

Predicting output and inflation using the entire yield curve

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Abstract

Many studies find that government bond yield spreads predict real economic activity and, to lesser extent, inflation. Most of these studies define the yield spread exogenously as the difference between two yields of specific maturities to predict output and inflation. In this paper, I propose a different approach, which allows using information contained in the entire term structure of US Treasury yields to predict US real GDP growth and inflation. In particular, I modify the Diebold-Li (2006) dynamic yield curve model, which is based on the Nelson-Seigel (1987) three latent factor framework, to model the entire yield curve, real GDP growth, and inflation jointly. I find that this yield curve-macro model produces better out-of-sample real GDP forecasts than those produced by the simple yield spread model. Using the dynamic yield curve model provides a more accurate depiction of the predictive power of the yield curve and uses information contained in the entire yield curve, while the yield spread forecasting model uses only information in the difference between two yields of arbitrarily chosen maturities and also possibly overfits in sample. With regard to inflation, I do not find robust improvement in forecasts based on the dynamic yield curve model over the yield spread model and there is evidence of a weakening relationship between the yield curve and inflation in post-1982 period.

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1. Introduction

There are numerous papers which explore the question: “What information does the yield spread contain about future real economic activity and inflation?” These studies are based on the intuition that when agents price assets they take into account expectations about future states of the economy, and therefore interest rates potentially contain useful information about future economic growth and inflation. Estrella and Hardouvelis (1991) find evidence that the US government bond yield spread contains information about future US real economic growth at horizons of up to four years. Estrella and Mishkin (1997) confirm that the yield spread has predictive power for real economic activity in the United States and in number of European countries. The research of Mishkin (1990a, 1990b) based on a Fisher equation decomposition of the nominal interest rate into the real interest rate and expected inflation finds that the yield spread contains information about future changes in US inflation.

In most of the previous literature on the predictive power of yield curve for real economic activity and inflation, researchers use simple OLS regressions of future output or inflation changes on a yield spread defined as a difference between specific long-term government bond rate and short-term T-bill rate. Although this approach has an advantage in its simplicity, it does not have enough flexibility to use all the information contained in the entire term structure of interest rates. Moreover, its results reflect the choice of yield spread measure, and apparent predictability could be due to data mining to find the best in-sample fitting measure. Mishkin’s (1999a,1990b) analysis of predictive power of the yield spread for the change in inflation suggests that its predictive power is sensitive to the choice of maturities used for the yield spread. Ang, Piazzesi, and Wei (2006) (APW) report that yield spreads with different choices of long-term yield maturities also have predictive power for the output. Thus, the choice of maturities for the yield spread for predicting macro variables is not trivial.

In this paper, I propose an approach, which allows using information contained in the entire yield curve and not limiting only to information in exogenously defined yield spread to predict real GDP growth and inflation. In particular, I examine the predictive power of the entire yield curve for real output and inflation by jointly modeling macro variables and yield curve using Diebold and Li’s (2006) (DL) dynamic yield curve model, which is based on the Nelson-Siegel (1987) (NS) three latent factor framework. The choice of the NS model for this study is driven

by its parsimony and good out-of-sample performance. This model describes the entire yield curve structure only by three factors. DL introduce dynamics to evolution of these three factors to show that the NS model has good in-sample fit and produces good forecasts of future yields at long horizons relative to other standard simple models. Use of DL's dynamic model for predicting macro variables has two advantages over the standard yield spread framework: (i) the model contains information about the entire yield term structure and (ii) macro variables can be modeled jointly with yields in a parsimonious way using endogenously defined three factors. Thus, this approach avoids the problem of exogenous choice of the yield spread for predicting macro variables and allows extracting more information from the entire yield curve than from the yield spread only. Another potential choice of term structure modeling would be affine arbitrage-free class of models, which is popular in finance literature. However, as reported by Duffee (2002), arbitrage-free models produce poor out-of-sample forecasts of future yields.

APW study predictive power of short-term yield and yield spread for real GDP growth using affine arbitrage-free dynamic yield curve model. Their approach is based on modeling real GDP growth jointly with exogenously defined short-term yield and yield spread with imposed no-arbitrage constraints on the pricing of bonds. They show that the dynamic way of modeling evolution of yields and GDP improves forecasts of GDP growth over the OLS yield spread model. However, in contrast to the previous findings, they find that the short-term interest rate has more predictive power for the GDP growth than the yield spread.

The focus of my analysis is to find out whether using the entire yield curve and endogenously defined factors improves forecasts of output and inflation over the simple OLS framework with arbitrarily defined yield spread. I perform out-of-sample forecast comparisons for the real GDP growth and inflation using root mean square errors (RMSE) from the dynamic yield curve model and the standard yield spread models based on OLS regressions of GDP growth and inflation-difference on yield spread. I consider various versions of the dynamic yield curve model where real GDP growth and inflation are explained by different yield factors and their lagged values to analyze marginal effect of those factors for forecasting performance.

I find that the dynamic yield curve model significantly improves out-of-sample forecasts of real GDP growth over those produced by the standard yield spread model for all horizons. Thus, using the dynamic yield curve model provides a more accurate depiction of the predictive power

of the yield curve and uses information contained in the entire yield curve compared to the OLS yield spread model. The OLS yield spread model has good in-sample fit, while it has poor out-of-sample performance. This result may be partially explained by a tendency of the OLS regression to overfit in sample.

The results of predicting inflation based on the dynamic yield curve model do not suggest robust improvement over the OLS yield spread model. Forecasting performances of both the dynamic yield curve and the OLS yield spread models relative to each other and to $AR(p)$ are sensitive to the choice of out-of-sample period. My results suggest that the predictive power of the yield curve for inflation weakened in post-1982 period. Thus, the yield curve should be used for predicting inflation with caution.

The rest of this paper is organized as follows. Section 2 and 3 describe data and standard yield spread OLS forecasting models and reports predictive power of these models for output and inflation. Section 4 describes the dynamic yield curve model and reports estimation results. Section 5 reports out-of-sample forecasting results and compares various versions of dynamic yield curve and yield spread models. Section 6 concludes.

2. Data

The interest rate data are on monthly average zero-coupon yields on US government bonds for maturities 3, 6, 12, 24, 36, 60, 84, 120 months obtained from FRED database. The yields are constant maturity rates, except for 3 and 6 month maturities that are secondary market rates¹. Yield data for the maturities 3, 12, 36, 60, 120 months are from 1953:04 to 2007:12, for 6 months from 1959:01 to 2007:12, for 24 months from 1976:07 to 2007:12, for 84 months from 1969:07 to 2007:12². Quarterly data on real GDP and inflation from 1952:Q1 to 2007:Q4 are also from FRED database. Real GDP data are seasonally adjusted and chained in 2000 prices. To measure inflation I use personal consumption expenditure (PCE) index. In general the PCE and

¹ I use secondary market rate data for 3 and 6 month maturities because the constant maturity rate data for these maturities are substantially shorter than the period I use for this study. I compared secondary market 3 and 6 month maturity yield series with the constant maturity rate series for common periods and found that the dynamics of series are close to each other, and therefore I believe this heterogeneity in data should not significantly affect my results.

² Yields are not extrapolated to the same periods as the focus of this study is predictive power of yields on macro variable using available information. Monthly data on yields are transformed to quarterly frequency by using data for last month of quarters.

CPI measures of inflation are close to each other. The advantage of PCE inflation is that it is a chain weighted series, while CPI index is based on fixed weights. Annualized real GDP growth and inflation rates are calculated from the indices using log transformation factorized by 400.

Table1 shows descriptive statistics of yields, real GDP growth, and inflation.

Table 1

Descriptive statistics of yields and macro variables

Maturities (months)	Period	Mean	Std. Dev.	Min.	Max.	ADF
3	1953-M04 : 2007-M12	5.11	2.79	0.64	16.30	-2.69
6	1959-M01 : 2007-M12	5.63	2.68	0.92	15.52	-2.12
12	1953-M04 : 2007-M12	5.67	2.96	0.82	16.72	-2.04
24	1976-M07 : 2007-M12	6.89 ¹	3.15	1.23	16.46	-1.21
36	1953-M04 : 2007-M12	6.07	2.82	1.47	16.22	-1.92
60	1953-M04 : 2007-M12	6.26	2.75	1.85	15.93	-1.77
84	1959-M07 : 2007-M12	7.41 ¹	2.56	2.84	15.65	-1.25
120	1953-M04 : 2007-M12	6.46	2.68	2.29	15.32	-1.60
RGDP growth	1953-Q2 : 2007-Q4	3.14	3.66	-11.02	15.46	-10.51 *
Inflation (PCE based)	1953-Q2 : 2007-Q4	3.42	2.46	-1.18	11.74	-2.38

RGDP growth and Inflation are log transformed series factored by 400. The Augmented Dickey-Fuller (ADF) unit root test is based on SIC lag selection. The critical values for rejection of hypothesis of a unit root are: -3.440 at 1% level and -2.866 at 5% level. The hypothesis that yields and inflation have a unit root cannot be rejected at 5% level. The hypothesis that real GDP growth has a unit root is rejected at 1% level, denoted by an asterisk.

¹ Average yields of 24 and 84 month bonds are higher than those of 36 and 120 month respectively because of difference in periods.

3. Motivation

3.1. Predictive power of the yield curve for output

The standard explanation for why yield spread may predict economic growth is based on the expectation hypothesis. Under this theory the term structure of interest rates is determined by agents' expectation of future short-term interest rates, and therefore any current long-term interest rate is an average of expected future short-term rates. If a monetary contraction sends the current short-term rate higher than the expected future short-term interest rate, then today's investment and consumption will decline causing decline in future economic growth. Conversely, if a monetary expansion produces low current short-term interest rate leading to higher economic growth in future, then future short-term interest rate is expected to increase. Thus, in theory the yield curve contains information about future economic growth. The term-premium for holding long-term bonds is also a component that contributes to determination of the term structure of interest rates in addition to expectation factor. Ang, Piazzesi, and Wei (2006) suggest that the expectation hypothesis component of the term structure of interest rates is

the main driving force for output predictability. Hamilton and Kim (2002) suggest that the term-premium, in addition to expectation component, is also important for output prediction.

Most previous studies of predictive power of yield curve for real economic activity have employed OLS regressions of real GDP growth rates on the yield spread defined as a difference between interest rates on the long-term (10 years) treasury bonds and short-term (3 month) treasury bills:

$$g_{t,t+k} = \alpha_{0,k} + \alpha_{1,k}(y_t(120) - y_t(3)) + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (1)$$

where $g_{t,t+k}$ is annualized real GDP (RGDP) growth rate defined as:

$$g_{t,t+k} = 400/k(\ln RGDP_{t+k} - \ln RGDP_t) \quad (2)$$

$y_t(120)$ and $y_t(3)$ are interest rates on 10-year and 3-month treasury bonds and bills respectively.

Figure 1 which plots the yield spread defined as above and annualized real GDP growth rate over next four quarters suggests existence of a positive correlation between real GDP growth and the yield spread.

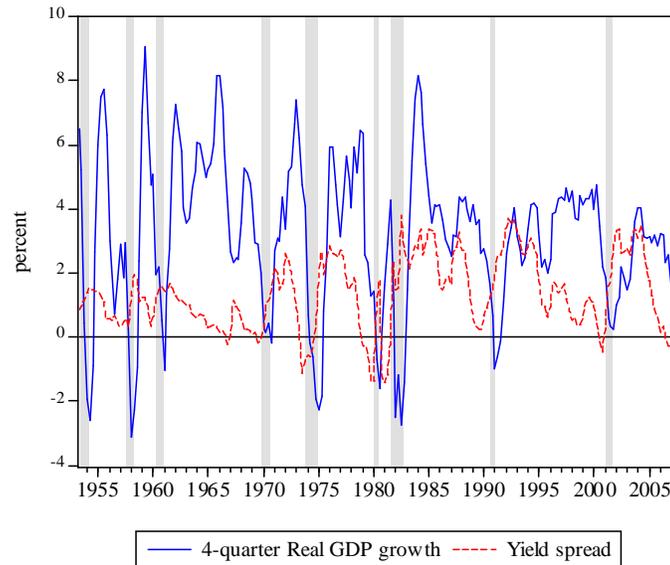


Figure1. Four-quarter real GDP growth rate and yield spread. Shaded areas are NBER recession dates.

Table 2 reports estimation results of OLS regressions of future real GDP growth on the spread only according to equation 1, spread and one period lagged real GDP growth, short rate only (defined as 3 month T-bills interest rate), and spread and short rate for the period from 1953:Q2 to 2007:Q4. The estimates for the yield spread coefficient from the yield spread only regression is statistically significant for all horizons up to 12 quarters ahead and *adjusted R²* are substantially higher for 4 and 8 quarter horizons than those for 1 and 12 quarter horizons. The estimate for the yield spread coefficient remains robust to controlling for one period lagged real GDP growth, which increases *adjusted R²* only at the one quarter horizon. It can be explained by short-term persistence of real GDP growth. I also consider explanatory power of short-term interest rate for future real GDP growth. Although the short-term interest rate is statistically significant in the short-term rate only regression, *adjusted R²* of this regression is lower than one from the yield spread only model. The yield spread remains strongly statistically significant after controlling for short-term rate up to 8 quarters ahead, while short-term rate remains significant only at 4 quarter ahead. These results, which are in line with previous research findings on predictive power of yield spread for output, suggest that the yield spread may be used to predict real output.

Table 2
 Parameter estimates: OLS regressions of k-quarter ahead annualized real GDP growth on yield spread
 Sample period: 1953Q2-2007Q4

horizon k-quarters	Spread		Spread Lagged g			Short Rate		Spread Short Rate		
	$\alpha_{1,k}$	R ² adj.	$\alpha_{1,k}$	$\alpha_{2,k}$	R ² adj.	$\alpha_{1,k}$	R ² adj.	$\alpha_{1,k}$	$\alpha_{2,k}$	R ² adj.
1	0.600 (0.239)	0.03	0.606 (0.227)	0.171 (0.067)	0.06	-0.263 (0.123)	0.04	0.640 (0.276)	-0.184 (0.133)	0.07
4	0.757 (0.188)	0.15	0.756 (0.189)	-0.007 (0.051)	0.14	-0.276 (0.097)	0.10	0.704 (0.230)	-0.189 (0.094)	0.21
8	0.556 (0.136)	0.16	0.554 (0.137)	-0.029 (0.039)	0.16	-0.173 (0.081)	0.09	0.474 (0.170)	-0.113 (0.077)	0.18
12	0.313 (0.119)	0.08	0.310 (0.121)	-0.032 (0.030)	0.09	-0.086 (0.071)	0.03	0.255 (0.138)	-0.054 (0.070)	0.07

In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors

3.2. Predictive power of the yield curve for inflation

According to the expectation hypothesis the term structure of interest rates should also contain information about expected future inflation. Mishkin (1990a, 1990b) finds that the short-term yield spread has no predictive information about future inflation, while long-term spread can

predict long-term inflation. His approach is based on Fisher decomposition of nominal interest rate on real interest rate and expected inflation:

$$y_t(m) = r_t(m) + E_t\pi_{t,t+m} \quad (3)$$

where $y_t(m)$ is m-maturity interest rate, $r_t(m)$ ex-ante annualized real interest rate over the next m periods, $E_t\pi_{t,t+m}$ is annualized expected inflation rate over the next m periods with $\pi_{t,t+m}$ defined as:

$$\pi_{t,t+m} = 400/m(\ln PCE_{t+m} - \ln PCE_t) \quad (4)$$

Assuming that expectations are rational and real interest rate is constant over long period the equation, Mishkin (1990a) derived the following empirical form for inflation-difference:

$$\pi_{t,t+m} - \pi_{t,t+n} = \alpha_{0,mn} + \alpha_{1,mn}(y_t(m) - y_t(n)) + u_t \quad u_t \sim N(0, \sigma_u^2) \quad (5)$$

Thus, this framework suggests that yield spread should predict the change in future inflation. Since my study focuses on out-of-sample forecast of inflation in levels I employ the specification used by Stock and Watson (2003) in their inflation forecasting research:

$$\pi_{t,t+k} - \pi_t = \alpha_{0,k} + \alpha_{1,k}(y_t(m) - y_t(n)) + u_t \quad (6)$$

where

$$\pi_t = 400(\ln PCE_t - \ln PCE_{t-1}) \quad (7)$$

Table 3 reports estimation results of equation (6) for various combinations of forecasting horizons and definitions of yield spread. The regression with 3-month maturity interest rate as short-end of the yield spread produces statistically insignificant estimates of the yield spread coefficient and very low or even negative *adjusted R*² at all horizons (not reported here to save space). The estimates for the spread become statistically significant with short-end of the spread changed from 3-month yield to 1 year yield. The coefficient for spread $\alpha_{1,k}$ is statistically significant for long-term forecast horizons from 12 to 20 quarters ahead. The highest *adjusted R*² is at 16-20 quarter horizons. For the horizons 8 quarter ahead and shorter (not reported here to save space) the spread is not statistically significant and *adjusted R*² is very small. The estimates for long-term horizons remain relatively robust to controlling for lags of inflation-difference.

Thus, these results suggest that predictive power of yield spread for inflation is concentrated at long horizons and it is sensitive to the choice of yield spread.

Table 3

Parameter estimates: OLS regression of inflation-difference on yield spread. Sample period: 1953Q2-2007Q4

Spread	on yield spread only		on yield spread and lagged inflation-difference		
	$\alpha_{1,k}$	R^2 adjusted	$\alpha_{1,k}$	α_2 (for lag)	R^2 adjusted
<i>inflation-difference=12 quarter average inflation - 1 quarter inflation</i>					
5years - 1 year	0.561 (0.252)	0.06	0.506 (0.258)	-0.354 (0.071)	0.12
10years - 1 year	0.423 (0.182)	0.07	0.388 (0.187)	-0.357 (0.070)	0.11
<i>inflation-difference=16 quarter average inflation - 1 quarter inflation</i>					
5years - 1 year	0.745 (0.283)	0.10	0.683 (0.288)	-0.364 (0.075)	0.15
10years - 1 year	0.554 (0.207)	0.10	0.512 (0.213)	-0.368 (0.074)	0.15
<i>inflation-difference=20 quarter average inflation - 1 quarter inflation</i>					
5years - 1 year	0.792 (0.324)	0.11	0.724 (0.330)	-0.369 (0.079)	0.15
10years - 1 year	0.582 (0.245)	0.10	0.535 (0.252)	-0.374 (0.079)	0.14

The BIC method suggests one lag for reported specification and sample period. Lagged inflation-difference is defined as $\Delta\pi_{t-1}=\pi_{t-1}-\pi_{t-2}$. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors

My results are in line with findings of Mishkin (1990b) and Estrella and Mishkin (1997). Mishkin (1990b) reports the highest predictive power of yield spread for the change in inflation at horizons 16 and 20 quarters ahead. Estrella and Mishkin (1997) finds that yield spread remains statistically significant after controlling for the lag of inflation-difference at 11-20 quarter horizons with the highest R^2 at 16 and 20 quarter horizons.

Thus, the above analysis suggests that the yield spread contains information about future economic growth and long-term inflation. These OLS forecasting regressions use only an exogenously defined yield spread, which contains only limited information about the entire yield curve. Results are sensitive to the choice of yield maturities used to determine yield spread. Also parameter estimates reflect potential OLS overfitting in sample.

4. Model and estimation results

This section describes the yield curve model that I use to jointly model the entire term structure of interest rates and the macro variables. By employing endogenously defined yield factors for output and inflation modeling I avoid the need to exogenously define the yield spread.

4.1. The dynamic yield curve model

I consider three latent factor dynamic yield curve model developed by Deibold and Li (2006) (DL), which is based on the Nelson-Seigel (1987) (NS) framework. In this dynamic model yields are represented by the following functional form:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \underbrace{\left(\frac{1 - \exp(-\lambda_t \tau)}{\lambda_t \tau} \right)}_{L_2(\tau, \lambda_t)} + \beta_{3t} \underbrace{\left(\frac{1 - \exp(-\lambda_t \tau)}{\lambda_t \tau} - \exp(-\lambda_t \tau) \right)}_{L_3(\tau, \lambda_t)} \quad (8)$$

where $y_t(\tau)$ is an interest rate of zero-coupon bond with maturity τ month at period t ; $\beta_{1t}, \beta_{2t}, \beta_{3t}$ are three latent dynamic factors interpreted as level, slope, and curvature of the yield curve; and λ_t is a parameter responsible for fitting yield curve at different maturities. Small values of λ_t fit curve better at long maturities, while large values at short maturities. Similar to DL, to fit the yield curve, λ_t is set to be time invariant for simplicity, and therefore I drop its time subscript. $L_2(\tau, \lambda)$ and $L_3(\tau, \lambda)$ denote the loadings for factors β_{2t}, β_{3t} respectively. The loading for factor β_{1t} is 1.

The choice of the NS framework model is motivated by its parsimony and good out-of-sample forecast performance. The alternative yield curve model to consider for this study would be affine arbitrage-free yield curve model. However, as reported by Duffee (2002), arbitrage-free yield curve models perform poorly out-of-sample. The NS framework models the entire panel of interest rates by three key latent factors with imposed exponential structure of loadings. DL show that this model can generate all possible yield curve shapes, has good in sample fit, and produces out-of-sample forecasts of future yields better than many simple standard yield models at 6 months and longer horizons. DL show that β_{1t} factor is highly correlated with yields of different maturities, and therefore is interpreted as level factor; $-\beta_{2t}$ factor is highly correlated with the yield spread; and β_{3t} is correlated with empirical curvature of the yield curve. In this model all three latent factors are assumed to be stationary, which is standard assumption. I show later that this model is also flexible for incorporating macro variables.

Diebold, Rudebusch, and Aruoba (2006) (DRA) show that the panel of yields can be casted in the state-space form. The measurement equation of the state space system is given as follows:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & L_2(\tau_1, \lambda) & L_3(\tau_1, \lambda) \\ 1 & L_2(\tau_2, \lambda) & L_3(\tau_2, \lambda) \\ \vdots & \vdots & \vdots \\ 1 & L_2(\tau_n, \lambda) & L_3(\tau_n, \lambda) \end{pmatrix} \begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \end{pmatrix} \quad \varepsilon_t \sim N_n(0, R) \quad (9)$$

The variance-covariance matrix of measurement errors of this equation is denoted as R . Similar to DRA, I assume that derivations of yields of different maturities are independent from each other, and therefore R is a diagonal matrix, which is also standard assumption for this model.

The latent factors are modeled as a Gaussian first order vector autoregressive process:

$$\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} \beta_{1t-1} \\ \beta_{2t-1} \\ \beta_{3t-1} \end{pmatrix} + \begin{pmatrix} q_{1t} \\ q_{2t} \\ q_{3t} \end{pmatrix} \quad q_t \sim N_3(0, Q) \quad (10)$$

Q denotes the variance-covariance matrix of error-term of the state equation (10). I allow for correlations among shocks to the latent factors, therefore, Q is unrestricted full positive definite matrix.

I estimate the model using one-step Kalman filter maximum-likelihood procedure, which produces more efficient inferences than those from the two-step estimation procedure applied by DL and Ang, Piazzesi, and Wei (2006). In-sample estimation of the dynamic yield curve model is based on quarterly yield data for the period from 1953Q2 to 2007Q4.

The largest estimated eigenvalue of the Φ matrix is close to unity (0.974). The Augmented Dickey-Fuller (ADF) tests for unit root of $\beta_{1t}, \beta_{2t}, \beta_{3t}$ suggest that β_{1t} may have unit root with *p-value* 0.575 while β_{2t}, β_{3t} are stationary with *p-values* 0.002 and 0.000 respectively. The ADF tests for unit root of all yields reported in the last column of Table1 indicate that all yields may have unit roots. Cointegration tests, using the Johansen (1988,1991) methodology suggests that the yields are cointegrated with each other³. Based on these results, I also consider the version of the model where yields are assumed to be cointegrated unit root processes⁴. Forecast results for

³ The cointegration test suggests that elements of the vector of 3, 12, 36, 120 month yields are cointegrated with each other at 5 percent level. Also the ADF test for a unit root suggests that inflation may have unit root. The cointegration test also suggests that inflation and yields are cointegrated at 5 percent level.

⁴ In the unit root specification, assumptions that β_{1t} is unit root and β_{2t}, β_{3t} are stationary achieved through restricting ϕ_{11} to 1 and ϕ_{21} and ϕ_{31} to zeros. These restrictions are close to point estimates of $\phi_{11}, \phi_{21},$ and ϕ_{31} in the stationary specification of the model.

macro variables are close to each other in the stationary and unit root specifications and there is no dominant model⁵. Therefore, I report only results in stationary specification of the model.

4.2. Dynamic yield curve model with macro variables.

This subsection describes the way of incorporating macro variables into the dynamic yield curve model. Since the macro variables are correlated with yields and yields are described by three factors, the macro variables have to be correlated with yield factors of the model⁶. Therefore I modify the yield curve model to jointly model yields with real GDP growth rate and inflation-difference using three yield factors. Previous analysis suggested that lagged real GDP growth and inflation-difference improve forecasts of macro variables, and therefore I also control for lags of the macro variables in this model.

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_n) \\ g_t \\ \Delta\pi_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \mu_g \\ \mu_\pi \end{pmatrix} + \begin{pmatrix} 1 & L_2(\tau_1, \lambda) & L_3(\tau_1, \lambda) \\ 1 & L_2(\tau_2, \lambda) & L_3(\tau_2, \lambda) \\ \vdots & \vdots & \vdots \\ 1 & L_2(\tau_3, \lambda) & L_3(\tau_3, \lambda) \\ \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{22} & \gamma_{23} \end{pmatrix} \begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \beta_{3t} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \gamma_3 g_{t-1} \\ \sum_{k=1}^p \gamma_{4k} \Delta\pi_{t-k} \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_n) \\ \varepsilon_t(g) \\ \varepsilon_t(\pi) \end{pmatrix} \quad (12)$$

$$\varepsilon_t \sim N_{n+2}(0, \Sigma)$$

where g_t and $\Delta\pi_t$ are real GDP growth rate and inflation-difference defined as:

$$g_t = 400(\ln RGDP_t - \ln RGDP_{t-1})$$

$$\Delta\pi_t = \pi_t - \pi_{t-1} \quad \text{where } \pi_t = 400(\ln PCE_t - \ln PCE_{t-1})$$

Macro variables enter only into the measurement equation while the transition equation remains the same as in the dynamic yield only model. Thus, in this setting macro variables are modeled only by the yield latent factors, which are mainly defined by term structure of interest rates due to rich panel of yields. This approach focuses on interaction in the direction from yields to macro variables.

⁵ The unit root yield curve model produces lower RMSE than the stationary model at long horizons. Forecasting performances of the models for macro variables relative to each other are mixed.

⁶ DRA find evidence of yield curve effects on macro variables based on analyses of impulse response functions and variance decompositions. They do not study forecasting performance of the macro-yield model. They model macro variables as additional factors in the state dynamics of the yield curve model.

To ensure consistency with the Fisher equation framework inflation-difference in the dynamic yield curve model does not include the level factor as it is presumed to be canceled out due to differencing of inflation levels and high persistence of the level factor⁷. I also considered a version of the model where inflation enters into the model in levels. My results in this case (not reported here to keep analysis concise) suggest that the model with inflation in levels produces worse in sample fitting and out-of-sample forecasting results than the model with inflation in differences. Therefore, I focus my analysis using only the model with inflation in differences. This result is consistent with Stock and Watson (2003).

4.3. Estimation results of the dynamic yield curve model with macro variables

Table 4 reports estimates of the macro variables' parameters in the dynamic yield curve model. The negative sign of statistically significant estimate of slope coefficient γ_{12} for real GDP growth is consistent with interpretation of β_{2t} as a minus the slope of the yield curve. Statistical insignificance of coefficient γ_{11} in the stationary model can be explained by high persistence of β_{1t} factor relative to the variability of real GDP growth rate. This suggests that the level of yields might not be important for modeling real GDP growth after controlling for the yield slope. The estimate of the coefficient for lagged real GDP growth, denoted as γ_3 , is statistically significant and its value is comparable with the estimate in *AR(1)* model suggesting that the autocorrelation component remain important after controlling for yield factors. With regards to inflation differences, the estimate of slope coefficient γ_{22} is also statistically significant, although it has positive sign indicating a counterintuitive negative relationship between inflation-difference in yield slope⁸. However, the value of the coefficient and its statistical significance is sensitive to controlling for the curvature factor and lags of inflation-difference, indicating weak relationship between yield curve and inflation at short horizons. The BIC method suggests including two lags in the dynamic yield curve model with inflation differences.

⁷ To check validity of this assumption, I also estimated the model with inflation-difference modeled by all three factors. The point estimate of the coefficient for the level factor is small in magnitude and statistically insignificant.

⁸ The OLS yield spread model also produces negative sign of the coefficient for the yield spread at short horizons, although the estimate is statistically insignificant. In order to check whether the negative sign on slope of the yield curve is driven by changes in the short-term interest rate as a result of monetary policy reaction to the current level of inflation, I also estimated the yield curve model with controlled short-term interest rate. The sign does not change after controlling for the short-term interest rate and changing the specification from contemporaneous to one period ahead forecasting.

Table 4 : Dynamic yield curve model: estimates of loading factors for macro variables

Real GDP growth					Inflation-difference				
γ_{11}	γ_{12}	γ_{13}	γ_3	μ_{gdp}	γ_{22}	γ_{23}	γ_{41}	γ_{42}	μ_{inf}
-0.153	-0.365	0.263	0.303	2.549	0.164	-0.064	-0.459	-0.263	0.300
(0.118)	(0.171)	(0.148)	(0.064)	(0.796)	(0.061)	(0.050)	(0.066)	(0.066)	(0.130)

4.4. In-sample results

Table 5 reports statistics of measurement errors of yields, real GDP growth, and inflation-difference from in-sample fit by the dynamic yield curve, OLS yield spread, and univariate autoregressive models. All models with real GDP growth control for its one period lagged value and models with inflation-difference for its lagged values with the number of lags determined by BIC method.

The dynamic yield curve model has better fit of real GDP growth at 1 quarter horizon while the OLS yield spread model has better fit at long horizons. The models have similar fit of inflation-difference at short horizons, while the OLS yield spread model has better fit at long horizons.

The fit of real GDP growth and inflation by the OLS yield spread model at long horizons is explained by the forecasting specification of the model and the nature of OLS regression to minimize square residuals. Specifically, the OLS yield spread model has an advantage in in-sample fit over the dynamic yield curve model, because former is a forecasting model at targeted horizons, while the dynamic yield curve model fits the current data⁹.

Also since the OLS regression minimizes square of errors of fit at a targeted horizon, it may cause overfitting the data, further affecting out-of-sample performance of the forecasting OLS yield spread model. Both the dynamic yield curve model and the OLS yield spread model have better fitting results than those produced by univariate autoregressive models.

⁹ To check this point, I estimated the dynamic yield curve model with the specification changed to be more like a forecasting model. Even with forecasting specification at one period ahead and iteration for longer horizons, in-sample fit by the forecasting dynamic yield curve model improved over the results of the OLS yield spread models for most of horizons. Despite the obvious advantage of forecasting specification of the dynamic yield curve model, I use the contemporaneous version of the model for this study as it uses all available current information for out-of-sample forecasting. Out-of-sample forecasting results suggest that the contemporaneous model outperforms the model with forecasting specification.

Table 5
 Statistics of measurement errors of yields and macro variables

maturity& macro variable	Dynamic Model		OLS		AR(p)	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
3	0.01	1.01				
12	0.17	1.01				
36	0.02	0.83				
60	- 0.01	0.74				
120	0.02	0.61				
<i>RGDP growth</i>						
1 quarter ahead	0.00	3.43	0.00	3.55	0.00	3.48
4 quarter ahead	0.05	2.23	0.00	2.16	0.06	2.30
8 quarter ahead	0.07	1.59	0.00	1.48	0.09	1.64
<i>Inflation (in levels from models with inflation in differences)</i>						
1 quarter ahead	0.01	1.21	0.00	1.21	0.00	1.21
8 quarter ahead	0.02	1.46	0.00	1.46	- 0.03	1.48
16 quarter ahead	0.05	1.63	0.00	1.56	- 0.04	1.67

The dynamic yield curve model include one lag of real GDP growth and lags of inflation-difference determined by BIC. OLS denotes (i) a regression of RGDP growth on spread and one lag RGDP growth and (ii) a regression of inflation-difference on yield spread and lags of inflation-difference. AR(p) denotes AR(1) for RGDP growth and AR(p) for inflation-difference.

5. Out-of-sample forecasting results

5.1. Forecasting procedure and notation

I perform out-of-sample forecasts of real GDP growth and inflation for the period from 1990:Q1 through 2007:Q4. To compare forecast performance of models I use root mean square errors (RMSE) relative to one from the benchmark model. Following Stock and Watson (2003), I also use RMSE from $AR(1)$ and $AR(p)$ models as benchmarks for real GDP growth and inflation, respectively, where the number of lags of inflation-difference is determined by the BIC method.

To generate RMSE statistic from the dynamic yield curve model I use the following procedure. First I estimate parameters of the state-space model using Kalman filter method and then recursively forecast yields, real GDP growth, and inflation for 1 to 20 quarters ahead. Next I add one more observation to the data and repeat estimation of the model parameters and forecasting of the variables. This procedure produces $73-k$ observations of k -quarter-ahead out-of-sample forecasts for k from 1 to 20 quarter horizons. I report forecast results for real GDP growth only for horizons up to 12 quarters where according to in-sample estimation results the yield spread has most of its predictive power. For inflation forecasting, given that the OLS model

predicts inflation at long horizons and policy interest in long-term inflation forecasts, I look at results up to 5 years.

I compare out-of-sample forecast performance of two classes of models: the dynamic yield curve models and the OLS yield spread models. In each class of models I consider several models with different explanatory variables for real GDP growth and inflation. I denote class of dynamic yield curve models as NS and the OLS yield spread models for real GDP growth as G , and the OLS yield spread models for inflation-difference as ΔINF . To denote specification of a model in each class of models I list explanatory variable used to model real GDP growth and inflation in parentheses. For example, the notation $NS(g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3, \Delta\pi lags))$ means that this is the dynamic yield curve model with real GDP growth modeled by β_2, β_3 factors and its one period lag, inflation rate modeled in differences by latent factors β_2, β_3 , and lags.

5.2. Forecasts of real GDP growth

In this subsection I analyze real GDP forecast performance of the dynamic yield curve model relative to the OLS yield spread model. Table 6 reports RMSEs of different versions of the dynamic yield curve and OLS yield spread models. I analyze forecast performance in two ways. First, I analyze effect of different explanatory factors for forecasting real GDP growth. Second, I compare forecasts from relevant dynamic yield curve and OLS yield spread models.

5.2.1. The effect of yield factors and other variables

First, the dynamic yield curve model with lagged real GDP growth has lower RMSEs than those from the model without lagged real GDP. Most of the improvement is observed at short horizons. Similarly, in the OLS yield spread model adding lagged real GDP growth improves forecasts at short horizons. The positive effect of the autoregressive component on short-term horizon forecasts reflects short-term persistence of real GDP growth.

Second, RMSEs from models with curvature factor β_3 are smaller than those from models without this factor in real GDP growth modeling at all horizons. It may be explained by the fact that the curvature factor is the most volatile factor among all three factors and it may contain

some information useful for predicting business cycles in addition to slope factor β_{2t} which is more persistent than real GDP growth. Thus, although the gain from adding the curvature factor to the slope factor for modeling real GDP growth is small, it still allows extracting additional information contained in yield curve for real GDP modeling, while the OLS yield spread model does not contain this information.

Table 6
Out-of-sample forecasts of real GDP growth rate: Root Mean Square Error Ratios
Out-of-sample period 1990:Q1-2007:Q4

	Forecast horizon		k-quarters ahead	
	1	4	8	12
<i>Dynamic yield curve model</i>				
NS(g(β_2))	1.052	1.063	1.042	1.030
NS(g(β_2, β_3))	1.037	1.057	1.026	1.004
NS(g(β_2, g_{t-1}))	1.013	1.027	1.029	1.029
NS(g($\beta_2, \beta_3, g_{t-1}$))	1.008	1.020	1.007	0.997
NS(g($\beta_2, \beta_3, g_{t-1}$), $\Delta\pi(\beta_2, \beta_3)$)	1.008	1.020	1.007	0.997
<i>OLS yield spread model</i>				
G(spread)	1.130	1.365	1.366	1.197
G(spread, g)	1.076	1.323	1.382	1.208
G(spread, g, π)	1.082	1.353	1.454	1.303
G(short rate, spread)	1.161	1.462	1.501	1.290
Unconditional in-sample mean	1.038	1.062	1.056	1.039

NS and G denote the dynamic yield curve and OLS yield spread models respectively.
Denominator is RMSE from AR(1). Lowest RMSE ratios within each class of models are in bold.

Third, the effect of modeling real GDP jointly with inflation on forecasts is negligible: RMSEs from the dynamic yield curve model with and without inflation are very close to each other. This is related to the fact that although inflation and real GDP growth are modeled jointly, inflation is not an explanatory variable for real GDP growth, and yield factors are mainly identified by the panel of yields. In the OLS yield spread model adding inflation makes RMSEs worse. This may be explained by correlation of inflation and yields, so that adding inflation may cause multicollinearity producing less precise estimates.

5.2.2. *Does the dynamic yield curve model forecast output better than the OLS yield spread model?*

To answer the question whether the dynamic yield curve model improves forecasts of real GDP growth over the OLS yield spread model I compare RMSEs from the following pairs of models with comparable explanatory variables for real GDP growth: $NS(g(\beta_2))$ and $G(\text{spread})$; $NS(g(\beta_2, g_{t-1}))$ and $G(\text{spread}, g)$; $NS(g(\beta_2, \beta_3, g_{t-1}))$ and $G(\text{spread}, g)$. Table 6 reports noticeably lower RMSEs from the dynamic yield curve models than from OLS yield spread models for all horizons. The Diebold-Mariano (1995) (DM) test of forecast accuracy comparison, reported in Table 7, suggests that these differences in RMSEs are statistically significant. This result remains robust to controlling for lagged real GDP growth in both models. Thus, I conclude that the dynamic yield curve model forecasts real GDP growth better than the OLS yield spread model.

Table 7
Diebold-Mariano test for real GDP growth forecast accuracy comparison

Models	Forecast horizon			
	1	4	8	12
NS($g(\beta_2)$) against G(spread)	-0.708 *	-1.380 *	-0.820 *	-0.274 *
NS($g(\beta_2, g_{t-1})$) against G(spread, g)	-0.552 *	-1.308 *	-0.894 *	-0.294 *
NS($g(\beta_2, \beta_3, g_{t-1})$) against G(spread, g)	-0.591 *	-1.334 *	-0.940 *	-0.343 *
NS($g(\beta_2, \beta_3, g_{t-1})$) against AR(1)	0.067	0.076	0.015	-0.005
G(spread, g) against AR(1)	0.658 *	1.410 *	0.955 *	0.338 *

NS and G denote the dynamic yield curve and OLS yield spread models respectively. The null hypothesis of the test is mean of square loss-differential of two compared models is zero, against alternative that it is negative. An asterisk indicates statistical significance of estimates at 5 percent level. Significantly negative (positive) value of the estimate indicates that the first model produces more (less) accurate forecasts than compared model. The test is based on Newey-West heteroskedasticity and autocorrelation consistent std.errors.

There are several sources of this gain. First, the yield curve model describes the macro variables by endogenously and dynamically determined factors using the information in the entire yield curve, while the OLS forecasting model uses only information on difference between yields for two specific maturities. Modeling macro variables by endogenously determined factors

avoids the problem of dependence of results on arbitrary choice of the maturities for the yield spread¹⁰. Real GDP forecasts based on the slope factor are significantly better than those from the OLS model based on exogenously defined spread. Therefore, the dynamic yield curve model, through endogenously determined factors, extracts more information from the entire yield term structure than the OLS yield spread model for real GDP forecasting.

Second, although the main gain for real GDP forecasting comes from endogenously determined slope factor and dynamic structure of the model, there is also additional gain from using curvature factor.

Third, as I noted earlier, the OLS regression for targeted forecasting horizon may cause over fitting of in-sample data due to its least square nature of the inferences. This point is supported by the fact that the OLS yield spread model performs even worse than the simple unconditional in-sample mean of the real GDP growth, while the OLS yield spread model has the best in-sample fit. Thus, poor out-of-sample performance of the OLS yield spread model indicates that the yield curve is less useful for GDP forecasting than in-sample OLS regression suggests.

5.2.3. *Do the OLS yield spread and dynamic yield curve models forecast output better than the simple autoregressive model?*

It is important to note that neither the dynamic yield curve nor OLS yield spread models can beat the $AR(1)$ model in out-of-sample period. The DM test, reported in Table 7, suggests that the $AR(1)$ model produces on average significantly smaller forecast errors than the OLS yield spread model. Although the differences in RMSEs from the dynamic yield curve model and $AR(1)$ are small in magnitude and statistically insignificant, the dynamic yield curve model still cannot improve over $AR(1)$. This result can be explained in three ways.

First, overfit of data by the OLS regression described earlier may cause underperformance of the OLS yield spread model relative to $AR(1)$. Second, there is evidence for structural instability in yield curve and output relationship reported in the literature. Haubrich and Dombrosky (1996) and Dotsey (1998) find a decline in predictive ability of the yield curve for output in period after 1985. Estrella, Rodriguez, and Schich (2003), using the test for unknown break date, also find

¹⁰ Ang, Piazzesi, and Wei (2006) reports that RMSEs from the OLS model with long-end of the yield spread at the longest maturity are not always lower than those with shorter maturities.

some evidence of structural instability in yield spread and industrial production relationship in 1983. To analyze the issue of structural stability of the relationship between yield curve and output, I perform out-of-sample forecasts of real GDP using the OLS yield spread model based on shorter in-sample period 1982:Q4 to 1997:Q4. The choice of the beginning of in-sample period is motivated by the last major change in monetary policy in October 1982. The first two panels in Table 8 report RMSE ratios from the OLS yield spread model based on full sample and sub-sample periods. The RMSEs from the OLS yield spread model based on post monetary policy regime shift period noticeably declined. Thus, this result suggests a regime shift in the relationship between the yield curve and output.

Table 8
Out-of-sample forecasts of real GDP growth rate: Root Mean Square Error Ratios
Different in-sample and out-of-sample periods

	Forecast horizon		k-quarters	
	1	4	8	12
<i>In-Sample period: 1953Q2-1997:Q4; Out-of-sample period 1998-2007</i>				
NS($g(\beta_2, \beta_3, g_{t-1})$)	0.995	0.987	0.962	0.932
G(spread, g)	1.045	1.251	1.235	1.152
<i>In-Sample period: 1982Q4-1997:Q4; Out-of-sample period 1998-2007</i>				
G(spread, g)	1.035	1.056	1.029	0.970
<i>In-Sample period: 1953Q2-1970:Q4; Out-of-sample period 1971-1990</i>				
G(spread)	0.992	0.883	0.815 *	0.878 **
G(spread, g)	0.989	0.882	0.825 *	0.888

NS and G denote the dynamic yield curve and OLS yield spread models respectively. Denominator is RMSE from AR(1). RMSEs from AR(1) based on period 1982:Q4-1997:Q4 are higher than those from 1953:Q2-1997:Q4, therefore former is used for the second panel. One and two astericks indicate statistical significance of forecast improvements at 5 and 10 percent levels respectively based on the Diebold-Mariano (1995) test.

I also estimate the dynamic yield curve model based on sample period 1953-1997 and perform out-of-sample forecasts. Although this sample change does not fully address structural change in parameters, it still should reduce bias of parameter estimates because the sample period contains more post regime shift observations. Even after this partial adjustment in parameters for regime shift, the dynamic yield curve model forecasts output better than $AR(1)$ at all horizons¹¹. The dynamic yield curve model outperforms the OLS yield spread model that is

¹¹ The DM test of forecast accuracy reports statistical insignificance of all improvements over $AR(1)$ model performance, which might be related to short out-of-sample period.

estimated based on post monetary policy regime shift period. This also confirms robustness of the result that the dynamic yield curve model forecast output better than the OLS yield spread model. The point that the OLS regression overfit data in sample is supported by the result that the OLS yield spread model cannot beat $AR(1)$ in most of horizons even after addressing structural break in parameters, while the dynamic yield curve model outperforms $AR(1)$ in 1998-2007 period.

Third, the predictive power of yield spread for real GDP is concentrated at periods of large changes in business cycles. Previous research findings show that the yield spread is relatively good predictor of recessions. Meantime, $AR(1)$ prediction has good performance at periods of low volatility. The period 1990-2007 on average had substantially more observations with relatively low volatility in real GDP growth, than in previous years. Even two recessions within this period were not as deep as those in preceding years. Thus, $AR(1)$ has advantage over the yield models in considered out-of-sample period. The results are opposite if I consider 1971-1990 as out-of-sample period for the OLS yield spread model. This period is characterized by high volatility in business cycles and substantial changes in real GDP growth. The ratios of RMSEs from the OLS yield spread model to those from $AR(1)$ reported in the third panel of Table 8 suggests that the OLS model produces better forecasts than $AR(1)$ in periods of large fluctuations in real GDP. Since the dynamic yield curve also uses yield information for predicting output, presumably it would have outperformed the $AR(1)$ in that out-of-sample period¹².

5.2.4. *Summary of out-of-sample forecasting of output*

Out-of-sample forecast analysis of output can be summarized by the following points. First, the dynamic yield curve model strongly outperforms the OLS yield spread model. This result can be attributed to i) the dynamic structure of the yield curve model; and ii) extraction of more information from the term structure of interest rates than exogenously defined yield spread used in the OLS yield spread model. Second, there is evidence for structural change in relationship between yield curve and real GDP growth. Third, forecasting performance of both yield models relative to $AR(1)$ are sensitive to the choice of out-of-sample period. In general, the yield curve is

¹² A similar comparison for the period 1970-1990 with the dynamic yield curve model is not performed because of short sample size relative to the number of parameters in the dynamic yield curve model.

less useful for out-of-sample predicting real GDP than predictive power suggested by in-sample OLS regression due to its overfit.

5.3. Forecasts of inflation

5.3.1. The effect of yield factors and lags of inflation-difference

Table 9 reports inflation forecast results produced by the dynamic yield curve and the OLS yield spread models. Adding inflation lags in both the dynamic yield curve and the OLS yield spread models noticeably improves forecasts, which is explained by high persistence of inflation. Adding curvature factor to the slope factor for inflation modeling also improves forecasts. In the OLS yield spread model I define the spread as the difference between 5 year and 1 year interest rates, as suggested by in-sample fitting results.

Table 9
Out-of-sample forecasts of Inflation: Root Mean Square Error Ratios
Out-of-sample period 1990:Q1-2007:Q4

	Forecast horizon			k-quarters ahead		
	1	4	8	12	16	20
<i>Dynamic yield curve model</i>						
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3)$)	1.145	1.205	1.215	1.220	1.214	1.128
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3, \Delta\pi \text{ lags})$)	0.978	0.962	0.990	0.977	0.956	0.878
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \Delta\pi \text{ lags})$)	0.986	0.993	1.053	1.064	1.051	0.945
<i>OLS yield spread model</i>						
$\Delta\text{INF}(\text{spread})$	1.140	1.160	1.193	1.398	1.522	1.677
$\Delta\text{INF}(\text{spread}, \Delta\pi \text{ lags})$	0.993	0.996	1.067	1.279	1.418	1.585
Unconditional in-sample mean of $\Delta\pi$	1.139	1.155	1.093	1.075	1.060	1.062

NS and ΔINF denote the dynamic yield curve and OLS yield spread models respectively.

Denominator is RMSE from AR(1). Lowest RMSE ratios within each class of models are in bold.

5.3.2. Does the dynamic yield curve model forecast inflation better than the OLS yield spread model?

The dynamic yield curve model with lags of inflation-difference produces lower RMSEs than those from the OLS yield spread model at all horizons. The DM test, reported in Table 10, suggests that the differences in forecasts at one quarter ahead and long horizons are statistically

significant. However, the result is opposite in shorter out-of-sample period 1998-2007, reported in Table 12. Thus, I conclude that the dynamic yield curve model does not produce robust improvement of inflation forecast over the OLS yield spread model.

Table 10

Test for Inflation forecast accuracy comparison: PCE Inflation, out-of-sample period 1990-2007

Models	Forecast horizon k-quarters ahead					
	1	4	8	12	16	20
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3)$) against $\Delta\text{INF}(\text{spread})$	0.013	0.091	0.045	-0.444	-0.913	-1.687 *
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3, \Delta\pi \text{ lags})$) against $\Delta\text{INF}(\text{spread}, \Delta\pi \text{ lags})$	-0.035 *	-0.056	-0.134	-0.649 **	-1.189 **	-1.909 *
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3, \Delta\pi \text{ lags})$) against $\text{AR}(p)$	-0.053	-0.063	-0.017	-0.044	-0.093	-0.251
$\Delta\text{INF}(\text{spread}, \Delta\pi \text{ lags})$ against $\text{AR}(p)$	-0.017	-0.007	0.117	0.605 **	1.096 *	1.658 *

NS and ΔINF denote the dynamic yield curve and OLS yield spread models respectively.

The null hypothesis of the test is mean of square loss-differential of two compared models is zero, against alternative that it is negative. One and two asterisks indicate statistical significance of estimates at 5 and 10 percent levels respectively.

Significantly negative (positive) value of the estimate indicates that the first model produces more (less) accurate forecasts than compared model. The test is based on Newey-West heteroskedasticity and autocorrelation consistent std.errors.

5.3.3. Do the yield spread and dynamic yield curve models forecast inflation better than the simple autoregressive model?

The OLS yield spread model after controlling for lags of inflation-difference has similar performance as $\text{AR}(p)$ at short horizons, but at long horizons $\text{AR}(p)$ significantly outperforms the OLS yield spread model, where, according to in-sample results, the predictive power of yield spread suppose to be concentrated. The dynamic yield curve model after controlling for lags of inflation-difference produces lower RMSEs than the $\text{AR}(p)$ model at all horizons; however, all of these differences are statistically insignificant. Also $\text{AR}(p)$ outperforms both models in shorter out-of-sample period 1998-2007. Thus, the dynamic yield curve model does not produce robust improvement of inflation forecast over $\text{AR}(p)$. Next, I analyze the possible reason of this result.

Since inflation is relatively responsive to monetary policy, there are might be structural changes in the yield curve and inflation relationship. Mishkin (1990b) finds that the predictive power of the yield spread for future long-term inflation was stronger in pre-1979 period. Estrella, Rodriguez, and Schich (2003), using known break dates of monetary regime changes, find

evidence of structural breaks in yield spread and inflation relationship in October 1979 and October 1982. In-sample estimate of the coefficient for the yield spread, reported in Table 11, becomes statistically insignificant and its magnitude noticeably declined in the period 1982-1997, and *adjusted R²* declined from more than 10 percent to almost zero. These results suggest that the relationship between yield spread and inflation significantly weakened in post-1982 period.

Table 11

Parameter estimates: OLS regression of inflation-difference on yield spread only. Two in-sample periods

Horizon in quarters	Sample period 1953Q2-1997Q4		Sample period 1982Q4-1997Q4	
	yield spread $\alpha_{1,k}$	R ² adjusted	yield spread $\alpha_{1,k}$	R ² adjusted
16	0.834 (0.334)	0.11	0.268 (0.283)	0.00
20	0.951 (0.369)	0.13	0.374 (0.314)	0.02

In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors

To reduce the effect of this structural instability, I perform out-of-sample forecasts of inflation for the period 1998:Q1-2007:Q4 using the OLS yield spread model based on in-sample period 1982:Q4 to 1997:Q4, reported in Table 12. The OLS yield spread model forecasts based on post-1982 in-sample period are still worse than $AR(p)$ at long horizons. Also the dynamic yield curve model performs worse than the OLS yield spread model and $AR(p)$ in this out-of-sample period. Thus, these results confirm a weakening of relationship between inflation and the yield curve in post-1982 period. These results are in line with Stock and Watson (2003), who find that yield spread is not useful and stable for predicting inflation after controlling for lagged inflations and does not improve over a simple univariate autoregressive model. They report that the yield spread model over performed $AR(p)$ in 1971-1984, while noticeably under performed in 1985-1999 in four quarter ahead forecasts.

5.3.4. Summary of out-of-sample forecasting of inflation

Analysis of out-of-sample forecast of inflation can be summarized by the following points. First, the forecasting performance of the dynamic yield curve model relative to the OLS yield spread and the $AR(p)$ models is sensitive to out-of-sample period and there is no dominant model. Second, there is evidence of a weakening relationship between yield curve and inflation

in post-1982 period. Third, the out-of-sample forecasting performance of the OLS yield spread model is sensitive to the choice of the yield spread. Thus, my results suggest yield curve is not robust predictor of inflation.

Table 12
Out-of-sample forecasts of inflation: Root Mean Square Error Ratios

	Forecast horizon			k-quarters		
	1	4	8	12	16	20
<i>In-Sample period: 1953Q2-1997:Q4; Out-of-sample period 1998-2007</i>						
NS($g(\beta_2, \beta_3, g_{t-1}), \Delta\pi(\beta_2, \beta_3, \Delta\pi \text{ lags})$)	0.994	1.006	1.125	1.172	1.227	1.260
Δ INF(spread, $\Delta\pi$ lags)	1.003	1.013	0.927	0.898	0.819	0.977
<i>In-Sample period: 1982Q4-1997:Q4; Out-of-sample period 1998-2007</i>						
Δ INF(spread, $\Delta\pi$ lags)	0.966	0.911	1.050	1.101	1.176	1.403 *

NS and Δ INF denote the dynamic yield curve and OLS yield spread models respectively.

Denominator is RMSE from AR(p). RMSEs from AR(p) based on period 1982:Q4-1997:Q4 are used in both panels

All errors are in terms of level of inflation. Number of lags determined by BIC method. One and two astericks indicate statistical significance of forecast improvements at 5 and 10 percent levels based on the Diebold-Mariano (1995) test.

6. Conclusion

Most studies that consider predictive power of the yield curve for real GDP growth and inflation use a simple structure with exogenously defined yield spread as the predictive variable. This approach limits information contained in yields for predicting output and inflation only to the difference between two yields of specific maturities. In this paper, I jointly model real GDP growth, inflation, and yields using dynamic three latent yield factors. I find that the dynamic yield curve model produces better out-of-sample forecasts of real GDP growth than the OLS yield spread model for all horizons. Thus, using the dynamic yield curve model provides a more accurate depiction of the predictive power of the yield curve and uses information contained in the entire yield curve relative to the yield spread model. Also poor out-of-sample performance of the OLS yield spread model indicates that yield curve is less useful for output forecasting than in-sample regression suggests. Good predictive power suggested by in-sample OLS regression may be explained by a tendency to overfit in sample.

Forecasting performances of both the dynamic yield curve and the OLS yield spread models relative to autoregressive model with respect to output are sensitive to the choice of out-of-

sample period. I suggest two main explanations to this result. First, there is evidence of structural change in relationship between the yield curve and output in post mid-1980s. Second, the yield curve is a good predictor of large changes in output, while the autoregressive models are good in predicting real GDP growth in periods of its low volatility.

With regard to inflation, I do not find robust improvement in forecasts based on the dynamic yield curve over the standard OLS yield spread model. Also neither model does well compared to $AR(p)$. This can be partly explained by a weakening of yield curve and inflation relationship in post-1982 period.

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