A Structural Explanation of Comovements in Inflation across Developed Countries ☆

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Abstract

Inflation is highly positively correlated across the industrialized countries even though the cross-country correlation in money growth rate is negligible. This has been a puzzle. I show that a two-country new-Keynesian sticky-prices model driven by monetary and productivity shocks is capable of explaining this fact. The structure of this model generates cross-country correlations of inflation, output and consumption that appear to closely correspond to the data. Additionally, this model can explain the internal correlation between inflation and output.

JEL Classification : E31; E32; F41; F47

Keywords: sticky prices, international comovement, inflation, money, productivity

1. Introduction

During the last years, economists have expressed a deep interest in studying the international comovement in inflation across countries, particularly industrialized countries. In particular, Ciccarelli and Mojon (2008) and Neely and Rapach (2008) are interested in the role that global inflation movements play as an attractor of domestic inflation. Furthermore, Wang and Wen (2007) are interested in finding the sources and

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mechanisms explaining the observed international comovements in inflation. In the cross-country investigation of inflation dynamics prepared by Wang and Wen (2007), they find for the period covered between 1977 Q1 and 1998 Q4 that the average cross-country correlation in inflation for developed countries is high and positive, although the cross-country correlation in money growth rate is near zero. This finding is a puzzle, at least for people who believe in the quantity theory of money. In addition, they conclude that standard new Keynesian sticky-information and sticky-price models driven only by monetary shocks are not able to explain the highly positive cross-country correlation in inflation when the monetary shocks are uncorrelated across developed countries. In this paper, I update the cross-country correlations in inflation, output and money growth rate calculated by Wang and Wen (2007) until 2008 Q1. As we will see these updated empirical findings do not differ substantially from the calculated by Wang and Wen (2007). Afterwards, I set up a two-country new Keynesian sticky price model, which has the same modelling framework presented by Wang and Wen (2007), but with four different features which are imperfect substitution between home and foreign goods, home bias consumption, inflation's inertia and both monetary and productivity shocks as an uncertainty sources. With the solution of this model, I can generate, first, a highly positive cross-country correlation in inflation across developed countries, even though when zero cross-country correlation in the money growth rate process across these countries is assumed[1], second a positive cross-country correlation in output with values that do not differ substantially from those observed in the data from the industrialized countries, and third a positive inner-correlation between output and inflation. All these three results agree with the observed data between 1977Q1 and 2008Q1.

The remainder of the paper is organized as follows. I present the stylized facts that describe the puzzle found by Wang and Wen (2007). Then, I set up and calibrate the model with the four features mentioned above. Next, I explain how the assumptions of imperfect substitution and home bias consumption can generate a positive cross-

[1]These countries are Australia, Canada, Japan, United Kingdom and United States. However, the results obtained in this paper do not differ so much from shorter data samples, which include more countries

country correlation in inflation between two symmetric countries when productivity shocks are assumed as a sole uncertainty's source in this model. Since productivity shocks as a unique uncertainty's source cannot generate a positive inner-correlation between output and inflation, I describe the mechanism of how this model can generate this inner-correlation by assuming monetary shocks as a unique uncertaninty's source. As I mentioned above, having monetary shocks as unique uncertainty's source cannot generate a highly positive cross-country correlation in inflation, I explain how the assumption of inflation's inertia and both productivity and monetary shocks can generate jointly a positive cross-country correlation in inflation, consumption and output, and a positive inner-correlation in inflation and output. Finally, I present the unconditional cross-country correlations in inflation, output, and consumption generated by this model's solution under the following three different scenarios. First when productivity is the sole uncertainty's source, second when money growth rate is the only uncertainty's source and third when both productivity and money growth rate are the uncertainty's sources in this model.

2. Stylized Facts

Wang and Wen (2007) calculated the cross-country correlation in inflation from 1977 Q1 until 1998 Q1. I updated this through 2008 Q1. As we can see, Table 1, which displays data from 1977Q1 until 2008Q1 [2], shows that the cross-country correlation in inflation between the industrialized countries is still very high and positive. In the same way, Table 2 shows that some cross-country correlations in M1 growth rate are near zero and some of them are negative.

Table 3 shows that the cross-country correlation in output is positive but not as high as the cross-country correlation in inflation across these countries. Moreover, Table 4 shows a very low domestic correlation between inflation and output.

From Tables 1 and 2, it can be inferred that the average cross-country correlation in inflation across these countries is equal to 0.6 while the average cross-country cor-

[2] I do not include the European Union Countries, since many of them adopted the Euro from 1999.

Table 1: Cross-country correlation in inflation (mean = 0.6084)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	0.6320	0.4500	0.4576	0.5248
Canada	0.6320	1.0000	0.6020	0.6024	0.7756
Japan	0.4500	0.6020	1.0000	0.6543	0.6705
UK	0.4576	0.6024	0.6543	1.0000	0.7150
USA	0.5248	0.7756	0.6705	0.7150	1.0000

Table 2: Cross-country correlation in money growth rate (mean = 0.015)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	-0.0860	-0.1593	0.1019	0.0845
Canada	-0.0860	1.0000	0.0281	0.0399	0.0387
Japan	-0.1593	0.0281	1.0000	0.1380	0.0966
UK	0.1019	0.0399	0.1380	1.0000	-0.1289
USA	0.0845	0.0387	0.0966	-0.1289	1.0000

relation in money growth rate is equal to 0.0154. So the puzzle of highly positive cross-country correlation in inflation accompanied with a negligible cross-country correlation in money growth rate still holds. For the purpose of this research, this puzzle will be called the Wang and Wen's puzzle.

3. The Model

3.1. Environment

- Households live infinite number of periods; they consume a basket of final goods which can be domestic or imported.

Table 3: Cross-country correlation in Output (mean = 0.3472)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	0.0469	0.6252	0.3270	0.4914
Canada	0.0469	1.0000	0.0981	0.0791	0.1260
Japan	0.6252	0.0981	1.0000	0.5314	0.5773
UK	0.3270	0.0791	0.5314	1.0000	0.5700
USA	0.4914	0.1260	0.5773	0.5700	1.0000

Table 4: Domestic Correlation between Output and Inflation (mean = 0.2147)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Correlation	0.1961	0.1780	0.0702	0.2595	0.3698

- There exists imperfect substitution in the consumption of domestic and foreign goods.
- Households are endowed with $\bar{l}$ units of time, which they can spend on leisure or labor.
- Households are the owner of all firms.
- Only final goods are tradable.
- Intermediate goods firms are producing in a monopolistically competitive market.
- Intermediate goods' factors are produced in a perfectly competitive market.
- There is a regime of floating exchange rate.
- Money supply is determined by an exogenous stochastic process for the money growth rate.
- Total factor productivity is determined by an exogenous stochastic process.

3.2. Households

The representative household chooses $\{C_t, N_t, B(s^{t+1}), M_{t+1}\}$ which maximizes its lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[C_t^{1-\sigma}/(1-\sigma) - \psi N_t^{1+\eta}/(1+\eta) \Big] \tag{1}$$

Subject to:

$$T_t + B_t + M_t + \Pi_t + P_t W_t N_t \geq P_t C_t + \sum_{s^{t+1}}^{\infty} Q(s^{t+1}|s^t) B(s^{t+1}) + M_{t+1} \tag{2}$$

$$M_t + T_t + B_t \geq P_t C_t + \sum_{s^{t+1}}^{\infty} Q(s^{t+1}|s^t) B(s^{t+1}) \tag{3}$$

Taking: $M_t, T_t, P_t, \Pi_t, M_t, W_t, Q(s^{t+1}|s^t)$ as given.

C_t is the composed consumption at date t, N_t represents the worked hours, T_t is a lump sum transfer of flow of money that the representative household receives from the Government, M_t is the representative households money holdings in home currency carried over from the last period, B_t are the nominal bonds expressed in domestic currency, P_t denotes the consumer's price index, W_t is real the real wage (deflated by using the consumer price index) and $Q(s^{t+1}|s^t)$ is the price at t of a bond that next period would yield $B(s^{t+1})$ and Π_t is the intermediate goods producer's profit. The variables for the foreign country are denoted with star.

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The first order conditions from the representative houeshold's maximization problem are as follows:

$$C_t : C_t^{-\sigma} - \lambda_t - \mu_t = 0 \tag{4}$$

$$N_t : -\psi N_t^{\eta} + \lambda W_t = 0 \tag{5}$$

$$M_{t+1} : -\lambda/P_t + \beta E_t\Big[\lambda_{t+1} + \mu_{t+1}/P_{t+1}\Big] = 0 \tag{6}$$

$$B(s^{t+1}) : \Big[\lambda_t + \mu_t/P_t\Big]Q(s^{t+1}|s^t) - \beta\kappa(s^{t+1}|s^t)\Big[\lambda(s^{t+1}|s^t) + \mu(s^{t+1}|s^t)/P(s^{t+1}|s^t)\Big] = 0 \tag{7}$$

At each period t, the representative household chooses $C_{H,t}$ and $C_{F,t}$ which minimizes its total expenditure:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \tag{8}$$

Given, that they have chosen $\{C_t\}_{t=0}^{\infty}$ previously.[3]

Subject to:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \tag{9}$$

As we can see (9) is an implication of assuming imperfect substitution in consumption between domestic and foreign goods.

$C_{H,t}$ denotes the home consumption of the domestic final good, $C_{F,t}$ denotes the home consumption of the foreign final good, $P_{H,t}$ and $P_{F,t}$ are their prices respectively.

The main result of this intratemporal minimization problem is that the economy's price level can be expressed as a function of the prices of home and foreign goods. That is,

$$P_t = \phi P_{H,t}^{\gamma} P_{F,t}^{1-\gamma} \tag{10}$$

Where: $\phi = \left[(\gamma / \ 1 - \gamma)^{1-\gamma} + (1 - \gamma / \ \gamma)^{\gamma} \right]$

3.3. Firms

3.3.1. Final Good

Each country produces a single final good through the following production function:

$$Y_{H,t} = \left(\int_0^1 (Y_{H,t}(i))^{\zeta - 1/\zeta} di \right)^{\zeta/\zeta - 1} \tag{11}$$

[3]Since the representative household preferences are separable on every period , I can solve this intratemporal minimization problem independently from the intertemporal maximization problem described previously.

$Y_{H,t}(i)$ is the intermediate i good which is non tradable and ζ measures the elasticity of substitution among the intermediate goods, $Y_{H,t}(i)$.

The optimization problem of the final good producer is to find the optimal set of inputs $Y_{H,t}(i)$ maximizing:

$$P_{H,t}Y_{H,t} - \int_0^1 P_{H,t}(i)Y_{H,t}(i)di \tag{12}$$

Subject to (11)

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$$Y_{H,t}(i) : P_{H,t}Y_{H,t}^{1/\zeta}Y_{H,t}^{-1/\zeta}(i) - P_{H,t}(i) = 0 \tag{13}$$

3.3.2. *Intermediate good firms*

Each intermediate good i is produced by a single monopolistically competitive firm according to the following technology:

$$Y_{H,t}(i) = A_{H,t}N_t(i) \tag{14}$$

$$A_{H,t} = A_{H,t}^{\rho} \exp^{\epsilon_t^A} \tag{15}$$

$A_{H,t}$ is the Total Factor Productivity (TFP) , which is the same for every i firm and ϵ_t^A is a stochastic process which follows a normal distribution with zero mean and constant variance, σ_A^2.

3.3.3. *Price Setting*

Following Calvo (1983), I assume that each individual firm resets its price with probability $(1-\theta)$ each period independently of the time elapsed since its last price adjustment. Thus, each period a measure $(1-\theta)$ of (randomly selected) firms reset their prices, while a fraction θ keep their prices unchanged.

Let $P_{H,t}(i)$ denotes the price set by a firm i adjusting its price in period t. Let $\overline{P}_{H,t}(i)$ denotes the price set by a firm i adjusting its price in period t. Under the Calvo price setting structure, $P_{H,t+k}(i) = \overline{P}_{H,t}(i)$ with probability θ^k for k = 0,1,2,3 ...

Then, the firm's optimal price setting model is written as follows:

$$\overline{P}_{H,t}(i) = \arg\max \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} E_t \left[\overline{P}_{H,t}(i) - MC_{h,t}(i) \right] \tag{16}$$

Subject to (13) and taking $Y_{H,t+k}$ as given.

3.3.4. Price Index Dynamics

Under the assumed price-setting structure, the dynamics of the domestic price index is described by the following equation:

$$P_{H,t} = \left[\theta P_{H,t-1}^{1-\zeta} + (1-\theta) \check{P}_{H,t}^{1-\zeta} \right]^{1/1-\zeta} \tag{17}$$

In order to introduce persistence in the New-Keynesian Phillips Curve, I use the same approach as Gali and Gertler (1999). Consider a fraction of firms, $(1-\chi)$, that follow the optimal updating price rule, $\overline{P}_{H,t}$ and a fraction of firms χ that follow a backward looking adjustment process,$P_{h,t}^b$. That is,

$$\check{P}_{H,t} = \left[\chi (P_{H,t}^b)^{1-\zeta} + (1-\chi)(\overline{P}_{H,t})^{1-\zeta} \right]^{1/1-\zeta} \tag{18}$$

$$P_{H,t}^b = \overline{P}_{H,t-1}(1+\pi_{t-1}) \tag{19}$$

3.4. Government

The government transfers to individuals a lump sum transfer of flow of money.

$$T_t = M_t U_{t+1} \tag{20}$$

where the gross money growth rate follow the subsequent stochastic process:

$$U_t = (U_t)^{\varsigma}(\overline{U})^{1-\varsigma}\exp^{\epsilon_t^U} \tag{21}$$

$\bar{U}$ is the steady state money growth rate, ϵ_t^U denotes the stochastic process which follows a normal distribution with zero mean and constant variance, σ_U^2.

Since both countries are identical, then the mathematical expressions and parameterization that are described above are the same for the other country.

3.5. Equilibrium Conditions

$$Y_{H,t} = C_{H,t} + C^*_{F,t} \tag{22}$$

$$Y_{H,t} = C^*_{H,t} + C_{F,t} \tag{23}$$

$$\tau_t = (1 + \pi_{t+1})\tilde{M}_{t+1} - \tilde{M}_t \tag{24}$$

$$\tau^*_t = (1 + \pi^*_{t+1})\tilde{M}^*_{t+1} - \tilde{M}^*_t \tag{25}$$

Where: $\tau_t = T_t/P_t$ and $\tilde{M}_t = M_t/P_t$

4. Model Predictions and Results

4.1. Calibration

Table 5 presents the calibration of this model which largely follows Wang and Wen (2007) with the addition of two parameters: the expenditure share on domestic goods γ which comes from allowing imperfect substitution in consumption between domestic and foreign goods, and the probability of adjusting the price based in the past period optimal reset price χ.

Table 5: Baseline Calibration

Parameter	Source
$\sigma = 0.05$	Wang and Wen (2007)
$\eta = 0.05$	Wang and Wen (2007)
$\beta = 0.99$	Wang and Wen (2007)
$\varsigma = 0.60$	Wang and Wen (2007)
$\gamma = 0.81$	Calibrated by the author
$\theta = 0.80$	Gali and Gertler (1999)
$\chi = 0.85$	Gali and Gertler (1999)
$\sigma_A = \sigma_{A^*} = 0.085$	Backus et al. (1992)
$\sigma_U = \sigma_{U^*} = 0.85$	Backus et al. (1992)
$\rho(\epsilon^A, \epsilon^{A^*}) = 0.258$	Backus et al. (1992)
$\rho(\epsilon^U, \epsilon^{U^*}) = 0.00$	Wang and Wen (2007)

Note: Estimates of the standard errors for the shock component of M1 growth rate were obtained from the updated database that I used for computing the cross-country correlations presented in Tables 1 and 2. The symbol $\rho(x, y)$ represents the unconditional correlation between two stochastic processes.

4.2. Predicted Unconditional Correlations

In this subsection, I present the results implied by this model when it is driven by a) only productivity shocks, b) only monetary shocks and c) both monetary and productivity shocks.

4.2.1. Productivity Shocks Only

One important result provided by this model is its capability of generating cross-country correlation in inflation and output close to the observed data when productivity shocks in each country are the sole uncertainty's source, see Table 6. This interesting result is a consequence of the following three assumptions: imperfect substitution in consumption between foreign and home goods, home bias consumption and the presence of only productivity shocks processes in this model. To clarify the role of these assumptions behind of this result, consider a world economy with only two countries USA and UK, then suppose that there is a one-time transitory but persistent shock to USA's productivity process. In response to this shock, USA's output jumps up causing that USA's final goods will be cheaper than UK's final goods. This change in the relative prices is reflected in a jump up of USA's terms of trade (a jump down in UK's terms of trade). Since I assume home bias cunsumption, USA's total consumption basket is cheaper than UK's total consumption basket, that is a jump up in USA's real exchange rate (obviously a jump down in UK's real exchange rate). As a result of the risk sharing condition, this change in USA's real exchange rate implies a jump up in USA's total consumption and a jump down in UK's total consumption. On the other hand, this fall in UK's consumption shifts UK's labor supply curve to the right, which in turn generates a fall in UK's real wage (see equation (C.16) in the context of UK). Inasmuch as the real marginal cost in both countries is a positive function of the domestic real wage and the terms of trade, and a negative function of the domestic productivity process, UK's real marginal cost jumps down since both UK's terms of trade and UK's real wage have jumped down. This jump in UK's real marginal cost causes a fall in UK's domestic goods inflation. (see equation (C.19), in the context of UK)

Moreover, since the effect of USA's terms of trade is lower than the effect of both USA's productivity and USA's real wage on USA's real marginal cost, then this latter variable jumps down and in turn USA's domestic goods inflation falls.

Since I have assumed imperfect substitution in consumption between home and foreign goods, then total inflation is a function not only of domestic goods inflation but also a function of foreign goods inflation. Therefore, it is necessary to consider the dynamics of foreign goods inflation of each country. As long as foreign goods inflation is a positive function of terms of trade growth rate, it is expected a fall in UK's foreign goods inflation and hence in the UK's total inflation. However, in USA the story is different because in response of this USA's productivity shock, USA's terms of trade jumps up and therefore USA's foreign goods inflation does increase.

Inasmuch as, USA's productivity shock has a stronger impact than USA's terms of trade on USA's real marginal cost, then domestic goods inflation falls. Since, I assume home bias consumption, then USA's total goods inflation also falls.

Therefore, we should expect a highly positive cross-country correlation in inflation between these two countries.

Furthermore, to put this model in perspective with the standard international RBC models, I assume a moderate positive cross-country correlation in productivity shocks between the two countries, assumed in this model, to generate a positive cross-country correlation in consumption. The cross-country correlation in consumption generated by this model is resported in Table 6 which is similar to the reported by Backus et al. (1992).

4.2.2. Monetary Shocks Only

When I assume productivity shocks alone, the model does not capture the inner correlation of inflation and output observed in the data. That is, the model does not generate a Phillips curve. To understand how this model generates the positive relationship between inflation and output, I put, initially, in the model money growth rate shocks as a unique uncertainty source. To be consistent with the observed data, I assume that this shock is uncorrelated between the two cuntries, assumed in this model. As we can see in Table 7 with this shock the model is able to generate a positive inner-correlation between inflation and output but is not able to generate a positive cross-country correlation in inflation between the two countries, assumed in this model.

The mechanism which explains this result is the following. Consider a world economy with only two countries USA and UK, then suppose that there is an one-time transitory but persistent shock to USA's money growth rate. This cause a jump up in USA's total consumption in view of the households have more money to spend in consumption goods. Since in this model is assumed home bias consumption, the raise in USA's domestic consumption is higher than the raise in USA's foreign consumption, USA's terms of trade jump up, this in turn causes a real depreciation of the dollar with respect to the sterling pound. This real depreciation of the dollar (or real appreciation of the sterling pound) causes a jump down in UK's consumption since the risk sharing condition holds.

Inasmuch as UK's consumption fall is larger than UK's real wage fall, UK's agents need to work more hours to increase their output and therefore improve their exports value. Since the real wages and the terms of trade in UK jumped off, UK's real marginal cost also jumps down and in turn UK's inflation decreases.

In USA the story is different. The jump up in USA's consumption shift backward the labor supply curve which leads a raise in the real wage. This jump in the USA's real wage causes a jump up in USA's worked hours and therefore in USA's output. Since there is a real depreciation of the dollar with respect to the sterling pound and a higher real wage, the USA's real marginal cost jumps up also. This jump in USA's real marginal cost causes an increment in USA's inflation.

Therefore, the model captures an inner positive correlation between USA's inflation and USA's output.[4]. However, with this monetary shock alone, the model is not able to generate the positive cross-country correlation in inflation between USA and UK.

4.2.3. Productivity and Monetary Shocks

In order to generate a jointly high and positive cross country correlation in inflation across countries, and a positive inner correlation between inflation and output, I assume the presence of both monetary and productivity shocks in this model. Also I show the role playing by the assumption of inflation's inertia in this model to generate these two important results.

As we saw above, the productivity shocks in this model generates a highly positive cross-country correlation in inflation but the monetary shocks in this model generates a negative cross-country correlation in inflation and a highly postive inner-correlation between inflation and output.

In addition to have both productivity and monetary shocks, it is importnat to see how the degree of inflation's inertia can affect the model's results. For example, when

[4]Also we can see that this experiment shows a negative inner correlation between UK's inflation and UK's output. However, if I generate a money supply shock in UK , I will obatain the phillips curve for UK. In the stochastic simulation's outcomes reported in Table 7, I assume uncorrelated monetary shocks for each country.

the degree of inflation's inertia,χ,takes lower values, the effect of monetary policy in inflation is very high.[5] Therefore, when I include both the productivity and the monetary shocks in this model, the cross-country correlation in inflation is negative. In addition, it is expected that the model generates a high inner corralletion between inflation and output because the effects of monetary policy on inflation are higher than the effect of productivity shocks on inflation.

However, when the degree of inflation's inertia,χ,takes high values, the effect of monetary policy in inflation is very low. Therefore, in Table 8, I show that if I include both the productivity and the monetary shocks in this model, the positive cross-country correlation in inflation is still high and positive but lower than the case in which the model has only productivity shocks. Also, with these two shocks the model still presents a positive inner correlation in inflation and oputput but lower than the case in which the model has only monetary shocks.

One remaining issue is the relative size of the cross-country correlation in inflation and the cross-country correlation in output. In the data the cross-country correlation in inflation is higher than the cross-country correlation in output, but in my model the opposite is true. Future work might usefully focus on addressing this issue.

5. Conclusion

One of the new challenges for the central banks is to identify what kind of domestic shocks affect the world economy and how they are transmitted to the rest of the world. In particular, we are interested in how the inflationary shocks are transmitted between countries.

In this paper I have presented a very simple two-country new-Keynesian model in which the inclusion of imperfect substitution between home and foreign consumption, home bias consumption, inflation's inertia and the existence of productivity shocks as well as monetary shocks are key for solving the Wang and Wen (2007)'s puzzle of

[5]Inflation's inertia is the result of having a fraction of firms which adjust their prices following the backward looking rule represented by equations 18 and 19

the joint occurrence of positive cross-country correlation in inflation and a near-zero cross-country correlation in money growth.

Although this model adequately captures the signs and magnitude of the cross country-correlations in inflation, output, and consumption, this model tends to generate a stronger positive cross country correlation in output than inflation. Future work is needed to investigate whether this model can generate a stronger positive cross country correlation in inflation than in output.

Appendix A. Description of Data

This appendix describes the data source and range. From the (IFS) database, I obtained series of Consumer Price Index (CPI), Real Gross Domestic Output (GDP) and Money Supply, (M1). All these data are available for these five countries. The range goes from 1977 Q1 to 2008 Q1. The cross-country correlation in inflation is computed from the quarterly percent change in the CPI. The cross-country correlation in output is computed from the percent deviation of the GDP from its long run trend, which is obtained through the Hodrick and Prescott (1997) filter.

Appendix B. Model's Structure

Efficient Risk Sharing Condition

In this subsection, I present the derivation of the Efficient Risk Sharing Condition implied by the households' intertemporal maximization of this model:

By substituting (4) in (7), we get:

$$C_t^{-\sigma}/P_tQ(s^{t+1}|s^t) - \beta\kappa(s^{t+1}|s^t)\Big[C^{-\sigma}(s^{t+1}|s^t)/P(s^{t+1}|s^t)\Big] \tag{B.1}$$

Since these two countries are the same, then we have:

$$(C_t^*)^{-\sigma}/E_tP_t^*E_tQ^*(s^{t+1}|s^t) - \beta\kappa(s^{t+1}|s^t)\Big[(C^*)^{-\sigma}(s^{t+1}|s^t)/P(s^{t+1}|s^t)\Big] \tag{B.2}$$

By assuming perfect substitution between home and foreign bonds, we obtain:

$$Q(s^{t+1}|s^t) = E_t Q^*(s^{t+1}|s^t) \tag{B.3}$$

Then by susbsituting (B.2) and (B.3) in (B.1), we get:

$$C_t^{-\sigma}/(C_t^*)^{-\sigma} RER_t = C^{-\sigma}(s^{t+1}|s^t)/(C^*)^{-\sigma}(s^{t+1}|s^t) \tag{B.4}$$

Where: $RER_t \equiv E_t P_t^*/P_t$ is the real exchange rate and E_t is the nominal exchange rate.

If we assume that initially the two economies are perfectly symmetric (i.e. in state s they have the same prices and marginal utility), then (B.4) implies:

$$RER_t = C^{-\sigma}(s^{t+1}|s^t)/(C^*)^{-\sigma}(s^{t+1}|s^t) \tag{B.5}$$

which is nothing but the ***Efficient Risk Sharing Condition***.

Intratemporal Optimization Problem

At each period t, the representative household chooses $C_{H,t}$ and $C_{F,t}$ which minimizes its total expenditure:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \tag{B.6}$$

Given, that they have chosen $\{C_t\}_{t=0}^{\infty}$ previously.[6]

Subject to:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \tag{B.7}$$

[6]Since the representative household preferences are separable on every period , I can solve this intratemporal minimization problem independently from the intertemporal maximization problem described previously.

As we can see (B.7) is an implication of assuming imperfect substitution in consumption between domestic and foreign goods.

Where: $C_{H,t}$ denotes the home consumption of the domestic final good, $C_{F,t}$ denotes the home consumption of the foreign final good, $P_{H,t}$ and $P_{F,t}$ are their prices respectively.

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$$C_{H,t} : P_{H,t} - \nu_t \gamma C_{H,t}^{\gamma-1} C_{F,t}^{1-\gamma} = 0 \tag{B.8}$$

$$C_{F,t} : P_{F,t} - \nu_t (1-\gamma) C_{H,t}^{\gamma} C_{F,t}^{-\gamma} = 0 \tag{B.9}$$

$$\nu_t : C_t - C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} = 0 \tag{B.10}$$

Where: ν is the Lagrange multiplier associated to (9).

The main implication of this intratemporal minimization problem is:

$$P_t = \phi P_{H,t}^{\gamma} P_{F,t}^{1-\gamma} \tag{B.11}$$

Where: $\phi = \left[(\gamma / \ 1-\gamma)^{1-\gamma} + (1-\gamma / \ \gamma)^{\gamma} \right]$

Price Setting

Following Calvo (1983) I assume that each individual firm resets its price with probability $(1-\theta)$ each period, independently of the time elapsed since its last price adjustment. Thus, each period a measure $(1-\theta)$ of (randomly selected) firms reset their prices, while a fraction θ keep their prices unchanged.

Let $P_{H,t}(i)$ denote the price set by a firm i adjusting its price in period t. Let $\overline{P}_{H,t}(i)$ denotes the price set by a firm i adjusting its price in period t. Under the Calvo price setting structure, $P_{H,t+k}(i) = \overline{P}_{H,t}(i)$ with probability θ^k for k = 0,1,2,3 ...

Then, the firm's optimal price setting model is written as follows:

$$\overline{P}_{H,t}(i) = argmax \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} E_t \left[\overline{P}_{H,t}(i) - MC_{h,t}(i) \right] \tag{B.12}$$

Subject to (13)

Taking $Y_{H,t+k}$ as given.

F.O.C

$$\sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[\left((1-\zeta) \left(\overline{P}_{H,t}(i)/P_{H,t} \right)^{-\zeta} + \zeta \left(\overline{P}_{H,t}(i)/P_{H,t} \right)^{-\zeta-1} RMC_{H,t}(i) \right) Y_{H,t+k} \right] = 0 \tag{B.13}$$

$$\sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[\left(\left(\overline{P}_{H,t}(i)/P_{H,t} \right) - (\zeta/\zeta - 1)\, RMC_{H,t}(i) \right) Y_{H,t+k} \right] = 0 \tag{B.14}$$

Where: $RMC_{H,t}(i) = MCh,t(i)/P_{H,t}$.

Since all firms resetting prices in any given period and having identical technology they will choose the same price, I henceforth drop the i subscript.

$$\sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[\left(\left(\overline{P}_{H,t}/P_{H,t} \right) - (\zeta/\zeta - 1)\, RMC_{H,t} \right) Y_{H,t+k} \right] = 0 \tag{B.15}$$

Where: $\zeta/1-\zeta$ denotes the intermediate firms' mark-up.

Appendix C. Equations implied by the model

This appendix presents the system of dynamic equations implied by this model under Flexible Prices and under Sticky Prices.

Flexible Prices

- From (4),(5) and (6) we have:

$$\frac{\psi}{\beta} N_t^{\eta} C_{H,t+1|t}^{\sigma} = W_t \tag{C.1}$$

- Recalling (9) we have:

$$C_t = C_{H,t}^{\gamma} C_{F,t}^{1-\gamma} \tag{C.2}$$

- Recalling (B.4), we have

$$RER_t = C_t^{-\sigma} / (C^*)_t^{-\sigma} \tag{C.3}$$

- From (B.13) and (B.14) we have:

$$C_{H,t}/C_{F,t} = \left(\frac{\gamma}{1-\gamma}\right) TOT_t \tag{C.4}$$

  Where: $TOT_t = \frac{P_{F,t}}{EP^*_{H,t}}$. Note that $TOT_t = \frac{P_{F,t}}{P_{H,t}}$ since LOOP holds.

- By dividing (10) from its first lagged values, we have:

$$(1 + \pi_t) = (1 + \pi_{H,t})^{\gamma} (1 + \pi_{F,t})^{1-\gamma} \tag{C.5}$$

- Recalling (14) and assuming that all intermediate goods forms have the same technology, we have:

$$Y_{H,t} = A_{H,t} N_t \tag{C.6}$$

- Recalling (eq:shA)

$$A_{H,t} = A_{H,t}^{\rho} \exp^{\epsilon_t^A} \tag{C.7}$$

- The real marginal cost associated to the technology described in (14), is defined as: $RMC_{H,t} \equiv P_t W_t / P_{H,t} A_{H,t}$ and the terms of trade is defined as: $TOT_t \equiv P_{F,t}/P_{H,t}$.

  Since this model does not determine the optimal path for nominal variables such as price levels, we have to rewrite the real marginal cost as follows:

  By using the definions of RMC_t and TOT_t described above and (10), we have:

$$RMC_{H,t} = \phi \frac{W_t}{A_{H,t}} TOT_t^{1-\gamma} \tag{C.8}$$

- Since prices are flexible, by (B.15) the optimality condition for intermediate goods firms reduces to:

$$RMC_{H,t} = (1 - \zeta/\zeta) \tag{C.9}$$

- Usign the definition of TOT_t and dividing it by tis forst lagged value, wee have:

$$(1 + \pi_{F,t}) = \frac{TOT_t}{TOT_{t-1}} \tag{C.10}$$

- From (20) and (24) we have:

$$\frac{\tilde{M}_{t+1}}{\tilde{M}_t} = \frac{U_{t+1}}{1 + \pi t + 1} \tag{C.11}$$

- Recalling (21), we have:

$$U_t = (U_t)^{\varsigma} (\overline{U})^{1-\varsigma} \exp^{\epsilon_t^U} \tag{C.12}$$

- Recalling (22), we have:

$$Y_{H,t} = C_{H,t} + C^*_{F,t} \tag{C.13}$$

- By using the equilibrium condition (24), the household's budget constraint (2), the consumer price index (10), the definition of terms of trade and the fact that $\Pi_t \equiv P_{H,t}Y_{H,t} - P_t W_{H,t} N_t$, we obtain the equilibrium CIA constraint:

$$Y_{H,t} = M_{t+1}(1\tilde{+}\pi_{t+1})\phi TOT_t^{1-\gamma} \tag{C.14}$$

- By using the above definition of real exchange rate, the consumer price index of each country and the LOOP, we obtain the following relationship between the real exchange rate and the terms of trade as follows:

$$RER_t = \frac{\phi}{\phi^*} TOT_t^{2\gamma-1} \tag{C.15}$$

Sticky Prices

By log-linearizing the above system of equations around the steady state and by including sticky prices, we have the following dynamic system of linear rational expectations equations:

$$\eta n_t + \sigma c_{t+1|t} = w_t \tag{C.16}$$

$$c_t = \gamma c_{H,t} + (1-\gamma)c_{F,t} \tag{C.17}$$

$$c_{H,t} - c^*_{F,t} = \frac{1}{\sigma} rer_t \tag{C.18}$$

$$c_{H,t} = c_{F,t} + tot_t \tag{C.19}$$

$$\pi = \gamma\pi_{H,t} + (1-\gamma)\pi_{F,t} \tag{C.20}$$

$$y_{H,t} = a_{H,t} + n_t \tag{C.21}$$

$$a_t = \rho a_{t-1} + \epsilon_t^A \tag{C.22}$$

$$rmc_t = w_t - a_{H,t} + (1-\gamma)tot_t \tag{C.23}$$

By log-linearizing (17)-(19) and (B.15) around zero inflation and by solving the system of equations implied by this log-linearization, which is described in Gali and Monacelli (2005) and Gali and Gertler (1999)), we obtain the NKPC:

$$\pi_{H,t} = \frac{(1-\chi)(1-\theta)(1-\beta\theta)}{\Delta} rmc_t + \frac{\beta\theta}{\Delta}\pi_{H,t+1|t} + \frac{\chi}{\Delta}\pi_{H,t-1} \tag{C.24}$$

$$\pi_{F,t} = tot_t - tot_{t-1} \tag{C.25}$$

$$m_{t+1} = m_t + u_{t+1} - \pi_{t+1} \tag{C.26}$$

$$u_t = \varsigma u_{t-1} + \epsilon_t^U \tag{C.27}$$

$$y_{H,t} = c_{H,t} + c^*_{F,t} \tag{C.28}$$

$$y_{H,t} = m_{t+1|t} + \pi_{H,t+1|t} + (1-\gamma)tot_t \tag{C.29}$$

$$rer_t = (2\gamma - 1)tot_t \tag{C.30}$$

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Table 6: Predicted Correlations

Source of uncertainty: Productivity shocks

γ	χ	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	0.7746	0.7690	0.2373	-0.0511
0.66	0.50	0.8057	0.7879	0.2820	-0.0488
0.66	0.85	0.9351	0.9262	0.6936	-0.0401
0.76	0.00	0.5356	0.5720	0.1225	-0.0510
0.76	0.50	0.5666	0.5888	0.1474	-0.0489
0.76	0.85	0.7806	0.7873	0.4906	-0.0403
0.81	0.00	0.4463	0.4826	0.1247	-0.0508
0.81	0.50	0.4707	0.4958	0.1418	-0.0487
0.81	**0.85**	**0.6779**	**0.6881**	**0.4164**	**-0.0403**
Data		0.6000	0.3472	0.3200[c]	0.2147

Note: a) The symbol $\rho(x, y)$ represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).

Table 7: Predicted Correlations

Source of uncertainty: Money growth rate shocks

γ	χ	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	-0.4779	0.2991	-0.4361	0.8519
0.66	0.50	-0.3650	0.2341	-0.4911	0.9030
0.66	0.85	0.1530	0.4568	-0.2754	0.4086
0.76	0.00	-0.2147	0.1610	-0.3496	0.9620
0.76	0.50	-0.1348	0.1253	-0.3812	0.9673
0.76	0.85	0.2151	0.3189	-0.1944	0.3752
0.81	0.00	-0.1385	0.1116	-0.2811	0.9812
0.81	0.50	-0.0842	0.0868	-0.3041	0.9703
0.81	**0.85**	**0.1801**	**0.2508**	**-0.1437**	**0.3543**
Data		0.6000	0.3472	0.3200[c]	0.2147

Note: a) The symbol $\rho(x, y)$ represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).

Table 8: Predicted Correlations

Source of uncertainty: Productivity and Money growth rate shocks

γ	χ	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	-0.0115	0.5883	-0.1006	0.3970
0.66	0.50	0.1365	0.5949	-0.0906	0.3800
0.66	0.85	0.6513	0.8031	0.3196	0.0992
0.76	0.00	0.1681	0.4160	-0.0844	0.3928
0.76	0.50	0.2464	0.4317	-0.0653	0.3578
0.76	0.85	0.5944	0.6636	0.2653	0.0843
0.81	0.00	0.1834	0.3439	-0.0425	0.3816
0.81	0.50	0.2350	0.3605	-0.0236	0.3426
0.81	**0.85**	**0.5184**	**0.5739**	**0.2471**	**0.0767**
Data		0.6000	0.3472	0.3200[c]	0.2147

Note: a) The symbol $\rho(x, y)$ represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).