest rates on bank loans remained at their pre-crisis levels until well into 2009. To illustrate this point, Figure shows the average interest rate charged on newly originated adjustable rate mortgages relative to the federal funds rate. The combination of a drop in lending and an increase in loan rates (relative to the cost of funds) suggests that the driving force behind the asymmetry is a leftward shift of the supply curve, rather than a fall in demand for bank loans. This phenomenon was not specific to the 2007-2009 financial crisis, and can be observed (albeit to a lesser degree) during every falling rate period over the past twenty years.

The asymmetric adjustment of bank loan rates (as well as deposit rates) is well documented (Hannan and Berger, 1991; Lim, 2001; Fuertes and Heffernan, 2006). Neumark and Sharpe (1992) found that the asymmetry in the market for bank deposits can in part be explained by concentration: banks with market power are slow to adjust deposit rates upward, but quick in lowering them. Explanations for loan rate asymmetry are very limited. Hoffman and Mizen (2004) explain loan rate asymmetry in a model where banks can only adjust their rates every other period, and face a fixed cost of rate adjustment. Because the opportunity cost of not adjusting rates is quadratic, the range of changes in the reference rate for which it is too costly to adjust the loan rate is larger for downward move-

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1 Source: Board of Governors of the Federal Reserve System, Statistical Releases, Schedule H.8.
2 This is not to say that monetary policy was not effective during this period. As Mishkin (2009) argues, to state that monetary policy was ineffective one should first establish a counterfactual without monetary easing. In this paper, I do not intend to partake in this discussion. My focus, instead, is on the observable transmission, rather than effectiveness, of expansionary versus contractionary monetary policy.
3 Borio and Fritz (1995), however, found evidence of asymmetric pass-through of the monetary policy rate into bank loan rates only in Germany and Japan in a sample of developed countries over the period 1984-1994.
ments in the reference rate than for upward changes. Banks should therefore not adjust their rates in falling rate periods unless the drop in the reference rate is very large. General equilibrium models that include an active banking sector either do not yet capture this asymmetry (e.g. Kumhof et al., 2010), or seek to capture it through a similar cost of rate adjustments (Gerali et al., 2010; Roger and Vlček, 2011). However, if adjustment costs fully explained the asymmetry, then given technological improvements and the high frequency of rate adjustments by banks, one would expect to observe a decline in asymmetry over recent decades. Instead, as can be seen in Figure 1 the asymmetry has worsened over the past twenty years.

Another potential explanation for the asymmetry, which would be particularly relevant when falling rate periods coincide with financial crises and/or recessions, is the pricing of risk. Although Collin-Dufresne et al. (2001) show that the time variation of credit spreads in the market for industrial bonds seems to be independent of most proxies for credit risk, Angbazo (1997) shows that in the United States bank loan market, banks include default risk and interest rate risk premia in their loan rates. If banks expect higher default rates during crisis periods and recessions, which coincide with expansionary monetary policy, they are likely to increase the default risk premium in their loan rates, effectively raising loan rates relative to their marginal cost of funding (which is influenced by the monetary policy rate). This may indeed be a plausible explanation of asymmetric pass-through in addition to the effects of regulatory requirements. In the loan rate regressions in Section 5.1 control for credit risk on bank loans (as measured by the fraction of nonperforming and past-due loans relative to total assets), and the empirical results still corroborate the importance of capital requirements in banks’ loan rate decisions as predicted by the model.

3 The Model

This section shows how the model is set up, and is divided into three subsections: 3.1 outlines the basics of the bank, while 3.2 discusses the capital and liquidity requirements imposed by the regulator. Finally, the bank’s optimization problem is shown in 3.3.
3.1 The Bank

The bank in this continuous time model has market power in the market for loans (Slovin and Sushka, 1983; Claessens and Laeven, 2004; Carbó et al., 2009). The bank determines loan rate \( r_L \), and the quantity of loans \( L \) then follows from the loan demand function,

\[
L^d_t = \gamma_0 - \gamma_1 r^L_t + \gamma_2 Y_t + \varepsilon^L_t,
\]

such that \( L^d = L \) at all times. This loan demand specification follows Chami and Cosimano (2010) and Zicchino (2006). The quantity of deposits, as well as the return on liquid assets are exogenously determined.

Loans \( L \) and liquid assets \( T \) constitute the bank’s total assets, while deposits \( D \) are the only liabilities of the bank. Net worth, or bank capital \( K \), is the sum of retained earnings and outstanding equity. The balance sheet identity is therefore

\[
L + T = D + K.
\]

The bank generates income from loans at the interest rate it charges on loans \( r^L \), and receives a return on liquid assets at rate \( r^T \). The bank incurs a marginal cost on lending \( c_L \) (e.g. a monitoring cost). It also pays interest rate \( r^D \) on deposits, which is related to \( r^T \). As such, the accounting profits of the bank are similar to Freixas and Rochet (1997, p. 45). In addition, the bank faces increasing, convex costs on issuing new equity \( G(q) \), where \( q \) is the amount of newly issued equity, and increasing, concave benefits from paying out dividends on its outstanding equity shares, \( H(r^E) \), where \( r^E \) is the value of dividends paid out per dollar of outstanding equity. Hence, profits at time \( t \)

4 Specifically, \( r^D = r^T - \frac{\nu_k}{1 - \kappa_1} \frac{(1 - \kappa_0) \kappa_1}{1 - \kappa_0} \), where \( \nu_k \) is the shadow value of capital, \( V_K \) is the marginal value of capital, and \( \kappa_0, \kappa_1 \) are the marginal risk-based capital requirements imposed by the regulator. As we will see later, \( \nu_k = 0 \) for banks that are not capital constrained. The deposit rate therefore equals the return on liquid assets for all banks, except for capital constrained banks. Hence, \( r^D \) is neither a state variable nor a control variable, but instead follows from the model.

5 Existing literature has provided many arguments why raising external equity is costly, such as the adverse-selection cost of equity (Myers and Majluf, 1984), the ex post verification cost of debt (Diamond, 1984; Gale and Hellwig, 1985), and the fact that underwriting costs are higher for equity than for debt (Aiyar et al., 2012). Alternatively, in the context of ‘too-big-to-fail’ banks, one may think of this cost of raising additional equity as the loss in value of a bailout subsidy. O’Hara and Shaw (1990) have shown that the value of the subsidy related to a bailout of creditors is inversely related to a bank’s solvency for the largest banks.
are defined as
\[
\pi(K_t, \xi_t) = \left( r^L_t - c^L_t \right) L_t + r^T_t T_t - r^D_t D_t - G(q_t) + H(r^E_t),
\]
By the balance sheet identity (2), liquid assets can be viewed as a balance sheet residual, \( T = D + K - L \). Thus, the profit function can be written as
\[
\pi(K_t, \xi_t) = \left( r^L_t - c^L_t \right) L_t + r^T_t (D_t + K_t - L_t) - r^D_t D_t - G(q_t) + H(r^E_t).
\] (3)

The bank chooses the interest rate on loans, \( r^L \), dividends, \( r^E \), and the amount of newly issued equity shares, \( q \). The vector of exogenous state variables (\( \xi \)) captures a measure of the aggregate economy (such as real gross domestic product (\( Y \)), the shock to the demand for loans (\( \epsilon^L \)), and the rate of return on liquid assets (\( r^T \)), as well as supply of deposits (\( D \)), such that \( \xi_t = [Y_t, \epsilon^L_t, r^T_t, D_t]' \). Its law of motion is given by
\[
d\xi_t = \mu(\xi) dt + \sigma(\xi) dW_t,
\] (4)
where \( \sigma(\cdot) \) is the standard deviation, and \( W \) is a standard Brownian motion. This law of motion is analogous to a random walk with drift \( \mu(\cdot) \).

The evolution of capital is given by the accumulation of retained earnings (profits less dividends on outstanding equity) plus the market value of newly issued equity (\( q \)), such that
\[
dK_t = [\pi_t - r^E E_t + q_t] dt,
\] (5)
where \( dx_t = x_{t+\Delta} - x_t \). Dividends and share issuance cannot be negative (i.e. no share repurchases), so \( r^E \geq 0 \) and \( q \geq 0 \).

\[6\] There are several benefits to incorporating dividends and new equity issuance separately, rather combining the two in a change-in-equity variable, which would be positive for issuance and negative for repurchases. With separate dividends, it is possible for banks to pay out more in dividends than the value of outstanding equity (i.e. more than $1 dividends per dollar of outstanding equity). Moreover, there is no danger of the nonnegativity constraint on outstanding equity to become binding after share repurchases. Finally, there is no mathematical benefit to having a single equity issuance/repurchase variable, as combining a concave benefit of repurchasing shares with a convex cost of issuing new equity will result in a piecewise function which is non-differentiable at zero, which would be practically equivalent to having two separate functions for issuance and repurchases.
3.2 Regulatory Constraints

Bank in this model faces two regulatory constraints. The first is the liquidity constraint, similar to the liquidity coverage ratio (LCR) proposed in Basel III (Basel Committee on Banking Supervision, 2011). The LCR indicates the fraction of short-term liabilities a bank should back up with liquid assets, including reserves and certain securities, such as Treasuries. Since deposits are the only liabilities in this model,

\[ T_t \geq \lambda D_t, \]  

where the liquidity requirement \( 0 \leq \lambda \leq 1 \). One specific example of a liquidity requirement is the reserve requirement. As of 2012, the reserve requirement in the United States is 10% for banks with at least $71 million in net transaction accounts.\(^7\) The relevance of reserve requirements has been diminishing, particularly since 1994. In January 1994, regulators allowed banks to use deposit sweep programs – an accounting technique through which banks can significantly reduce their reported transaction accounts. As a result, most banks found their reserve requirements non-binding (Anderson and Rasche, 2001). More general, self-imposed liquidity constraints tend to lie around 10 or 15% of total deposits. Over the period 2001Q1-2012Q1, the median ratio of liquid assets to total deposits is 12.1% for the 125 largest bank holding companies.

The other regulatory constraint is the capital constraint. Table I shows the current regulatory capital requirements in the United States. The capital requirements should be multiplied by risk weights\(^8\) to obtain the marginal risk-weighted capital requirements. Since this model only has two asset classes, the capital constraint is modeled as

\[ K_t \geq \kappa_0 T_t + \kappa_1 L_t, \]  

where \( 0 \leq \kappa_0 < \kappa_1 \leq 1 \) are the marginal capital requirements on liquid assets and loans, respectively.

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\(^8\) Examples of current risk weights are 0% for Treasury securities, 20% for interbank loans, 50% for secured loans, 100% for unsecured loans, and 200% for junk bonds (FDIC, “Rules and Regulations,” Part 325, Appendix A).
Table 1: Risk-Based Capital Requirements in the United States

<table>
<thead>
<tr>
<th></th>
<th>Total Risk-Based Capital Ratio</th>
<th>Tier 1 Risk-Based Capital Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well capitalized</td>
<td>≥ 10%</td>
<td>≥ 6%</td>
</tr>
<tr>
<td>Adequately capitalized</td>
<td>≥ 8%, &lt; 10%</td>
<td>≥ 4%, &lt; 6%</td>
</tr>
<tr>
<td>Undercapitalized</td>
<td>&lt; 8%</td>
<td>&lt; 4%</td>
</tr>
</tbody>
</table>


Substituting $T = D + K - L$ into the regulatory constraints, the liquidity and capital constraints can be simplified to

$$K_t \geq L_t - (1 - \lambda)D_t \quad \text{and}$$
$$K_t \geq \frac{K_1 - K_0}{1 - K_0}L_t + \frac{K_0}{1 - K_0}D_t,$$

respectively. The loan amounts at which each constraint will be exactly binding, $L_\lambda$ and $L_K$, are thus

$$L_{\lambda,t} = K_t + (1 - \lambda)D_t \quad \text{and}$$
$$L_{K,t} = \frac{1 - K_0}{K_1 - K_0}K_t - \frac{K_0}{K_1 - K_0}D_t.$$

Appendix A shows that a bank will run into its capital constraint before its liquidity constrained if its ratio of capital to assets is low. Alternatively, a bank with a high capital-to-assets ratio is likely to become liquidity constrained before it would become capital constrained.

### 3.3 Value of the Bank

Following Hennessy (2004), the bank maximizes its value, which is formulated as

$$V(K_0, \xi_0, t) \equiv \max_{r_t, x_t, \ell_t} \mathbb{E}\left[\int_0^\infty e^{-m_t}\pi[K_t, \xi_t)] dt\right] \quad \text{s.t. (8) and (9),}$$

(12)
where $m$ is the market discount rate. The Bellman equation for this problem is

$$m \cdot V(K_t, \xi_t) = \max_{r^L_t, r^E_t, q_t} \pi(K_t, \xi_t) + \frac{1}{dt} E[dV_t]$$

s.t. (8) and (9),

with the corresponding Lagrangean

$$m \cdot V(K_t, \xi_t) = \max_{r^L_t, r^E_t, q_t} \pi(K_t, \xi_t) + \frac{1}{dt} E[dV_t]$$

$$+ v_{\lambda,t} \cdot (K_t - L_t + (1 - \lambda) D_t) + v_{\kappa,t} \cdot \left( K_t - \frac{\kappa_0}{1 - \kappa_0} D_t - \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} L_t \right),$$

(13)

where $v_{\lambda}$ and $v_{\kappa}$ are the Lagrange multipliers for the liquidity constraint (8) and the capital constraint (9), respectively. These Lagrange multipliers can be interpreted as the shadow values of liquid assets and capital. By Ito’s Lemma, the Lagrangean can be rewritten as

$$m \cdot V(K_t, \xi_t) = \max_{r^L_t, r^E_t, q_t} \pi(K_t, \xi_t) + \frac{1}{dt} E[dV_t]$$

$$+ v_{\lambda,t} \cdot (K_t - L_t + (1 - \lambda) D_t) + v_{\kappa,t} \cdot \left( K_t - \frac{\kappa_0}{1 - \kappa_0} D_t - \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} L_t \right).$$

(14)

### 4 Analyzing the Dynamic Banking Model

In this section I characterize the optimal behavior of the bank. The first step is to derive the optimality conditions for the control variables in subsection 4.1. As it turns out, the optimality condition for the loan rate depends on the shadow values of liquidity and capital, which are derived in 4.2. The theoretical analysis is completed in 4.3, where the effects of monetary policy actions on bank behavior are shown.

#### 4.1 Optimal Bank Behavior

The first step in characterizing the optimal behavior of the bank is to differentiate the value function (14) with respect to the choice variables, $r^L$, $r^E$, and $q$. The first order condition of the value function
w.r.t. \( r^L \) yields

\[
\frac{\partial \pi(K_t, \xi_t)}{\partial r_t} - v_{L,t} \frac{\partial L_t}{\partial r_t} - v_{K,t} \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} \frac{\partial L_t}{\partial r_t} + \frac{\partial \pi(K_t, \xi_t)}{\partial r_t} V_{K,t} = 0
\]

\[
\Rightarrow L_t - \gamma_1 (r_t^L - c^L - r_t^T) + v_{L,t} \gamma_1 + v_{K,t} \gamma_1 \left( \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} + [L_t - \gamma_1 (r_t^L - c^L - r_t^T)] V_{K,t} = 0. \tag{15}
\]

Using the loan demand function, (1), the optimal loan rate, \( r^L^* \), can be expressed in terms of parameters (regulatory requirements and loan demand elasticities), exogenous variables (marginal costs, the interest rate on liquid assets, a loan demand shock, and real GDP), and the marginal value of capital, \( V_K \):

\[
r_t^L^* = \frac{\gamma_0 + \gamma_1 (c^L + r_t^T) + \gamma_2 Y_t + \epsilon_t^L}{2\gamma_1} + \frac{1}{2(1 + V_{K,t})} \left( v_{L,t} + v_{K,t} \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} \right). \tag{16}
\]

If neither the liquidity constraint nor the capital constraint is binding, i.e. \( v_\lambda = v_\kappa = 0 \), (16) reduces to \( r_t^L^* = \frac{\gamma_0 + \gamma_1 (c^L + r_t^T) + \gamma_2 Y_t + \epsilon_t^L}{2\gamma_1} \). Regardless of either constraint being binding, the optimal loan rate depends positively on the marginal cost of lending \( (c^L) \), and positively on the interest rate on liquid assets \( (r^L) \) as an opportunity cost. A binding liquidity or capital constraint, i.e. \( v_\lambda > 0 \) or \( v_\kappa > 0 \), respectively, would also drive up the interest rate on loans. This indicates that constrained banks are less competitive in the market for bank loans, which is in line with Berger and Bouwman’s (2011) finding that well-capitalized banks are able to increase their market shares during banking crises.

These results are intuitive, as a higher marginal cost of lending makes lending more expensive, requiring a higher return. The pass-through of the marginal cost of funds into the loan rate is 50% for an unconstrained bank. However, a liquidity or capital constrained bank will increase the spread between the loan rate and the marginal cost of funds. For example, if a bank is capital constrained, it will raise its interest rates on loans for two possible reasons: first, a higher interest rate will reduce the quantity of loans demanded, driving down the right hand side of (7), enabling the bank to meet and exceed its capital requirement with its current capital. Second, a larger spread between the loan rate and the marginal cost of funds will have a positive direct effect on profits. Keeping all else equal, higher profits would lead to higher retained earnings, which would increase bank capital, and thus help the bank move away from its capital constraint. In the case of the liquidity requirement being
binding, a higher interest rate on loans, keeping all else equal, in particular the size of total assets $(T + L)$, would reduce lending, freeing up cash for the bank to hold as liquid assets, and thereby meet and exceed its liquidity requirement. Cornett et al. (2011) have shown empirically that relatively illiquid banks reduced lending in an effort to increase asset liquidity during the recent financial crisis.

A bank can manage its capital by paying dividends and issuing new equity. The optimal conditions for the dividend rate $(r^E_*)$ and new equity issuance $(q^*)$ are

$$H'(r^E_*) = \frac{V_{K,t}E_t}{1 + V_{K,t}} \tag{17}$$

and

$$G'(q^*_t) = \frac{V_{K,t}}{1 + V_{K,t}}. \tag{18}$$

If we specify the functional forms of $G(\cdot)$ and $H(\cdot)$ as $G(q) = g_0q + \frac{1}{1 + g_1}(q)^{1 + g_1}$ and $H(r^E) = h_0r^E + \frac{1}{1 - h_1}(r^E)^{1 - h_1}$, where $g_1 > 0$ and $0 < h_1 < 1$, (17) and (18) become

$$r^E_* = \left( \frac{V_{K,t}E_t}{1 + V_{K,t}} - h_0 \right)^{-1/h_1} \tag{19}$$

and

$$q^* = \left( \frac{V_{K,t}}{1 + V_{K,t}} - g_0 \right)^{1/g_1}. \tag{20}$$

The optimal rate of dividends and the optimal amount of equity to issue both depend on the marginal value of capital, $V_K$. Appendix B shows that $V_K$ depends on the current values of bank capital and of the bank itself, and on the expected discounted value of future decisions by the bank. These optimal capital decisions are therefore dynamic, forward-looking decisions, as opposed to the optimal loan rate (as we will see below).

For the discussion of monetary policy in Section 4.3, it is useful to realize that neither the benefit of paying dividends nor the cost of issuing new equity is influenced by $r^T$ (which can be thought of as the monetary policy rate). Hence, expansionary monetary policy does not make it cheaper to raise additional capital.
4.2 The Shadow Values of Liquidity and Capital

The optimal loan rate, \( (16) \), depends on the shadow values of liquidity and capital, \( \nu_\lambda \) and \( \nu_\kappa \). To obtain the shadow value of liquid assets, consider three scenarios: i) Neither the capital constraint nor the liquidity constraint is binding (\( \nu_\lambda = 0, \nu_\kappa = 0 \)); ii) Due to a shock to loan demand of size \( \varepsilon_L^\lambda \), the liquidity constraint is exactly binding (\( \nu_\lambda = 0 \) exactly); iii) Due to a shock to loan demand greater than \( \varepsilon_L^\lambda \) the bank has been pushed over its liquidity constraint (\( \nu_\lambda > 0 \)).

Recall the value of the loan portfolio at which the bank is exactly liquidity constrained, \( (10) \). At this point \( \nu_\lambda \) still equals zero. Define \( r_L^{\lambda, t} \) and \( \varepsilon_L^{\lambda, t} \) as the interest rate on loans and the shock to loan demand that correspond to \( L^{\lambda, t} \). From the optimal loan rate, \( (15) \), with \( \nu_\lambda = 0 \), and \( \nu_\kappa = 0 \) we see that

\[
L^{\lambda, t} (1+V_{K,t}) = (r^{L, t} - c^L - r^T_t) \gamma_1 (1+V_{K,t}) .
\]

Use \( (10) \) for \( L^{\lambda, t} \), and from the loan demand equation \( (1) \), use \( r^{L, t} = \frac{1}{\gamma_1} (\gamma_0 + \gamma_2 Y + \varepsilon_L^{\lambda, t} - L^{\lambda, t} ) \) to substitute for the interest rate on loans and solve for the critical shock to loan demand, \( \varepsilon_L^{\lambda, t} \):

\[
\varepsilon_L^{\lambda, t} = 2 [K_t + (1-\lambda) D_t] \gamma_0 + \gamma_1 (c^L + r^T_t) \gamma_1 (1+V_{K,t}) - \gamma_2 Y_t ,
\]  \hspace{1cm} (21)

This is the size of the exogenous shock to loan demand that makes the liquidity constraint exactly binding.

In the third scenario the bank is liquidity constrained because of a shock to loan demand greater than the critical value, i.e. \( \varepsilon_L > \varepsilon_L^{\lambda, t} \). The shadow value of liquid assets has become positive (\( \nu_\lambda > 0 \)). Hence, \( (15) \) becomes

\[
L_t (1+V_{K,t}) + \nu_\lambda \gamma_1 = (r^{L, t} - c^L - r^T_t) \gamma_1 (1+V_{K,t}) .
\]

Substituting for \( r^{L, t} \) and solve for the loan demand shock gives

\[
\varepsilon_L^{L, t} = 2L_t + \frac{\nu_\lambda \gamma_1}{1+V_{K,t}} - \gamma_0 + \gamma_1 (c^L + r^T_t) - \gamma_2 Y_t .
\]  \hspace{1cm} (22)
Now subtract (21) from (22) and solve for the shadow value of liquid assets ($v_{λ}$). This expresses the shadow value of liquid assets in terms of the deviation from the critical loan demand shock:

$$v_{λ,t} = \frac{1 + V_{K,t}}{γ_t} \max \left[ 0, e^L_t - e^L_{λ,t} \right] - \frac{1 + V_{K,t}}{γ_t} (L_t - L_{λ,t}).$$

This can be simplified since a bank is not allowed to lend more than its critical value of loans once it is liquidity constrained, i.e. $L_t = L_{λ}$ even if $ε^L_t > ε^L_{λ,t}$. Hence,

$$v_{λ,t} = \frac{1 + V_{K,t}}{γ_t} \max \left[ 0, e^L_t - e^L_{λ,t} \right].$$  \tag{23}$$

By the definition of $e^L_{λ,t}$, any shock $ε^L_t > ε^L_{λ,t}$ will make the liquidity constraint binding, and will make $v_{λ} > 0$. Any shock $ε^L_t < ε^L_{λ,t}$, on the other hand, will leave the liquidity constraint not binding, so $v_{λ} = 0$. Therefore, this shadow value can be thought of as a European call option, with strike price (21) and with a marginal payoff of

$$\begin{cases} \frac{1 + V_{K,t}}{γ_t} & \text{if } e^L_t > e^L_{λ,t} \\ 0 & \text{if } e^L_t \leq e^L_{λ,t} \end{cases}$$

The shadow value of capital ($v_{κ}$) is derived in the same way, resulting in

$$v_{κ,t} = \frac{1 - κ_0}{κ_1 - κ_0} \frac{1 + V_{K,t}}{γ_t} \max \left[ 0, e^L_t - e^L_{κ,t} \right],$$  \tag{24}$$

where $e^L_{κ,t} = 2 \left( \frac{1 - κ_0}{κ_1 - κ_0} K_t - \frac{κ_0}{κ_1 - κ_0} D_t \right) - γ_0 + γ_t (c^L_t + r^T_t) - γ_2 Y_t,$  \tag{25}$$

and the marginal payoff is

$$\begin{cases} \frac{1 - κ_0}{κ_1 - κ_0} \frac{1 + V_{K,t}}{γ_t} & \text{if } e^L_t > e^L_{κ,t} \\ 0 & \text{if } e^L_t \leq e^L_{κ,t} \end{cases}$$

Optimality condition (16) showed that the shadow values of liquidity and capital directly affect the bank’s optimal loan rate decision. We can substitute the shadow values (23) and (24) into the optimality condition to see how a binding constraint affects the loan rate at the margin. After cancellations, and realizing that at most one constraint can be binding at the time,

$$r^L_t = γ_0 + γ_t \left( c^L_t + r^T_t \right) + γ_2 Y_t + e^L_t + \max \left[ 0, \frac{e^L_t - e^L_{λ,t}}{2γ_t}, \frac{e^L_t - e^L_{κ,t}}{2γ_t} \right].$$  \tag{26}$$
Hence, the liquidity constraint has no different marginal effect on the loan rate than the capital constraint. In both cases, the marginal effect of a loan demand shock on the loan rate is \( \frac{1}{\gamma_1} \). The only relevant distinction is the position of the critical shocks to loan demand, \( \varepsilon^L_{\lambda} \) and \( \varepsilon^L_{\kappa} \), as shown in Figure 2. By their definitions, (21) and (25) depend on the bank’s leverage through \( L_{\lambda} \) and \( L_{\kappa} \) (see Appendix A).

To see the differences in optimal loan rates between constrained and unconstrained banks more clearly, substitute the critical loan demand shocks (21) and (25) into (26). It follows that for a liquidity constrained bank the optimal loan rate is

\[
\begin{align*}
r^{L^*}_{LC,t} &= \frac{\gamma_0 + \gamma_2 Y_t + \varepsilon^L_t}{\gamma_1} - \frac{1}{\gamma_1} \left( K_t + (1 - \lambda) D_t \right),
\end{align*}
\]

(27)

where \( K + (1 - \lambda) D \) is the critical value of total loans \( L_{\lambda} \), by (10). The optimal loan rate for a capital constrained bank is

\[
\begin{align*}
r^{L^*}_{CC,t} &= \frac{\gamma_0 + \gamma_2 Y_t + \varepsilon^L_t}{\gamma_1} - \frac{1}{\gamma_1} \left( \frac{1 - \kappa_0}{\kappa_1 - \kappa_0} K_t - \frac{\kappa_0}{\kappa_1 - \kappa_0} D_t \right),
\end{align*}
\]

(28)

where \( \frac{1 - \kappa_0}{\kappa_1 - \kappa_0} K - \frac{\kappa_0}{\kappa_1 - \kappa_0} D \) is the critical value of total loans \( L_{\kappa} \), by (11). For unconstrained banks the optimal loan rate is

\[
\begin{align*}
r^{L^*}_{U,t} &= \frac{\gamma_0 + \gamma_1 (c^{L^*} + r^T) + \gamma_2 Y_t + \varepsilon^L_t}{2 \gamma_1}.
\end{align*}
\]

(29)

At the optimum, the loan rate decision in this model turns out to be a static one, because loans are of short (immediate) maturity, and therefore not a state variable. Compared to an unconstrained bank, loan rates of constrained banks depend negatively on the amount of capital the bank has. Hence, having less capital results in a higher loan rate for capital constrained banks as well as liquidity constrained banks. Moreover, loan rates of constrained banks relative to unconstrained banks are twice as sensitive to real GDP \( (Y) \) and loan demand shocks \( (\varepsilon^L) \). However, the marginal cost of lending \( (c^{L^*}) \) and the return on liquid assets \( (r^T) \) do not matter at all in loan rate decisions by constrained banks. This brings us to the pass-through from monetary policy into bank loan rates.
Figure 2: Loan Rate under Capital and Liquidity Requirements

\[ L_\lambda < L_\alpha \]

Loan Rate \( r^L \)

Liquidity Constrained

Capital Constrained

Unconstrained

Loan Demand Shock \( \epsilon^L \)

Unconstrained

Capital Constrained

Liquidity Constrained

17
4.3 Application to Monetary Policy

Similar to Allen et al. (2009), I consider the effects of monetary policy within the bank model, instead of introducing banks into a model of monetary policy. One can think of the interest rate on liquid assets \( r^T \) as the monetary policy rate. In the United States, the federal funds rate is the interest rate at which banks trade reserves, which are highly liquid assets. As such, \( r^T \) is the marginal opportunity cost of retail lending for a bank that acts as a lender in the interbank market, or a marginal cost of funding for a net borrower in the interbank market.

When engaging in traditional monetary policy by adjusting the federal funds rate, the Federal Reserve can affect bank loan rates. For a bank that is far from being constrained, (26) shows that lowering \( r^T \) will lower the optimal loan rate \( r^{L*} \) by half as much.\(^9\) Assume for simplicity that the loan demand shock faced by a bank right after a monetary policy action is equal to the loan demand shock beforehand. For constrained banks, as well as banks that are reasonably close to their requirements, the situation changes. For example, for a bank whose capital constraint is the first constraint to bind, a shock \( \varepsilon^L > \varepsilon^L_\kappa \) will make the shadow value of capital positive, and loan rate will be twice as sensitive to the loan demand shock \( \left( \frac{1}{2} \right) \) instead of \( \frac{1}{2} \), relative to the unconstrained case in (29). The same change applies to a binding liquidity constraint.

The transmission of monetary policy does not simply depend on the way banks set their loan rates relative to the monetary policy rate; monetary policy actions change the way banks set loan rates relative to the policy rate. As shown in Figure 3, expansionary monetary policy in the form of a drop in \( r^T \) causes the loan rate curve as a whole to shift downward. Moreover, reducing \( r^T \) lowers the critical values \( \varepsilon^L_\lambda \) and \( \varepsilon^L_\kappa \), by (21) and (25). In other words, expansionary monetary policy causes the ‘safe’ range of loan demand shocks (left of the critical value) to shrink. The shadow values of capital and liquidity hence become more valuable, which drives up the loan rate.

Moreover, (27) and (28) show that for any bank that was already constrained before any monetary easing, \( r^T \) has no effect on the loan rate whatsoever, resulting in zero pass-through for already-constrained banks. For banks that were initially unconstrained, but have become either liquidity

\(^9\)The pass-through for unconstrained banks is only half, since \( \frac{\partial r^L}{\partial r^T} = \frac{1}{2} \). This follows from substituting the loan demand equation into into the profit function, resulting in profits being quadratic in the loan rate.
Figure 3: Loan Rate under Expansionary Monetary Policy

or capital constrained due to the monetary easing, pass-through is limited, depending on the extent to which the actual loan demand shock exceeds the critical value. Maximum pass-through is only achieved by banks that were unconstrained before monetary easing, and still are unconstrained afterwards.

A monetary tightening through an increase in the policy rate is not as problematic. For a fixed loan demand shock, initially unconstrained banks will still be unconstrained after the tightening, and all will adjust their loan rates fully (unlike those initially unconstrained banks that became constrained after a drop in the policy rate). For banks that were initially constrained, some will become unconstrained and partially adjust their loan rates. Only the most constrained banks will still be constrained, and not adjust loan rates at all.

Incomplete pass-through, or a complete lack thereof, need not be permanent. As long as a constrained bank is able to grow its capital, its critical loan demand shocks (for both capital and liquidity) will gradually increase, meaning the bank will be able to absorb larger positive shocks to loan demand without being constrained. If this is the case, the bank’s loan rate will ultimately achieve maximum pass-through, but with a delay, as shown in Figure 3. If the new critical loan demand shock has in-
creased sufficiently such that $\varepsilon_{L,3}^L \geq \varepsilon_{L,1}^L$ (from Figure 3), pass-through is complete, although delayed. If $\varepsilon_{L,3}^L \leq \varepsilon_{L,1}^L$, full pass-through will not happen so long as the bank keeps experiencing loan demand shocks between $\varepsilon_{L,3}^L$ and $\varepsilon_{L,1}^L$. This is possible because the deposit rate falls along with the policy rate (or even more for capital constrained banks), since $r^D = r^T - \frac{\nu_1 - (1 - \kappa_1)\kappa_0}{1 - \kappa_0}$, which lowers the bank’s costs. Moreover, the increase in the loan rate may increase revenues. If profits increase sufficiently to have positive retained earnings ($\pi - r^F E$), capital will grow even without a new share issue.

Figure 4: Loan Rate after Increase in Capital

As we have seen, a monetary expansion may have unintended consequences for the health of the banking sector if it is subject to bank regulation in the form of capital and/or liquidity requirements. Even in non-crisis times, healthy banks experience a tightening of capital and liquidity requirements during falling rate periods, and may decide to increase their loan rates relative to their marginal cost of funding (or their marginal opportunity cost of lending).

Time-variant capital requirements may ameliorate these unintended consequences of expansive monetary policy. To see this, consider (25): $\frac{\partial \varepsilon_{L,3}^L}{\partial \kappa_0}, \frac{\partial \varepsilon_{L,3}^L}{\partial \kappa_1} < 0$ for all $0 \leq \kappa_0 < \kappa_1 \leq 1$. While a drop

\[10\text{It is also shown empirically by Cosimano and Hakura (2011) that higher marginal capital requirements lead to higher loan rates.}\]
in the monetary policy rate lowers the value of the critical loan demand shock, a (temporary) reduction in marginal capital requirements raises this value, hence offsetting the tightening of the capital constraint during expansionary monetary episodes. The counter-cyclical capital buffer as proposed in Basel III (Basel Committee on Banking Supervision, 2011) should serve this purpose, as long as the timing of this buffer coincides with monetary policy actions. This argument can be carried over to the liquidity requirement in the model, which corresponds to the proposed liquidity coverage ratio in Basel III. As it is designing the specifics of liquidity regulation, the Basel Committee on Banking Supervision may want to consider counter-cyclical liquidity requirements.

5 Estimation and Results

This section provides an empirical analysis of the predictions of the model. First, subsection 5.1 summarizes the testable predictions of the model. Next, 5.2 outlines the estimation strategy for these predictions, as well as the specification of loan demand, and 5.3 discusses the data. Finally, 5.4 shows the results.

5.1 Summary of Model Predictions

As discussed above, the liquidity requirement in the model is based on the proposed liquidity coverage ratio by the Basel Committee on Banking Supervision (2011). As such, banks currently do not face any regulatory liquidity requirements (other than reserve requirements, which were discussed in Section 3.2), so I skip liquidity requirements in the empirical part of this paper. The derivations and implications of the following predictions were discussed in Section 4. Table 2 summarizes the testable predictions of the model regarding loan rates. In addition to the hypotheses directly relating to loan rates, the model also predicts that banks are more likely to become constrained during falling rate periods (since a drop in the monetary policy rate translates into a tightening of the regulatory constraints) than during rising rate periods.

In Section 4 we saw that the optimal loan rate always depends positively on loan demand shocks ($\varepsilon^L$). However, since loan demand shocks cannot be observed, there is need for an empirical coun-
Table 2: Theoretical Predictions for the Loan Rate

<table>
<thead>
<tr>
<th>Effect of</th>
<th>Empirical counterpart</th>
<th>Loan Rate ($r^L$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon^L \uparrow$</td>
<td>Capital ratio ↓</td>
<td>Unconstrained $\uparrow$</td>
</tr>
<tr>
<td>$r_T \uparrow$</td>
<td>Rising federal funds rate</td>
<td>Capital constrained $\uparrow$</td>
</tr>
<tr>
<td>$f_{fr} \times (rising\ rate\ dummy)$</td>
<td>$-CR$</td>
<td></td>
</tr>
<tr>
<td>$r_T \downarrow$</td>
<td>Falling federal funds rate</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$f_{fr} \times (falling\ rate\ dummy)$</td>
<td></td>
<td>$\downarrow / 0$</td>
</tr>
<tr>
<td>$Y \uparrow$</td>
<td>Detrended real GDP ↑</td>
<td>$\uparrow / \downarrow$</td>
</tr>
</tbody>
</table>

NOTES: $\uparrow$, $\downarrow$, and 0 indicate a positive, negative, and zero effect, respectively. Moreover, $\uparrow (\downarrow)$ relative to $\uparrow (\downarrow)$ indicates a more positive (negative) effect. The model does not predict the sign of $\frac{\partial r^L}{\partial Y}$ (whether loan demand is cyclical or countercyclical), only relative magnitudes.

From Figure 2, it follows that at higher values for $\varepsilon^L$, a bank either moves closer toward its capital (or liquidity) constraint, or the constraint becomes more binding. Intuitively, this is the exact opposite of a bank’s capital ratio (bank capital relative to risk-weighted assets): a bank moves closer toward its capital constraint (or becomes more constrained) as the capital ratio falls. In terms of the model, it is also possible to relate an increase in the loan demand shock to a decrease in the capital ratio. Define the capital ratio as $\frac{K}{\kappa_0 L + \kappa_1 L}$. All else constant, an increase in $\varepsilon^L$ increases total loans, $L$, which drives up the denominator of the capital ratio. Hence, the capital ratio falls as the loan demand shock increases.

### 5.2 Estimation

This section outlines the approach to test the predictions of the model concerning the loan rate, summarized in Table 2, as well as the loan demand specification in (1). Ideally, one would regress the loan rate for bank $i$ at time $t$ on the variables in Table 2 such that

$$r^L_{it} = \beta_0 + \beta_1 (1 - cc_{it}) \varepsilon^L_{it} + \beta_2 (cc_{it}) \varepsilon^L_{it} + \beta_3 (1 - cc_{it}) (fall_{it}) FFR_t + \beta_4 (cc_{it}) (fall_{it}) FFR_t$$

$$+ \beta_5 (1 - cc_{it}) (rise_{it}) FFR_t + \beta_6 (cc_{it}) (rise_{it}) FFR_t + \beta_7 (1 - cc_{it}) Y_t + \beta_8 (cc_{it}) Y_t + bX_{it}, \quad (30)$$
where the dummy variable \( cc_{it} \) which equals 1 when bank \( i \) is capital constrained at time \( t \), and 0 otherwise, and the dummies for falling and rising rate periods, based on the federal funds rate, \( FFR \), are denoted by \( fall \) and \( rise \), respectively. The model predicts that \( 0 < \beta_1 < \beta_2 \), since capital constrained banks are more sensitive to the loan demand shock. Additionally, pass-through from the federal funds rate is lower for constrained banks, such that \( 0 < \beta_4 < \beta_3 \) during falling rate periods, and \( 0 < \beta_6 < \beta_5 \) for rising rate periods. Finally, the loan rate for constrained banks is predicted to be more sensitive to movement in the aggregate economy than for unconstrained banks, such that \( 0 < \beta_7 < \beta_8 \), where the aggregate economy, \( Y \), is Hodrick-Prescott detrended real GDP. Controls are in the matrix \( X \), both bank-specific (average operating expenses, lags of quarterly loan growth, the lagged ratio of nonperforming loans to total assets and its lagged growth rate, and a dummy for periods in which the bank acquired another institution) and macroeconomic (lags of quarterly real GDP growth and lags of quarterly inflation).

Since the loan demand shock, \( \varepsilon^L \), cannot be observed, I use the capital ratio as its empirical counterpart, as explained in the previous subsection. However, the capital ratio, \( CR \), is an imperfect measure of the loan demand shock. Assuming that the error \( e_{it}^{CR} = CR_{it} - \varepsilon_{it}^L \) has an average of zero (i.e. \( E( e_{it}^{CR} ) = 0 \)), this introduces classical measurement error into the regression, and the coefficient estimate on the capital ratio is subject to attenuation bias.\(^{11}\) This measurement error problem calls for two-stage least squares, where I instrument for the capital ratio.

Recall the evolution of capital, (5). The change in the value of capital is determined by the previous period’s dividends and share issue, as well as the loan rate (through profits). In (19) and (20) we saw that the optimal policies for dividends and share issues depend on the contemporaneous marginal value of capital (\( V_K \)), which in turn is a function of the contemporaneous value of capital, as shown in Appendix B. Moreover, for constrained banks, the optimal loan rate also depends on the value of capital, as seen in (27) and (28). Hence, a potential instrument for the current capital ratio is its lagged value. To justify the use of the lagged capital ratio as an instrument, it is important to note that in the model its elements (lagged capital, loans and liquid assets) are uncorrelated with the

\(^{11}\) Ordinary least squares estimates of (30) indeed show much smaller coefficient estimates for \( \beta_1 \) and \( \beta_2 \) compared to the two-stage least squares results provided in this paper, which indicate attenuation bias. OLS results are available upon request.
current loan rate. The first-stage regression is thus

$$ CR_t = \alpha_1 + \alpha_2 CR_{i(t-1)} + \alpha_3 (fall_t) FFR_t + \alpha_4 (rise_t) FFR_t + \alpha_5 Y_t + aX_{it} + v_t, \quad (31) $$

where $v$ is an error term. The predicted capital ratio, $\hat{CR}$, from (31) can then be used in the second-stage loan rate regression, such that

$$ r_{L}^{d} = \beta_0 - \beta_1 (1 - cc_{it}) \hat{CR}_{it} - \beta_2 (cc_{it}) \hat{CR}_{it} + \beta_3 (1 - cc_{it}) (fall_t) FFR_t + \beta_4 (cc_{it}) (fall_t) FFR_t + \beta_5 (1 - cc_{it}) (rise_t) FFR_t + \beta_6 (cc_{it}) (rise_t) FFR_t + \beta_7 (1 - cc_{it}) Y_t + \beta_8 (cc_{it}) Y_t + bX_{it} + u_{it}, \quad (32) $$

where $u$ is an error term.

Next, I test if the loan demand specification (1) is stable across falling and rising rate periods. If the interest rate semi-elasticity of loan demand were higher during falling rate periods than during rising rate periods, any effects of the asymmetry in pass-through from the federal funds rate into loan rates on the amount of lending may be dampened. To see this, consider two scenarios: (i) the federal funds rate increases by 100 basis points, and (ii) the federal funds rate decreases by 100 basis points. In the presence of asymmetric pass-through into loan rates, loan rates would rise, say, 100 basis points under (i), but fall by, say, 50 basis points under (ii). If the interest rate semi-elasticity of loan demand is constant across falling and rising rate periods, the ultimate result will be a relatively large drop in bank lending under (i) and a relatively small increase in lending under (ii). However, if the interest rate semi-elasticity of loan demand were higher during falling rate periods and lower when rates rise, this ultimate effect on bank lending would be dampened, or even disappear.

Estimating the loan demand function (1) creates an endogeneity problem, since the loan rate and the amount of bank lending are jointly determined in equilibrium. For this reason, I instrument for the loan rate using the predicted loan rate, $\hat{r}_{L}^{d}$, from (32). The loan demand regression is then

$$ \ln \left( L_{it}^{d} \right) = \delta_0 - \delta_1 \hat{r}_{it}^{d} + \delta_2 (rise_t) \hat{r}_{it}^{d} + \delta_3 (fall_t) \hat{r}_{it}^{d} + \delta_4 Y_t + \delta_5 X_{it}^{m} + u_{it}, \quad (33) $$

where $X^{m}$ is a matrix of macroeconomics controls (lags of quarterly real GDP growth, lags of quar-
terly inflation, and dummies for falling and rising rate periods), and $\nu$ is an error term.

5.3 Data

For the bank-specific variables I use bank holding company (BHC) data from the Federal Reserve Bank of Chicago. I use data at the holding company level rather than the commercial bank level since critical decisions regarding bank capital are generally made at the holding company level (Ashcraft, 2008). The data set ranges from 2001Q1 through 2012Q1. I use 98 of the largest 125 BHCs (as of 2011Q1). The largest 125 BHCs were responsible for 90% of total bank lending in the United States in 2011. Of these 125 BHCs, I dropped the ones with foreign parents, since these holding companies are eligible for exemptions regarding capital requirements. In addition, I dropped those institutions with missing data since 2002Q1. Measures of the macroeconomy, including real gross domestic product and the consumer price index, also at a quarterly frequency covering the full sample period, are taken from the Federal Reserve Economic Database (FRED).

The BHC data set primarily provides data on balance sheet and income statement items. Unfortunately, this does not include (marginal) interest rates. As an approximation for the interest rate on loans, I calculate the ratio of total interest income to total loans and securities, and the ‘loan rate’ throughout this section will refer to this approximation. By its nature, this ratio does not have much variation compared to marginal interest rates, which leads to low coefficient estimates in regressions where the loan rate is the dependent variable.

Table 3 shows some descriptive statistics of the BHC data. The first column covers the full sample period, whereas the next three columns cover the periods before, during, and after the financial crisis. Although the federal funds rate (and hence the marginal cost of funding or the marginal opportunity cost of lending) dropped sharply during the crisis, the loan rate (as approximated by average interest income) did not fall until after the crisis, indicating that banks were slow to adjust their rates downward (as seen in Figure 1).

In addition, capital ratios appeared largely unchanged during the crisis, with the percentage of

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12 To be precise, this consists of data from the forms FR Y-9C and FR Y-9LP.
13 In this section, I will use the terms ‘bank’ and ‘bank holding company’ interchangeably.
14 As a result, institutions such as Goldman Sachs and Morgan Stanley, which were initially investment banks, but turned into bank holding companies in 2008, are not included in the estimations.
Table 3: Descriptive Statistics of the Bank Holding Company Data

<table>
<thead>
<tr>
<th></th>
<th>2001Q1</th>
<th>2001Q1</th>
<th>2007Q3</th>
<th>2009Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2012Q1</td>
<td>-2007Q2</td>
<td>-2009Q2</td>
<td>-2012Q1</td>
</tr>
<tr>
<td><strong>Loan Rate (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.48</td>
<td>1.58</td>
<td>1.52</td>
<td>1.21</td>
</tr>
<tr>
<td>Median</td>
<td>1.44</td>
<td>1.54</td>
<td>1.49</td>
<td>1.21</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.36</td>
<td>0.36</td>
<td>0.34</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Total Risk-Based Capital Ratio (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>13.46</td>
<td>12.94</td>
<td>12.73</td>
<td>15.23</td>
</tr>
<tr>
<td>Median</td>
<td>12.92</td>
<td>12.32</td>
<td>12.31</td>
<td>15.07</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>3.39</td>
<td>3.44</td>
<td>2.63</td>
<td>3.14</td>
</tr>
<tr>
<td><strong>Tier 1 Risk-Based Capital Ratio (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Mean</td>
<td>11.40</td>
<td>10.92</td>
<td>10.59</td>
<td>13.12</td>
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<tr>
<td>Median</td>
<td>10.89</td>
<td>10.28</td>
<td>10.22</td>
<td>12.82</td>
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<tr>
<td>St. Dev.</td>
<td>3.58</td>
<td>3.70</td>
<td>2.76</td>
<td>3.24</td>
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<td><strong>% Obs. Capital Constrained</strong></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Total Risk-Based Capital Ratio</td>
<td>3.48%</td>
<td>3.46%</td>
<td>3.06%</td>
<td>3.85%</td>
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<tr>
<td>Tier 1 Risk-Based Capital Ratio</td>
<td>1.91%</td>
<td>1.69%</td>
<td>1.79%</td>
<td>2.54%</td>
</tr>
<tr>
<td><strong>Total # Observations</strong></td>
<td>4410</td>
<td>2548</td>
<td>784</td>
<td>1078</td>
</tr>
</tbody>
</table>

Notes: The loan rate is approximated by the ratio of total interest income over total loans and securities. A bank is categorized as ‘capital constrained’ if its relevant capital ratio is less than ‘well-capitalized’ (see Table I).
capital constrained, observations remaining roughly constant. Although the mean and median of capital ratios rose substantially after the crisis, so did the fraction of observations at which banks were less than well-capitalized. Figure 5 shows that the fraction of BHCs that are not well-capitalized rises during falling rate periods, and diminishes during rising rate periods, which is in line with the model prediction that banks are more likely to become capital constrained during falling rate periods than during rising rate periods.

Figure 5: Percentage of BHCs Capital Constrained across Falling and Rising Rate Periods, 2001Q1-2012Q1

NOTES: Falling and rising rate periods are based on the federal funds rate. Falling rate periods: 2001Q1-2003Q2 and 2007Q3-2008Q4; Rising rate period: 2004Q3-2006Q3. Bank holding companies are defined as capital constrained when they are less than well-capitalized (see Table 1).

15Since few holding companies were actually undercapitalized between 2001 and 2012, I define BHCs as capital constrained when they are less than well-capitalized (see Table 1).
5.4 Results

Table 4 reports the results for the first stage regression, (31). Its lagged value appears to be a significant and strong instrument. The marginal cost of funding, represented by the federal funds rate, has a negative impact on capital ratios, as the model predicted, regardless of falling or rising rate periods. Recall from (25) that the critical value of the loan demand shock at which the bank becomes capital constrained depends positively on the marginal cost of funding. An increase in the marginal cost of funding hence raises this critical value, lowering the shadow value of capital at any given point. At a lower shadow value, a bank desires a smaller amount of capital. More intuitively, one may imagine that when rates rise, banks become more eager to lend, and increase the denominator of the capital ratios (either risk-weighted assets, in the total and tier 1 risk-based capital ratios, or total assets, in the tier 1 leverage ratio), driving down the overall ratios.

Table 4: First-Stage Results for the Capital Ratio (31), 2002Q2-2012Q1

<table>
<thead>
<tr>
<th>Capital Ratio (CR)</th>
<th>Total Risk-Based</th>
<th>Tier 1 Risk-Based</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CR_{it}(t-1) )</td>
<td>0.891*** (0.016)</td>
<td>0.897*** (0.015)</td>
</tr>
<tr>
<td>((fall_t)) FFR(_t)</td>
<td>-0.063*** (0.013)</td>
<td>-0.061** (0.031)</td>
</tr>
<tr>
<td>((rise_t)) FFR(_t)</td>
<td>-0.046*** (0.014)</td>
<td>-0.013 (0.031)</td>
</tr>
<tr>
<td>( Y_t )</td>
<td>-0.687*** (0.179)</td>
<td>-0.704*** (0.226)</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>0.920</td>
<td>0.935</td>
</tr>
<tr>
<td>( F )-statistic</td>
<td>380.790</td>
<td>467.692</td>
</tr>
</tbody>
</table>

**NOTES**: Each regression is estimated with bank holding company fixed effects. Cross-section clustered standard errors are in parentheses. Intercept and coefficients on control variables are estimated, but not reported. ***, ** and * denote statistical significance at 10%, 5% and 1%, respectively.

Table 5 presents the second-stage results for the loan rate, (32). Each regression in this table is reported over two columns: the first column shows the results for unconstrained banks, and the second column shows the estimates when the capital constrained dummy \((cc_{it})\) equals 1. The bottom panel, ‘\( F \)-statistic for equality’, tests whether the coefficient estimates for constrained and unconstrained
bank holding companies are statistically equal. As explained in Section 5.3, because marginal loan rates are unavailable, I use the ratio of total interest income over total loans and securities. Due to the low variation in this variable the point estimates are likely to be underestimated.

For total and tier 1 risk-based capital, the results are very similar, and support almost all of the predictions of the model as outlined in Table 2. A drop in the capital ratio drives up loan rates, significantly more so for constrained banks than for unconstrained banks. Particularly, a drop of one percentage point in the capital ratio is followed by a 5 to 6 basis point increase in the loan rates for unconstrained banks, and a 6 to 9 basis point increase for capital constrained banks, although these point estimates are underestimated, as explained above. In addition, a 100 basis point drop in the federal funds rate translates into a 5 to 6 basis point reduction in bank loan rates for unconstrained banks, and weakens (though in most cases not statistically significantly so) when banks are capital constrained. Specifically, when tier 1 capital is used we see that the pass-through by bank holding companies that are constrained after a decrease in the federal funds rate is not statistically different from zero, as the model predicted. Qualitatively similar results are found for rising rate periods. The implied increase in the spread between loan rates and the federal funds rate is consistent with the finding of Saunders and Schumacher (2000) that bank regulations, including capital requirements, result in higher net interest margins. The only result that does not support the model is the fact that loan rates for constrained banks appear less (rather than more) sensitive to movements in GDP.

To see whether these results are driven by a particular period, I split the regressions into pre-crisis (2002Q2-2007Q2) and crisis/post-crisis (2007Q3-2012Q1) samples, reported in Table 6. Before the financial crisis started, BHCs did not seem to let capitalization play a significant role in loan rate decisions, unless it was tier 1 capital constrained. During rising rate periods, however, constrained banks adjusted their loan rates significantly less than unconstrained banks, which supports the prediction of the model. The falling rate periods show a sign reversal, where any bank, tended to raise its loan rate in response to a drop in the federal funds rate. Constrained banks increased the spread more than unconstrained banks, which is again in line with the model. Moreover, over this period it is clear that pass-through was less (even negative) during falling rate periods than during rising rate periods, as noted in Figure 1.
Table 5: Second-Stage Results for the Loan Rate (32), 2002Q2-2012Q1

<table>
<thead>
<tr>
<th></th>
<th>Loan Rate (Average Interest Income) ((r^L_t))</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Risk-Based CR</td>
<td>Tier 1 Risk-Based CR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unconstrained</td>
<td>Capital constrained</td>
<td>Unconstrained</td>
</tr>
<tr>
<td></td>
<td>((\cdot \times (1 - cc_{it})))</td>
<td>((\cdot \times (cc_{it})))</td>
<td>((\cdot \times (1 - cc_{it})))</td>
</tr>
<tr>
<td>(-\hat{CR}_d)</td>
<td>5.085***</td>
<td>6.415***</td>
<td>5.716***</td>
</tr>
<tr>
<td></td>
<td>(1.446)</td>
<td>(1.584)</td>
<td>(1.147)</td>
</tr>
<tr>
<td>((fall_t)) FFR_t</td>
<td>5.673***</td>
<td>5.307***</td>
<td>5.416***</td>
</tr>
<tr>
<td></td>
<td>(0.598)</td>
<td>(2.052)</td>
<td>(0.551)</td>
</tr>
<tr>
<td>((rise_t)) FFR_t</td>
<td>4.562***</td>
<td>3.561***</td>
<td>4.316***</td>
</tr>
<tr>
<td></td>
<td>(0.315)</td>
<td>(1.172)</td>
<td>(0.314)</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>42.595***</td>
<td>21.152</td>
<td>42.589***</td>
</tr>
<tr>
<td></td>
<td>(5.800)</td>
<td>(16.561)</td>
<td>(5.469)</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.656</td>
<td></td>
<td>0.661</td>
</tr>
<tr>
<td>(F)-statistic</td>
<td>62.116</td>
<td></td>
<td>63.348</td>
</tr>
<tr>
<td>(F)-statistic for equality</td>
<td></td>
<td></td>
<td>(F)-statistic for equality</td>
</tr>
<tr>
<td>(-\hat{CR}_d)</td>
<td>3.913**</td>
<td></td>
<td>39.280***</td>
</tr>
<tr>
<td>((fall_t)) FFR_t</td>
<td>0.035</td>
<td></td>
<td>2.774*</td>
</tr>
<tr>
<td>((rise_t)) FFR_t</td>
<td>0.691</td>
<td></td>
<td>1.288</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>2.085</td>
<td></td>
<td>0.232</td>
</tr>
</tbody>
</table>

Notes: Each two columns show the results of a single regression: is once estimated with the total risk-based capital ratio, and once with the tier 1 risk-based capital ratio. Each regression is estimated with bank holding company fixed effects. Cross-section clustered standard errors are in parentheses. Intercept and coefficients on control variables are estimated, but not reported. The null for the 'F-statistic for equality' is \(H_0: (\cdot \times (1 - cc_{it})) = (\cdot \times (cc_{it}))\). ***, ** and * denote statistical significance at 10%, 5% and 1%, respectively.
The second half of Table 6 shows that the predictions of the model are particularly well supported over the crisis/post-crisis subsample. Capitalization always plays a significant role in loan rate decisions, significantly more so for capital constrained banks than for unconstrained banks. Pass-through of the federal funds rate is positive, but smaller for constrained banks during falling rate periods (there were no rising rate periods between 2007Q3 and 2012Q1), and the loan rate charged by constrained banks is significantly more sensitive to fluctuations in GDP than for unconstrained banks. Note that the sign reversal does not contradict the model; a negative relationship between the loan rate and GDP implies that loan demand over this period was countercyclical, rather than cyclical.

The loan demand regression results are shown in Table 7. The results do not differ much between the two different ways in which I instrumented for the loan rate (either with total or tier 1 risk-based capital). The interest rate semi-elasticity of loan demand is between -1.1 and -1.3, which indicates that loan demand falls by 1.1 to 1.3 percent after a 100 basis point increase in the loan rate. In periods that the federal funds rate is moving (falling as well as rising rate periods), this semi-elasticity is slightly lower (in absolute terms). However, between falling and rising rate periods there is no significant difference in these semi-elasticities. Under the null hypothesis that the semi-elasticities are equal between falling and rising rate periods, the $t$-statistics are 1.051 and 1.182 for specifications (i) and (ii), respectively. This shows that the loan demand specification is stable across falling and rising rate periods, and that the asymmetry in interest rate pass-through results ultimately in an asymmetry in bank lending. The income elasticity of loan demand is positive for both specifications.

6 Conclusion

It has been widely documented that banks tend to increase the spread between their loan rates and the monetary policy rate during falling rate periods (by lowering rates less than the policy rate), while passing interest rate hikes through into loan rates more fully. In this paper I have shown that this asymmetry in interest rate pass-through can be explained by capital and liquidity requirements imposed on banks by regulators. If a bank experiences a binding capital (liquidity) constraint, the shadow value of capital (liquidity) increases. Holding capital (liquid assets) becomes more valuable, and as a result the bank will raise its loan rate relative to the monetary policy rate. The first benefit is
<table>
<thead>
<tr>
<th></th>
<th>2002Q2-2007Q2 Total Risk-Based CR</th>
<th>Tier 1 Risk-Based CR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unconstrained ((-\hat{CR}_d))</td>
<td>Capital constrained ((\cdot \times (1 - cc_{it})))</td>
</tr>
<tr>
<td>(-\hat{CR}_d)</td>
<td>1.490 (1.198)</td>
<td>1.550 (1.270)</td>
</tr>
<tr>
<td>((fall_i)FFR_t)</td>
<td>-1.461*** (0.411)</td>
<td>-2.542* (1.544)</td>
</tr>
<tr>
<td>((rise_i)FFR_t)</td>
<td>4.503*** (0.299)</td>
<td>3.633*** (0.496)</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>93.832*** (6.877)</td>
<td>50.732** (22.854)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2007Q3-2012Q1 Total Risk-Based CR</th>
<th>Tier 1 Risk-Based CR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unconstrained ((-\hat{CR}_d))</td>
<td>Capital constrained ((\cdot \times (1 - cc_{it})))</td>
</tr>
<tr>
<td>(-\hat{CR}_d)</td>
<td>4.001*** (1.075)</td>
<td>5.578*** (1.169)</td>
</tr>
<tr>
<td>((fall_i)FFR_t)</td>
<td>13.691*** (0.871)</td>
<td>12.457*** (1.593)</td>
</tr>
<tr>
<td>((rise_i)FFR_t)</td>
<td>-38.700*** (2.953)</td>
<td>-54.379*** (6.808)</td>
</tr>
</tbody>
</table>

Adj. \(R^2\) 0.791 0.790
\(F\)-statistic 64.796 64.483
\(F\)-statistic for equality 15.939*** 1.910
\(F\)-statistic for equality 9.175*** 0.815

**Notes:** Each two columns show the results of a single regression: \([32]\) is once estimated with the total risk-based capital ratio, and once with the tier 1 risk-based capital ratio. Each regression is estimated with bank holding company fixed effects. Cross-section clustered standard errors are in parentheses. Intercept and coefficients on control variables are estimated, but not reported. The null for the \(F\)-statistic for equality is \(H_0: (\cdot \times (1 - cc_{it}) = (\cdot \times (cc_{it})\). ***, ** and * denote statistical significance at 10%, 5% and 1%, respectively.
Table 7: Second-Stage Results for Loan Demand (33), 2002Q2-2012Q1

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{r}_{lt}^{\text{fall}})</td>
<td>-1.295***</td>
<td>-1.145***</td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(0.232)</td>
</tr>
<tr>
<td>(\hat{r}_{lt}^{\text{rise}})</td>
<td>0.453***</td>
<td>0.422***</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
<td>(0.125)</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>0.397***</td>
<td>0.359***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>(\text{Adj. } R^2)</td>
<td>1.728***</td>
<td>1.622***</td>
</tr>
<tr>
<td></td>
<td>(0.183)</td>
<td>(0.213)</td>
</tr>
<tr>
<td>(F)-statistic</td>
<td>1151.300</td>
<td>1128.800</td>
</tr>
</tbody>
</table>

NOTES: Column (i) uses the predicted loan rate from the ‘Total Risk-Based CR’ columns in Table 5, and column (ii) uses the predicted loan rate from the ‘Tier 1 Risk-Based CR’ columns. Each regression is estimated with bank holding company fixed effects. Cross-section clustered standard errors are in parentheses. Intercept and coefficients on control variables are estimated, but not reported. ***, ** and * denote statistical significance at 10%, 5% and 1%, respectively.

that the quantity of loans demanded will fall, enabling the bank to reduce its asset size (in the case of a binding capital constraint) or to move funds into liquid assets (if the liquidity constraint is binding). Second, the capital-to-asset ratio can be improved as loans will generate a higher return, which, all else equal, allows the bank to retain more earnings and thus increase capital.

In this paper, I empirically test the following predictions of the model: (i) capital constrained banks charge higher loan rates than unconstrained banks; (ii) the pass-through of the federal funds rate into bank loan rates is lower for capital constrained banks (in falling as well as rising rate periods); and (iii) the loan rates of capital constrained banks are more sensitive to fluctuations in the aggregate economy. Regression estimates using bank holding company data over 2002-2012 support predictions (i) and (ii) of the model. Since the 2007, prediction (iii) is also supported. In addition, it follows from the model that banks are more likely to become capital constrained during falling rate periods than during rising rate periods, which is also supported by the data. The facts that more banks are capital constrained during falling rate periods, and that capital constrained banks reduce interest rate pass-through, together explain the asymmetric pass-through between falling and rising rate periods.
The finding that traditional bank regulation induces this loan rate asymmetry can be viewed as an argument in favor of macroprudential, rather than microprudential, financial regulation (Hanson et al., 2010). In particular, regulators may want to adopt countercyclical capital and liquidity requirements (lower requirements during expansionary monetary episodes). The Basel Committee has already proposed a countercyclical capital buffer as part of the Basel III Accord. In its design of liquidity requirements, in particular the liquidity coverage ratio, the Committee may also want to consider countercyclical requirements. Additionally, it follows from the model that there may be a clash of interests between the monetary policy authority and bank regulators, as argued by Calomiris (2010).

References


Appendix A

In this appendix I show the relative importance of the capital and liquidity constraints. Constraints (8) and (9) are plotted in Figure 6. It can be seen that after a drop in the value of bank capital, banks with smaller loan portfolios relative to deposits, will become capital constrained first, whereas banks with a relatively large amount of loans will run into their liquidity constraint first. The value of loans at
which both constraints intersect in Figure 6 is denoted by $L_A$. This value can be derived by equating (8) and (9), yielding

$$L_A = \frac{\kappa_0 + (1 - \kappa_0)(1 - \lambda)}{1 - \kappa_1}D.$$  \hspace{1cm} (34)

Increasing either or both of the marginal capital requirements, $\kappa_0$ and $\kappa_1$, will increase $L_A$, meaning that the range of loans (relative to deposits) for which the capital constraint will be the first to bind increases. Similarly, increasing the marginal liquidity requirement, $\lambda$, lowers $L_A$, making the liquidity constraint the first constraint to bind for lower levels of loans.

Figure 6: Liquidity and Capital Constraints for $0 \leq \kappa_0 < \kappa_1 \leq 1$ and $0 \leq \lambda \leq 1$

Perhaps more intuitively, the order in which the liquidity and capital constraints will bind can be expressed in terms of leverage of the bank’s balance sheet. Leverage is the ratio of total assets to capital. For given values of capital, deposits and liquid assets, a bank will run into its capital
constraint before its liquidity constraint if \( L_\kappa < L_\lambda \). Using (10) and (11), \( L_\kappa < L_\lambda \) is\(^{16}\)

\[
D > \frac{1 - \kappa_1}{\kappa_1 - (\kappa_1 - \kappa_0) \lambda} K.
\]

(35)

To express this in terms of leverage, recall that total assets is made up of loans and liquid assets, and \( L + R = D + K \) at all times. Add \( K \) to both sides of (35), and it shows that if \( L_\kappa < L_\lambda \) it must be that

\[
\text{Leverage} > \frac{1 - (\kappa_1 - \kappa_0) \lambda}{\kappa_1 - (\kappa_1 - \kappa_0) \lambda}.
\]

(36)

where \( \text{Leverage} = \frac{\text{Total Assets}}{\text{Capital}} = \frac{L+R}{K} \). For \( \kappa_0 = 0 \), \( \kappa_1 = 0.10 \), and \( \lambda = 0.15 \), this would mean that a bank will run into the capital constraint before the liquidity constraint if its leverage is 11.6 to 1 or higher.

**Appendix B**

The purpose of this appendix is to derive the marginal value of capital, \( V_K \), following the same approach as Hennessy (2004). First of all, to simplify notation, define the infinitesimal generator \( B \). For an arbitrary, twice-differentiable function \( V \) of \((K, \xi)\), Ito’s lemma implies that

\[
E[dV] \frac{1}{dt} \equiv B[V] = (\pi(K,\xi) - rE + q)V_K + \mu(\xi)V_\xi + \frac{1}{2} \sigma^2(\xi)V_{\xi\xi}.
\]

(37)

Using generator \( B \), (13) can be written as

\[
m \cdot V(K,\xi) = \max_{r^f, r^k, q} \pi(K,\xi) + B[V]
\]

\[
+ v_\lambda \cdot (K - L + (1 - \lambda)D) + v_\kappa \cdot \left( K - \frac{\kappa_0}{1 - \kappa_0} D - \frac{\kappa_1 - \kappa_0}{1 - \kappa_0} L \right),
\]

where the last two terms disappear, since by the definition of Lagrange multipliers, either \( v_\lambda \) and \( v_\kappa \) are zero when the constraints are nonzero, or the constraints are binding, in which case the terms in

\(^{16}\)For this inequality to hold, it must be that \( \kappa_0 > \kappa_1 - \frac{\kappa_0}{\lambda} \). Under realistic circumstances, this will always be the case, as \( \kappa_1 - \frac{\kappa_0}{\lambda} < 0 \) and \( \kappa_0 \in [0, 1) \).
parentheses equal zero. Hence,

\[ m \cdot V(K, \xi) = \max_{r^E, r^q, q} \pi(K, \xi) + B[V]. \tag{38} \]

Since this Bellman equation holds identically in \( K \) for all \((K, \xi)\), the derivatives w.r.t. \( K \) of the left and right sides of (38) must match. Evaluating at the optimum and differentiating w.r.t. \( K \) gives

\[ m \cdot V_K(K, \xi) = \frac{\partial \pi^*(K, \xi)}{\partial K} + \frac{\partial \{ \pi^*(K, \xi) - r^E E + q^* \}}{\partial K} V_K + \left[ \pi^*(K, \xi) - r^E E + q^* \right] V_{KK} \]

\[ + \mu(\xi) V_{K\xi} + \frac{1}{2} \sigma^2(\xi) V_{K\xi\xi}. \]

By the envelope theorem \( \frac{\partial r^*_E}{\partial K} = \frac{\partial r^*_E}{\partial K} = \frac{\partial q^*}{\partial K} = 0 \), from which it follows that \( \frac{\partial \pi^*(K, \xi)}{\partial K} = r^T \). Hence,

\[ m \cdot V_K(K, \xi) = r^T + r^T V_K + B[V_K]. \tag{39} \]

Since

\[ K \cdot B[V_K] = K \left[ \left( \pi(K, \xi) - r^E E + q \right) V_{KK} + \mu(\xi) V_{K\xi} + \frac{1}{2} \sigma^2(\xi) V_{K\xi\xi} \right] \]

and \( B[K \cdot V_K] = (\pi(K, \xi) - r^E E + q) (V_K + K \cdot V_{KK}) + K \left[ \mu(\xi) V_{K\xi} + \frac{1}{2} \sigma^2(\xi) V_{K\xi\xi} \right] \),

we can use the fact that

\[ K \cdot B[V_K] = B[K \cdot V_K] - (\pi(K, \xi) - r^E E + q) V_K \]

as we multiply (39) by \( K \) to obtain

\[ m \cdot K \cdot V_K = K \cdot r^T (1 + V_K) + B[K \cdot V_K] - (\pi(K, \xi) - r^E E + q) V_K. \tag{40} \]
Subtracting (40) from (38) and rearranging gives

\[ B[V(K, \xi) - K \cdot V_K] - m \cdot [V(K, \xi) - K \cdot V_K] = K \cdot r^T (1 + V_K) - \pi(K, \xi) - (\pi(K, \xi) - r^E_t E + q) V_K. \]

(41)

Now define the Itô process \( \zeta \) as

\[ \zeta(K, \xi, t) = \exp(-mt) [V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)]. \]

By Itô’s lemma,

\[ d\zeta(K, \xi, t) = e^{-mt} \{ B[V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)] - m \cdot [V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)] \} dt \]

\[ + e^{-mt} \sigma(\xi) [V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)] dW_t. \]

(42)

Substituting (41) into (42) yields

\[ d\zeta(K, \xi, t) = e^{-mt} \{ K_t \cdot r^T_t (1 + V_K(K, \xi, t)) - \pi(K, \xi, t) - (\pi(K, \xi, t) - r^E_t E_t + q_t) V_K(K, \xi, t) \} dt \]

\[ + e^{-mt} \sigma(\xi) [V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)] dW_t. \]

(43)

Integrating \( \zeta \) up to the optimal default date of the bank, \( t_1 \), and taking expectations gives

\[ E[V(K, \xi, t_1) - K_{t_1} \cdot V_K(K, \xi, t_1)] = V(K, \xi, t_0) - K_0 \cdot V_K(K, \xi, t_0) \]

\[ + E \left[ \int_{t_0}^{t_1} e^{-mt} \{ K_t \cdot r^T_t (1 + V_K(K, \xi, t)) - \pi(K, \xi, t) - (\pi(K, \xi, t) - r^E_t E_t + q_t) V_K(K, \xi, t) \} dt \right] \]

\[ + E \left[ e^{-mt} \sigma(\xi) [V(K, \xi, t) - K_t \cdot V_K(K, \xi, t)] dW_t \right], \]

(44)

where the last term is a martingale with expectation zero, as long as the integral is well defined. In particular, the bank must choose a policy for the loan rate, dividends and share issues, such that \( \zeta \) does not increase faster in \( \xi \) than the probability of \( \xi \) decreases for increasing (decreasing) values of \( \xi \).

Substituting in the value matching and smooth pasting conditions at the default date, \( V(K, \xi, t_1) = 0 \).
and \( V_K(K, \xi, t_1) = 0 \). (44) simplifies to

\[
K_0 \cdot V_K(K, \xi, t_0) = V(K, \xi, t_0)
\]

\[
+ \mathbb{E} \left[ \int_{t_0}^{t_1} e^{-m_s} \left\{ K_t \cdot r_t^T (1 + V_K(K, \xi, t)) - \pi(K, \xi, t) - (\pi(K, \xi, t) - r_t^E_s + q_t) V_K(K, \xi, t) \right\} dt \right].
\]

(45)

At this point, we can find the current marginal value of capital by evaluating (45) at start date \( t_0 \) instead of \( t_0 \) and by dividing both sides by \( K_t \). Then evaluating the control variables at their optimal values shows that the current marginal value of capital, \( V_{K,t} \), depends not only on the current value of the bank and the current value of bank capital, but also on the expected optimal path for the bank between now (\( t \)) and default (\( t_1 \)):

\[
V_K(K, \xi, t) = \frac{V(K, \xi, t)}{K_t}
\]

\[
+ \frac{1}{K_t} \mathbb{E} \left[ \int_{t}^{t_1} e^{-m_s} \left\{ K_s \cdot r_s^T (1 + V_K(K, \xi, s)) - \pi^*(K, \xi, s) - (\pi^*(K, \xi, s) - r_s^{E^*} + q_s^*) V_K(K, \xi, s) \right\} ds \right],
\]

(46)

where, substituting the loan demand function (1) into total loans, \( L \),

\[
\pi^*(K, \xi, t) = (r_t^{L^*} - c^L) \left( \gamma_0 - \gamma_1 r_t^{L^*} + \gamma_2 Y_t + \epsilon_t^L \right)
\]

\[
+ r_t^T \left[ D_t + K_t - \left( \gamma_0 - \gamma_1 r_t^{L^*} + \gamma_2 Y_t + \epsilon_t^L \right) \right] - r_t^D D_t - G(q_t^*) + H \left( r_t^{E^*} \right),
\]

and \( r^{E^*}, q^* \) and \( r^{L^*} \) are given by (19), (20) and (26), respectively.