Developing price discovery measures within a volatility-based model

(Second Draft, 08/28/2012) *

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Abstract

Using a volatility-based model, our study extends the Hasbrouck (1995) information share methodology in three ways: uniquely determining the information share, proposing a time dependent information share, and adjusting the information share to volatility spillover among idiosyncratic shocks. When applying the model to Chinese stock index and index futures markets, we are able to discover the dual role played by liquidity and uncover the complex pattern of price discovery process. We also find that the market information shares experienced turbulent changes at the beginning of the infancy stage of the futures market.

JEL CODE: C32, C51, G13, G14

Keywords: price discovery; unique information share; time dependent information share; volatility spillover adjusted information share

* Previously circulated as “Use VECM-Structural GARCH to Study Price Discovery, with an Application to Chinese Stock Index and Stock Index Futures Markets”.

** We thank Professor Jian Yang for his inspiring seminar and generous help in providing data. Thank Jinfei Sheng and Lanlan Chu for their helpful suggestions. Special thanks to Hui Liu for his help in math.
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1. Introduction and literature review

When the same or similar assets are traded in multiple markets, we want to know where price discovery happens, that is, which market absorbs fundamental information first. Since these prices are based on the same fundamental values, according to law of one price, arbitrage should keep them close to each other. For this reason, VECM models are commonly used in price discovery studies, and the Hasbrouck (1995) information share is the most widely used framework out of them (e.g. Tse (1999), Huang (2002), Forte and Peña (2009)). It measures each market’s price discovery ability with information shares, defined as variance decomposition of the common stochastic trend innovation with respect to corresponding shocks identified through Cholesky factorization.

Using a volatility-based model, our paper extends the Hasbrouck (1995) information share framework in three ways.

First, it uniquely identifies the shocks, thereby determining unique Hasbrouck (1995) type information shares. Non-uniqueness of the information shares was perceived as the major drawback of Hasbrouck (1995) (see Baillie et.al (2002), de Jong (2002), Harris et.al (2002), Lehmann (2002)), because it impedes further quantitative analysis of price discovery with other variables, and sometimes even makes a clear-cut qualitative judgment infeasible (e.g. Huang (2002)). To address this shortcoming, Lien and Shrestha (2009) proposed a factorization structure based on the residuals’ correlation matrix, and Grammig and Peter (2008, 2010) proposed an identification method under the assumption that the residuals follow mixture of normal distribution. While they could both determine unique information shares, their underlying economics intuition was not year clear. Comparatively speaking, our identification method has the additional benefit of strong economics intuition besides
generating unique information shares measures. Moreover, it is further advantageous in its mild condition requirement, which makes it applicable in nearly every situation, even in those where the markets are so closely connected that Grammig and Peter (2008) may fail to work.

Second, we propose a time dependent information share by replacing the shock variances in Hasbrouck (1995) with their conditional variances, thereby naturally extending the static Hasbrouck (1995) measure into a dynamic one. Such a dynamic measure is desirable when one is interested in studying inter-temporal relationship between price discovery and other variables like liquidity, public information, and basis; or if one wants to identify price discovery trends during certain particular periods, such as the infancy stage or crisis period of a market. Previously, several studies have also proposed their versions of dynamic price discovery measures. Specially, Avino et.al (2011) proposed to measure time dependent information share with residual conditional variances. While similar to our method, theirs is plagued by the conditional correlation in residuals, which made their measure dependent on ordering, and forced them to use midpoint outcome as an approximation. By contrast, our model is immunized from this shortcoming. Taylor (2011) proposed a dynamic price discovery measure within a time varying coefficients model. Our method is different from his in our direct focus on messages in the residuals, i.e. the innovation generating part of the markets. Another dynamic information share model was Fricke and Menkhoff (2011), where they used ultra-high frequency data to determine daily information shares. Yet, as suggested by Grammig and Peter (2008), the high-level contamination from microstructure noise in the high frequency data might have biased their result. Since our model does not rely on ultra-high frequency data, we do not have such a problem.

Third, we propose a volatility spillover adjusted information share by further exploring the economic implication of volatility spillover among shocks, and the new measure offers a new perspective to understand price discovery process. Although volatility spillover and information share were both commonly used as cross-market information relationship indicators in previous studies, they were modeled separately,
and the former was typically interpreted as a short-run indicator that has no long-run effect (e.g. see Chan, Chan and Karoyi (1991), Tse (1999), Yang et.al (2001, 2011), Zhong et.al (2004)). We argue in this paper that volatility spillover does have long-run effect in term of its impact on information share, and Hasbrouck (1995) type information share, by ignoring it, essentially measures how each market contributes to price discovery through information interpretation, rather than information dissemination, yet these two functions are different, and differentiating between them could help us identity the pattern of and deepen understanding on price discovery process. To our limited knowledge, we are the first to explore the relationship between volatility spillover and information share, and differentiate information interpretation from information dissemination.

To illustrate the methodology, we revisit the study by Yang et.al (2011) on price discovery between Chinese stock index and index futures markets¹. Our new findings well demonstrate the model’s advantages. With the unique information share measure and the volatility spillover adjusted information share, we find that although the futures market can discover information more quickly, it is the spot market’s interpretation that constitutes the major part of the common efficient price. Combined with Yang et.al (2011)’s finding that when there is disparity between prices, futures price adjusts to the more “right” spot price. We uncover a complex pattern of the price discovery process. It is argued that the dual role played by liquidity, which was also found by other studies (e.g. Grammig and Peter (2010) and Yan and Zivot (2010)), be the driving force underlying this pattern. In addition, using our dynamic price discovery measure, we find that each market’s information share exhibited turbulent changes at the beginning and became relatively stable later on. Since the sample period corresponds to the infancy stage of the futures market. It is conjectured that such time series trend be due to market participators’ gradual learning of futures market as their investment tool.

The remainder of the paper is organized as follows: section 2 discusses model specification, identification and estimation. Then, we propose and discuss our new

¹ Deeply grateful to Professor Jian Yang for kindly providing the data.
price discovery measures in the following three sections: respectively the modified information share in section 3, the time dependent information share in section 4 and the volatility spillover adjusted information share in section 5. The empirical application is presented and discussed in section 6. At last, section 7 concludes.

2. Model specification, identification and estimation

2.1 Specification

When the same or similar assets are traded in n markets, according to law of one price, the dynamics of these n prices \( P_t = (P_{1,t} \ldots P_{n,t}) \) can be described by a VECM of order q with cointegration rank of n-1:

\[
\Delta P_t = \alpha (\beta^T P_{t-1} - Z) + \Gamma_1 \Delta P_{t-1} + \ldots + \Gamma_{q-1} \Delta P_{t-q+1} + B \mu_t
\]

(2.1)

where \( \alpha \) and \( \beta \) are both \( n \times n-1 \) matrices of rank n-1, with \( \alpha \) describing the adjustment coefficients, and \( \beta \) the n-1 independent cointegration relationship.

Using the first price as benchmark, \( \beta \) can be normalized as \( \beta^T = [l_{n-1}^T I_{n-1}] \), where \( l_{n-1} \) is a \( n-1 \) dimensional unitary vector, and \( I_{n-1} \) is a \( (n-1) \times (n-1) \) identity matrix. \( Z \) is a \( n-1 \) dimensional constant vector that captures the systematic price differences. \( \Gamma_1 \ldots \Gamma_{q-1} \) are \( n \times n \) matrices that describe the short run price dynamics.

The \( n \times n \) matrix \( B \) denotes the contemporaneous impact of the n dimensional shock vector \( \mu_t \) on the prices. The shocks have zero means, and their variance processes will be introduced later. Both \( B \) and \( \mu_t \) are identified from the reduced form VECM residuals \( \epsilon_t \), with \( \epsilon_t = B \mu_t \), and the identification method will also be introduced later.

Note that no zero restrictions are imposed on \( B \), implying that each shock is allowed to affect every price contemporaneously. Such a specification is particularly appealing for price discovery study, where the markets are so closely connected that it
makes no sense to assume, like the Cholesky case in Hasbrouck (1995), that one contemporaneously affects another, but not vice versa.

After specifying price level dynamics, we proceed to model shock variance processes. Specifically, we assume that they are contemporaneously and serially uncorrelated:

\[ E(\mu_i, \mu_j) = 0, E(\mu_i, \mu_{j,t-1}) = 0, E(\mu_i, \mu_{j,t-1}, \mu_j) = 0, \text{ for } i \neq j \text{ and } s \neq 0 \]  \hspace{1cm} (2.2)

In addition, their conditional variances follow GARCH(1,1)\(^2\) processes:

\[ \mu_i = \sqrt{h_{i,t}} \phi \]  \hspace{1cm} (2.3)

\[ h_t = (I - Vec(\Lambda_s) - Vec(\Phi)) + \Lambda \mu^2_{i,t} + \Phi h_{i,t} \]  \hspace{1cm} (2.4)

where the \( \phi \)s are independent stochastic processes with zero means and unitary variances. \( h_t = (h_{1,t}...h_{n,t}) \) is the \( n \) dimensional conditional variance vector of the shocks. \( \Lambda \) is a \( n \times n \) matrix, and \( Vec(\Lambda_s) \) is a \( n \) dimensional vector whose \( i \)th element is the sum of elements in the \( i \)th row of \( \Lambda \). \( \Phi \) is a \( n \times n \) diagonal matrix, and \( Vec(\Phi) \) is a \( n \) dimensional vector whose \( i \)th element is the \( i \)th diagonal element in \( \Phi \). \( l \) is a \( n \) dimensional unitary vector. The term \( I - Vec(\Lambda_s) - Vec(\Phi) \) in equation (2.4) serves to normalize the unconditional variance of each shock to unity.

To ensure positive and finite second moments, all elements in \( \Lambda \) and \( \Phi \) should be non-negative, and all elements in the vector \( I - Vec(\Lambda_s) - Vec(\Phi) \) should be positive.

Note that when some of non-diagonal elements in \( \Lambda \) are positive, volatility of one shock can spill over to another. Besides contemporaneous and lead-lag price level interaction in equation (2.1), it forms another channel through which the markets may interact with each other.

2.2 Identification

As proved by Sentana and Fiorentini (2001), if \( B \) is of full rank, and if the conditional

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\(^2\) GARCH(1,1) is used because of its simplicity and good performance reported in many studies. More lags can be used in the model, and it will not influence the analysis below.
variances of the shocks are linearly independent (i.e. $\exists \bar{M} \in \mathbb{R}^n, M \neq 0 : M h_t = 0, \forall t$), the model is identified up to column permutation and column sign change in $B$. In light of the relationship between volatility and information flow (See Ross (1989), Fleming and Remolona (1999), Melvin and Yin (2000), and Kalev et.al (2004) for details)), it implies that as long as the amounts of information interpreted by different markets are not in constant proportion all the time, identification is achieved up to such degree.

To uniquely identify the model, we further assume that the first row of $B$ is positive, and that the shock associated with the $i$th market is the one whose corresponding column in $B$ has the largest absolute value at its $i$th element. While the former assumption is just a normalization method to address the column sign change in $B$, the latter can save us from the column permutation, and it implies that each shock have the largest contemporaneous effect on its own price, a quite plausible assumption.

Up till now, our model is fully identified. It can be seen that the identification condition is rather mild, which makes it applicable in nearly every situation---even in the cases where market prices follow each other so closely that Grammig and Peter (2008)'s model might fail to work.

As mentioned in the introduction, our identification method has the benefit of being rather intuitive, and its rationale goes as follows: changes in the amounts of information interpreted by different markets induce changes in the relative shock variances. Under the assumption of a stable contemporaneous price level relationship (a constant $B$), the latter changes in turn induce changes in residual correlations, through which the model is identified (See Rigobon and Sack (2003) for graphic illustration on how changes in residual correlation can help achieve identification).

2.3 Estimation

The model’s log likelihood function in Gaussian case is given by:

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3 In Grammig and Peter (2008)'s study on price discovery of Canadian stocks cross-listed in U.S. market, they successfully determined unique information shares for 67 stocks out of total 69 samples. Yet, the other two stocks' information shares cannot be uniquely determined because the relative variances across regimes are so similar that their identification method failed to work.
\[ L(\Theta) = -\frac{1}{2} \sum_{t=1}^{T} (n \log 2\pi + \log |\Omega_i| + \epsilon_i^T \Omega_i \epsilon_i) \]  \hspace{1cm} (2.5)

where \( \Theta \) includes all the parameters to be estimated. Following Tse (1999), we recommend a two-step estimation procedure. The first step is to estimate the reduced form VECM model equation (2.1). Then, in the second step, the structural GARCH parameters in equation (2.4) are estimated using maximum likelihood method. In this step, the residuals’ unconditional covariance matrix is targeted at \( \Omega \), their sample covariance matrix estimated from the first step. Through variance targeting, we are able to ensure consistency between shocks’ sample variances and their unconditional variances in GARCH, so that we can use them interchangeably\(^4\).

### 3. Modified information share

In the section, we will develop and discuss the modified information share, a uniquely-determined Hasbrouck (1995) type information share.

Transform equation (2.1) into its moving average representation and rearrange the terms, we get:

\[ P_t = P_0 + \Psi(1) \sum_{i=1}^{L} B_{\mu_i} + \Psi(1)^T B_{\mu_i} \]  \hspace{1cm} (3.1)

where \( \Psi(1) \) is a \( n \times n \) matrix. In this equation, the stochastic trend component \( \Psi(1) \sum_{i=1}^{L} B_{\mu_i} \) follows random walk and corresponds to the fundamental value concept, which is also known as efficient price. Because the \( n \) prices have the same fundamental value, \( \Psi(1) \) has identical rows.

Let \( \psi^T = (\psi_1 \ldots \psi_n)^T \) denote the common row vector in \( \Psi(1) \), then the innovation in the common stochastic trend (or the fundamental value) is given by:

\[ CI_t = \psi^T B_{\mu_i} \]  \hspace{1cm} (3.2)

From equation (3.2), it is clear that the innovation in the common stochastic trend, i.e.

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\(^4\) The DCC model by Engle (2002) uses similar targeting technique when estimating correlation parameters.
innovation in the fundamental value, is driven by the shocks, i.e. news interpreted by corresponding markets. The aim of price discovery study is to measure the proportion of changes in the former innovation driven by each individual shock. To this end, we first deduce from equation (3.2) the relationship between the variance of the innovation and that of the shocks, which is given by:

\[ V = \psi^T B \Delta_n B^T \psi = \sum_{i=1}^{n} \psi^T B_i B_i^T \psi \]  

(3.3)

where \( V \) is the variance of the innovation in the common stochastic trend. \( B_i \) is the \( i \)th column in \( B \). The \( n \times n \) diagonal matrix \( \Delta_n \) is the covariance matrix of the orthogonal shocks, and in our model, and it is normalized to identity (see equation (2.4)), which justifies the second equation in (3.3).

Then, following Hasbrouck (1995), we define a market’s modified information share\(^5\) as:

\[ MIS_i = \frac{\psi^T B_i B_i^T \phi}{\phi^T B B^T \phi} \]  

(3.4)

From equation (3.4), a market’s modified information share is its weight in the variance of the innovation in the common stochastic trend. Intuitively, it measures the proportion of fundamental information first interpreted by the market. From (3.3), it is easy to prove that the sum of all markets’ information shares equals one.

4. Time dependent information share

As our GARCH model can estimate the conditional variances of the shocks dynamically, we use them to estimate time dependent information shares. Similar to equation (3.5), a market’s time dependent information share is defined as:

\[ MIS_{ij} = \frac{\phi^T B_i h_j B_j^T \phi}{\phi^T BH_i B_i^T \phi} \]  

(4.1)

\(^5\) It is labeled “modified” so that we can distinguish it from Hasbrouck (1995), whose information share, unlike ours, is not unique and depends on the ordering of Cholesky factorization.
where $H_i$ is a diagonal matrix, whose $ith$ diagonal element is the conditional variance of the $ith$ shock, $h_{ii}$. Using the shock’s conditional variance, an indicator of the market’s dynamic information amounts, to replace its variance, an indicator of the market’s average information amounts over a period, our time dependent information share offers a natural dynamic extension to the static Hasbrouck (1995) information share. Accordingly, we can measure price discovery at every time spot instead of merely within a time interval. In addition, it is worth mentioning that these time dependent information shares are unique and not subject to influence from ordering. As for their sums, they have the desirable property of always equaling one.

5. Volatility spillover adjusted information share

In this section, we first demonstrate why we should consider volatility spillover among shocks when measuring information share, then we show how to adjust the latter to the former.

To see “why”, we first show how volatility spillover among shocks could affect the information share measures, that is, volatility spillover’s long run effect.

Take expectation on the both sides of equation (2.4) and rearrange its terms. Then, the (unconditional) variance of the $ith$ shock $h_i$ (the $ith$ diagonal element of $\Lambda_{\varnothing}$ in equation (3.3)) is given by:

$$h_i = \frac{1 - \Lambda_{\varnothing} - \Phi_i}{1 - \Lambda_{\varnothing} - \Phi_i} + \sum_{j \neq i} \frac{\Lambda_{\varnothing} h_j}{1 - \Lambda_{\varnothing} - \Phi_i}$$  \hspace{1cm} (5.1)

where the symbol $'$ denotes that the term $j=i$ is excluded, $\Lambda_{\varnothing}$ denotes the sum of the $ith$ row in $\Lambda$, and $\Phi_i$ denotes the $ith$ diagonal element in $\Phi$. From equation (5.1), it is clear that when some of the coefficients in its last term are positive, volatility spillover from the other shocks may increase the $ith$ shock’s variance, which, from equation (3.3) and (3.4), would in turn increase the $ith$ market’s Hasbrouck (1995) information share or modified information share.
Volatility spillover among shocks does not only affect the markets’ information shares, but also has its own economic implication. Specially, in light of the relationship between volatility spillover and information flow, it implies that the information first interpreted by a market into its shock may not necessarily first disseminates there, instead, it may flow from other markets. Thus, ignoring this point, the traditional information shares do not measure how much fundamental information first disseminates in a market. To measure it, we now turn to the “how” problem.

Let \( h_0 = (h_{0,1}, \ldots, h_{0,n}) \) denote the “own” variance vector, with \( h_{i,j} = \frac{1 - \Lambda_{i,j} \cdot \Phi_i}{1 - \Lambda_{i,i} \cdot \Phi_i} \), the first term in equation (5.1). These “own” variances are the shock variances without volatility spillover effect. Intuitively, they measure the amounts of information that first disseminates in a particular market. Further let \( \Pi \) denote the volatility spillover coefficient matrix in the last term of equation (5.1), with all diagonal elements zero and \( \Pi_{i,j} = \frac{\Lambda_{i,j}}{1 - \Lambda_{i,i} \cdot \Phi_i} \) (for \( i \neq j \)). The relation between \( h \) and \( h_0 \) is given by:

\[
h = h_0 + \Pi h
\]

Rearrange its terms, we have:

\[
h = (I_n - \Pi)^{-1} h_0
\]  \(6\)

From equation (5.3), the variances of the shocks are solely the function of the “own” variances. To gauge the impact of the ith “own” variance on every shock \( h_{j,i} = (h_{j,i,1}, \ldots, h_{j,i,n}) \), we need only to compute \((I_n - \Pi)^{-1} h_0\) with all the other “own” variances set to zero, and it is easy to prove that:

\[
\sum_{i=1}^{n} h_{j,i} = h.
\]  \(5.4\)

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\(6\) See it in another way, the volatility-spillover matrix \( \Pi \) in equation (3.8) works like a multiplier. Every time, the information initially disseminating in each market first flows to other markets. As a result, their shock variances increase accordingly. Since the expected “own” variance is \( h_0 = (h_{0,1}, \ldots, h_{0,n}) \), each shock’s expected variance increase by \( \Pi h_0 \). Then these additional variances (reinterpreted information) are further transmitted to each market, which enlarge their variances by \( (\Pi)^2 h_0 \). This feedback process continues indefinitely. At last, each market’s variance is given by \( h = \sum_{n=0}^{\infty} (\Pi)^n h_0 = (I_n - \Pi)^{-1} h_0 \), the same as equation (3.9).
Further from equation (3.3), the variance of common stochastic trend innovation can be expressed as a function of $h_{f,i}$, that is:

$$V = \psi^T \Delta_x B^T \psi = \sum_{i=1}^{n} \psi^T B \text{dia}(h_{f,i}) B^T \psi$$  \hspace{1cm} (5.5)$$

where $\text{dia}(h_{f,i})$ is a $n \times n$ diagonal matrix, whose ith diagonal element is $h_{f,i}$.

In equation (5.5), since $h_{f,i}$ is solely the function of $h_{n,i}$, the variance of common trend innovation is ultimately the function of $h_{n,i}$, too. This result implies that innovations in the fundamental value can be fully traced back to their original source markets. To measure the proportion of fundamental information first disseminating in the ith market, the volatility spillover adjusted information share is defined as:

$$VSAIS_i = \frac{\phi_i^T B[\text{dia}(h_{f,i})] B^T \phi}{\phi_i^T B B^T \phi}$$  \hspace{1cm} (5.6)$$

It is easy to prove that the sum of all the markets’ volatility spillover adjusted information share is one.\footnote{Since $\sum_{i=1}^{n} h_{f,i} = h = l$, $\sum_{i=1}^{n} \text{dia}(h_{f,i}) = \begin{pmatrix} h_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_n \end{pmatrix} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix}$, it follows that:

$$\sum_{i=1}^{n} VSAIS_i = \sum_{i=1}^{n} \frac{\phi_i^T B[\text{dia}(h_{f,i})] B^T \phi}{\phi_i^T B B^T \phi} = \sum_{i=1}^{n} \frac{\phi_i^T B B^T \phi}{\phi_i^T B B^T \phi} = 1.$$}

Here, we want to stress again the difference between volatility spillover adjusted information share, and the Hasbrouck (1995) information share and the above modified information share. Ignoring volatility spillover among shocks, the latter two measures how much fundamental information is first interpreted in a market, no matter where the information comes from, which by contrast is just the focus of the former. In practice, these two types of price discovery measures may produce qualitatively different results due to differences in markets’ information quality, as will be shown in the following empirical application section.

At last, it is worth mentioning that following similar steps as equation
(5.4)-(5.6), we can also quantify volatility spillover among residuals. Since it is not directly related as price discovery, we put it the Appendix. Yet, it should not be undervalued just because of its location. The quantification approach we employ marks a big step in volatility spillover study, and it should be useful when studying information flow and/or sources of instability among financial markets within VAR or other VECM framework, where no single common prices could be found, and thus the information share approach is infeasible.

6. Empirical application

6.1 Literature review

In section 6, we will apply our new measures to price discovery study on Chinese stock index and index futures markets. Previously, several studies have analyzed price discovery relationship and/or volatility spillover between stock index and index futures markets in other countries (e.g. Stoll and Whaley(1990), Chan, Chan and Karoyi (1991), Tse (1999), Alphonse (2000), Zhong et.al (2004), So and Tse (2006)). They often found that the futures market dominated the spot market in price discovery, and there exists bidirectional volatility spillover between the two markets. As for Chinese stock index and index futures markets, Yang et.al (2011) made a detailed study. Using five minutes data from April 16th, 2010 to July 30th, 2010 (the infancy stage of stock index futures market), they found bidirectional volatility spillover between these two markets. Yet, contrary to previous studies, the spot market, but not the futures market, was found to play the dominant role in price discovery. They suggested that the possible explanation for the spot market’s dominance might be the high entry barriers in stock index futures market due to regulation.

In Yang et.al (2011), the insignificance of adjustment coefficient of cointegration relationship for the spot market equation was taken as evidence of its leadership in price discovery. The rationale is that the market with the more “right” price does not adjust or adjusts slowly to disparity between prices, while the “wrong” market tend to adjust relatively quickly to the more “right price”. In Yang et.al (2011), the messages
in residuals were ignored. Using our new model and information share measures, we aim to complement their study by making further exploration along this line.

6.2 Data

We use the same data set as Yang et.al (2011). For ease of reading, the data and the construction method are reintroduced here.

The CSI 300 Index was created on April 16th, 2005. It consists of 300 large-capitalization and actively traded stocks listed on the Shanghai or Shenzhen Stock Exchanges, and represents about 70% of the total market capitalization of both stock exchanges. The CSI 300 Index futures contract was launched on April 16th, 2010 on the China Financial Futures Exchange. The expiration day of the CSI 300 index futures contact is the third Friday of the contract (delivery) month, and the contract (delivery) months include the current month, the next month, and the final months of the next two quarters.

The sample period is from April 16th, 2010, to July 30th, 2011. Since the nearby contract is the most actively traded, only data for the nearby futures contract are used. To construct continuous nearby futures price series, intraday prices for the nearby futures contract are used until the contract reaches the first day of the delivery month. Then, prices for the next nearby contract are used. Both the Shanghai and Shenzhen Stock Exchanges open from 9:30 a.m. to 11:30 a.m. and then from 1:00 p.m. to 3:00 p.m. (Beijing Time), while the trading hours of the CSI 300 index futures contract are from 9:15 a.m. to 11:30 a.m., and from 1:00 p.m. to 3:15 p.m. (Beijing Time). The last records of futures prices in a trading day are respectively registered at 11:25 a.m. for the morning session and 3:10 p.m. for the afternoon session. Spot and futures prices recorded before either the stock or futures exchange opens or after either of them closes are excluded, and there are 49 recorded spot and futures prices at five-minute intervals during each trading day. After eliminating weekends and holidays during which the trading was closed, a total of 3,528 five-minute observations are obtained. The returns of each series are calculated by taking first differences of the logarithms of prices. The summary statistics of returns are given in Table 1.
6.3 Estimation and discussion

The estimation follows the two-step method described in section 2.3. The first step involves testing and estimating the VECM model of equation 2.1, with the futures price as \( P_1 \) and the spot price as \( P_2 \). Following Yang et.al (2011), Augmented Dickey Fuller test suggests both price series follow I(1) process. The optimal lags length of the VECM model is four according to AIC criteria. Johansen (1991) trace test indicates the existence of one cointegration relationship between them. LR test cannot reject the theoretical cointegration relation \( \beta^T = (1,-1) \), and the constant \( Z \) is estimated to be 0.008. LR test on adjustment coefficients rejects the null hypothesis of \( \alpha_1 = 0 \) but cannot reject \( \alpha_2 = 0 \). Following Yang et.al (2011), the result \( \alpha_2 = 0 \) suggests that the spot market is dominantly better at following the right price. When there is disparity between prices, the futures market adjusts to the more “right” spot market, but not vice versa.

Based on the above test results, the reduced form VECM model is estimated with four lags and with restrictions of \( \beta^T = (1,-1) \), \( Z = 0.008 \) and \( \alpha_2 = 0 \). Estimation is performed using Cats in Rats software.

VECM model residuals are highly correlated with a correlation of 0.771. Using Cholesky factorization, the upper bound of the information share for the futures market is 59.7%, and the lower bound is zero. As the upper bound is larger than 50% yet the lower bound is less than 50%, we cannot make clear-cut qualitative judgment based on Cholesky factorization.

Preliminary estimation of the residuals’ conditional variance processes, with each using univariate GARCH (1, 1), suggests that they are quite different in conditional heteroskedasticity, see Figure 1. From the figure, it is clear that the ratios of the futures residual’s conditional variance to the spot residual’s conditional variance are

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8 As the same testing steps were performed in Yang et.al (2011), and the results are the same with theirs, to save space, it is not reported here. But they are available on request.
constantly changing. As residuals are linear combinations of shocks, it suggests that the relative shock variances are also changing. Thus, no surprise, the preliminary result suggests that our model is applicable here.

[Figure 1 Here]

The residuals from the first-step VECM model are used to estimate GARCH parameters as described in section 2.3. When estimating unrestrictedly, one of the volatility spillover coefficients is found to be negative (see Table 2). Since it is very small, this coefficient is excluded from the equation. Then a second round estimation is performed, whose result is reported in Table 3\(^9\). Compared to the unrestricted model, the estimated values show little changes, and they are all very significant. To be robust, we check whether there are still arch effects left in standardized residuals and standardized shocks, and the results indicate that the arch effects are all eliminated. As shock unconditional variances in GARCH (normalized to be unity) are used to replace shock sample variances in estimating the modified information share, we further check whether such a replacement is appropriate by looking at whether these unconditional variances are equal to one. The shock variances are respectively 0.998716 and 0.999969, which are not significantly different from one. Thus, we could proceed to estimate the modified information shares with shock unconditional variances.

[Table 2 Here]

[Table 3 Here]

To allocate shocks, invert matrix A to get the matrix B, which measures the magnitude of idiosyncratic shocks’ contemporaneous impact on each market. As \(|B(1,1)|>|B(2,1)|\), and \(|B(2,2)|>|B(1,2)|\), according to the assumption that each shock contemporaneously affects their own market larger than the other markets, the first set of conditional variance parameters \((\Lambda_{11}, \Phi_{11})\) and the associated column in matrix B correspond to the futures market idiosyncratic shock.

\(^9\) Initially, the second round estimation is performed by restricting this coefficient to be non-negative. However, due to rounding problem of RATS software, this value is still estimated to be negative. And still, it is very small. At last, to ensure the conditional variances positive, this coefficient is excluded from equation.
Simple algebraic calculation within B indicates that one percent increase in the futures price will contemporaneously increase the spot price by 0.558 percent, and one percent increase in the spot price will increase contemporaneous the futures price by 0.336 percent. Through these contemporaneous price level interaction, volatility from the residual of one market is “indirectly” transmitted to that of another market. This result is consistent with Yang et.al (2011), where strong interdependence in volatility is found between these two residuals. In term of “direct” volatility spillover, estimated results of shocks’ GARCH parameters suggest that lag volatility of futures idiosyncratic shock influences spot shock’s volatility, while the opposite does not hold. This result indicates that the spot market reinterprets information from the futures market, but not vice versa.

From equation (4.1), the modified information share for the futures market is 34.4%. Taken into account volatility spillover, from equation (5.6), the volatility spillover adjusted information share for futures market is 62.5%. When it comes to interpreting these results, the 34.4% modified information share for futures market indicates that spot market’s interpretation of information constitutes the major part of the efficient price. However, the 62.5% volatility spillover adjusted information share for futures market suggests that the major original source of common trend innovation variance is futures “own” variance. This indicates that slightly more information about the efficient price first disseminates in the futures market than in the spot market. Combined with $\alpha_2=0$, which implies that the spot market is dominantly better at following the right price (Yang et.al (2011)), these outcomes indicate a complex pattern of the price discovery process.

The possible reason underlying this pattern may go as follows. Since the futures market is more liquid, more information is first discovered here. Yet, the higher liquidity also brings itself more noise. At each time, its interpretation of information is more contaminated by the noise, and thus lacks accuracy. Due to this reason, the spot market will reinterpret the information flowing from the futures market than just mechanically following its price change. As a result, the efficient price is mainly
based on interpretation made by the spot market, which consists of both its own first
discovered information and the information flowing from the futures market and
reinterpreted there. Reflected in the modified information share, it indicates higher
information share for the spot market. Moreover, as the futures market is noisier,
when there is price disparity between these two markets, the noisier futures price
adjusts to the less noisy and more “right” spot price. Overall, the higher liquidity in
futures market plays a dual role: on the one hand, it facilitates information
dissemination and makes the futures market more quickly at discovering information.
On the other hand, it brings the futures market more noise and makes its information
interpretation and price lack accuracy, which in turn lessens its modified information
share and makes it follow the more “right” spot price when they are different.

The above explanation corroborates the dual role of liquidity in price discovery,
which was previously revealed by studies on other markets(Yan and Zivot (2010),
Grammig and Peter (2010)), and it is also well supported by the document in Yang
et.al (2011): “Over the first three months of trading, the average daily futures trading
turnover reached RMB 230.8 billion (approximately US$40 billion), which is more
than the average daily turnover of CSI 300 constituent stocks. In fact, the CSI 300
futures market has become one of the most actively traded futures contracts in the
world. However, the open interest remains very small and on average accounts for
only 7.7% of the futures trading volume, suggesting that trading volume is mainly
driven by speculative day trading.”

As for the time varying information share in equation (4.1), the plot for the
futures market’s information shares is given in Figure 2. The result indicates that the
market information shares exhibited turbulent changes in the first half of sample
period, and became relatively stable in the second half. As the sample period
corresponds to the infancy stage of the stock index futures market, this may be the
result of market participators’ gradual learning the use of the futures market as their
investment tool.

[Figure 2 Here]
7. Conclusion

Our study extends Hasbrouck (1995) information share in several ways. First, the contemporaneous price level interaction among markets is uniquely identified, which in turn enables a unique Hasbrouck (1995) type information share. Compared to previous approaches, our approach is more intuitive, and requires rather mild condition. Second, shock conditional variances are used to construct the time dependent information share, extending the traditional static measure to a dynamic one. This enables the study of inter-temporal relationship between the information share and other variables, and the study of price discovery relationship during some particular period, such as the infancy stage of a market. Lastly, a volatility spillover adjusted information share is proposed. While the traditional information share measures how each market’s interpretation of information contributes to the formation of the efficient price, the volatility spillover adjusted information share gauge how much information about the efficient price first disseminates in a particular market. It complements the traditional measure by offering another perspective, and helps us identify the pattern of and deepen understanding of price discovery process.

It is worth mentioning that our model requires rather mild application condition. Specially, as long as the amounts of information interpreted by each market are not in constant proportion all the time, it is applicable. The mild requirement makes the model applicable in nearly every situation, including those where other competing models may fail to work.

As an empirical application, we revisit the study by Yang et.al (2011) on price discovery between CSI stock index and index futures market. The result well demonstrates our approach’s advantages. With the uniquely determined modified information shares and the volatility spillover adjusted information shares, we are able to confirm the dual role played by liquidity and uncover the complex pattern of price discovery process. The result indicates that although the futures market is slightly better at quickly discovering new information, it is the spot market’s interpretation that constitutes the major part of the common efficient price. In addition, as revealed
by Yang et.al (2011), when there is disparity between prices, futures price adjusts to the more “right” spot price. The relatively high liquidity of the futures market may provide an explanation to the above result. Because the futures market is more liquid, more information about the efficient price first disseminates here. Yet, more noises are brought into the market along with the liquidity. The relatively high noise level makes its interpretation of information inaccurate, and thus a second round interpretation is executed by the spot market. At last, the spot market’s interpretation of information constitutes the major part of the efficient price. Moreover, since the futures price is less accurate, when there is disparity between prices, it adjusts to the more “right” spot price. In addition, we also find from our dynamic price discovery measure that the information share of each market shows turbulent changes in the first half of the sample period, and then becomes relatively stable in the second half. The time series pattern is conjectured to be a result of market participators’ gradual learning the use of futures market as their investment tool.

At last, we want to stress that our study is mainly methodological, and its application is not limited to study on price discovery between stock index and index futures markets. We hope that researchers will find it useful in other areas where price discovery and/or volatility spillover is the main concern. For example, the quantified volatility spillover approach in the appendix can be used within VAR or VECM models to study information flow and/or sources of instability among related financial markets, such as among domestic and/or international stock market(s), bond market(s) and monetary market(s).
Appendix: Volatility spillover in residuals

Volatility spillover in residuals can also be quantified following similar steps as deduction of volatility spillover information shares. From equation (3.3) and (5.4), the covariance matrix of residuals can be written as:

\[
\Omega = B \begin{pmatrix} h_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & h_n \end{pmatrix} B^T = BB^T = B[\sum_{i=1}^{n} \text{dia}(h_{f,i})]B^T = \sum_{i=1}^{n} B\text{dia}(h_{f,i})B^T \tag{A1}
\]

In equation (A1), the covariance matrix of residuals is a function of \( h_{f,i} \), which in turn is a function of \( h_{o,i} \). To compute the volatility spillover from the ith market to the jth market, one need to calculate how the presence of ith market’s “own” variance increase the variance of jth residual, and it can be computed as:

\[
VS_{i,j} = \sum_{r=1}^{n} B_{j,r}^2 h_{f,i,r} \tag{A2}
\]

where \( VS_{i,j} \) denotes volatility spillover from the ith shock to the jth market’s residual, where \( B_{j,r} \) denotes the corresponding element in B matrix, and where \( h_{f,i,r} \) is the rth element in \( h_{f,i} \) and denotes the impact from the ith own “variance” on the rth shock’s variance.

To estimate how much proportion this volatility spillover effect takes in the jth residual’s total volatility, one can simply take division between \( VS_{i,j} \) and the jth residual’s variance:

\[
RVS_{i,j} = \frac{\sum_{r=1}^{n} B_{j,r}^2 h_{f,i,r}}{\sum_{r=1}^{n} B_{j,r}^2} \tag{A3}
\]

As variance is well related with information, from information perspective, equation (A3) estimates how much proportion of information in the jth market’s residual flows from the ith market.
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index futures returns,” The Journal of Financial and Quantitative Analysis, 25:4,
441-468.


Roberto Rigobon and Brian Sack (2003): “Spillover across U.S. financial markets,” NBER working study 9640
Table 1: Summary statistics of the returns

<table>
<thead>
<tr>
<th></th>
<th>Nobs</th>
<th>Mean</th>
<th>Std</th>
<th>Skew</th>
<th>Kurt</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Futures return</td>
<td>3527</td>
<td>-0.000046</td>
<td>0.0023</td>
<td>-0.751</td>
<td>10.776</td>
<td>-0.024224</td>
<td>0.014573</td>
</tr>
<tr>
<td>Spot return</td>
<td>3527</td>
<td>-0.000052</td>
<td>0.0025</td>
<td>-0.320</td>
<td>15.950</td>
<td>-0.025822</td>
<td>0.019212</td>
</tr>
</tbody>
</table>

Notes:
Spot returns are the changes in the natural logarithms of the underlying CSI 300 index prices. Futures returns are the changes in the natural logarithms of the CSI 300 futures prices.
Table 2: Estimating result of structural GARCH without restriction

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistics</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(1,1)</td>
<td>497.2443339</td>
<td>8.0071366</td>
<td>62.10014</td>
<td>0.00000000</td>
</tr>
<tr>
<td>A(2,1)</td>
<td>-405.3161029</td>
<td>9.8231937</td>
<td>-41.26113</td>
<td>0.00000000</td>
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<tr>
<td>A(1,2)</td>
<td>-136.1106895</td>
<td>13.5063947</td>
<td>-10.07750</td>
<td>0.00000000</td>
</tr>
<tr>
<td>A(2,2)</td>
<td>683.6853874</td>
<td>2.6888958</td>
<td>254.26251</td>
<td>0.00000000</td>
</tr>
<tr>
<td>Λ_{11}</td>
<td>0.0528209</td>
<td>0.0056194</td>
<td>9.39981</td>
<td>0.00000000</td>
</tr>
<tr>
<td>Λ_{12}</td>
<td>-0.0058021</td>
<td>0.0018477</td>
<td>-3.14013</td>
<td>0.00168870</td>
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<tr>
<td>Φ_{11}</td>
<td>0.9302724</td>
<td>0.0083375</td>
<td>111.57726</td>
<td>0.00000000</td>
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<tr>
<td>Λ_{21}</td>
<td>0.0613377</td>
<td>0.0045457</td>
<td>13.49361</td>
<td>0.00000000</td>
</tr>
<tr>
<td>Λ_{22}</td>
<td>0.0660258</td>
<td>0.0057523</td>
<td>11.47819</td>
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<tr>
<td>Φ_{22}</td>
<td>0.7901307</td>
<td>0.0127381</td>
<td>62.02889</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>

Notes:

Λ_{12}, the coefficient that describes volatility spillover from the second shock to the first shock, is very small and negative. Moreover, it is not as significant as other estimates.
Table 3: Estimating result of structural GARCH with restriction $\Lambda_{12}=0$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistics</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{11}$</td>
<td>0.0449598</td>
<td>0.0030464</td>
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<td>0.00000000</td>
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<tr>
<td>$\Phi_{11}$</td>
<td>0.9402048</td>
<td>0.0037915</td>
<td>247.97632</td>
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<tr>
<td>$\Lambda_{21}$</td>
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<td>0.0070481</td>
<td>9.20937</td>
<td>0.00000000</td>
</tr>
<tr>
<td>$\Lambda_{22}$</td>
<td>0.0595820</td>
<td>0.0052653</td>
<td>11.31593</td>
<td>0.00000000</td>
</tr>
<tr>
<td>$\Phi_{22}$</td>
<td>0.7887511</td>
<td>0.0145877</td>
<td>54.06964</td>
<td>0.00000000</td>
</tr>
</tbody>
</table>

Notes:
Compared to Table 2, the restricted model shows no large changes in the estimated results of the coefficients. In addition, all estimated values are very significant.
Figure 1: Plot of residual’s conditional variance ratio (futures/spot)

Notes:
Conditional variances of futures market residual and spot market residual are each estimated with univariate GARCH (1, 1) model. The ratio is computed as futures market residual’s conditional variance divided by spot market residual’s conditional variance.
Figure 2: Plot of time varying information share for futures market

Notes:
As these two markets’ information shares always sum to one, the spot market’s dynamic information share is simply one minus that of futures market. For visual clarity, it is now shown in the figure.