EFFICIENCY WITH ENDOGENOUS INFORMATION CHOICE*

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Abstract

We study the efficiency of information acquisition decisions in models with dispersed information and strategic considerations. Our main result is that information choice is typically inefficient because agents do not fully internalize the effects of their information on others. This ex-ante suboptimality is obtained even in environments where information is used efficiently ex-post. We demonstrate this finding in 3 benchmark environments. In a beauty contest model à la Morris and Shin (1998), incentives to invest in information can diverge from the socially optimal level because the absolute level of the planner's welfare criterion is different from that of the private payoff function. In a RBC framework with dispersed information about technology shocks, distortions due to imperfect substitutability have no effect on incentives to respond to information, but distort the private value of information, leading to an inefficiently low level of information acquired in equilibrium. Finally, in a monetary model with nominal price-setting by heterogeneously informed firms, inefficiencies arise in both the use and the acquisition of information. Importantly, the latter persist even when the former are removed. We also discuss optimal policy response to address these inefficiencies.

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1 Introduction

Heterogeneity in information has been shown to have important implications in many applications, ranging from asset pricing to financial crises and nominal price-setting. Recent work exploring this idea has taken one of two broad approaches. The first[1] takes the information structure as exogenous and studies implications - both positive and normative. The other strand[2] explores the idea that information acquired by agents is endogenous and is a function of model primitives.

This paper falls into the second category and investigates the efficiency properties of equilibrium allocations in a setting where information is endogenous. It is well-known (for example, see Angeletos and Pavan 2007) that when information in exogenous, misalignment of private and social incentives to coordinate actions can lead to information being *used* in a socially sub-optimal manner.

The main contribution of this paper is to highlight a distinct source of inefficiency - one that arises only when the information structure is endogenous. This is the result of a wedge between the social and private *value* of information, which implies that the amount of information in equilibrium will, in general, not correspond to the socially optimal level. Importantly, we find that this property holds even in economies which are constrained-efficient under exogenous information, i.e. when there is no inefficiency in how agents respond to such information in equilibrium. In other words, even if the private incentives to *use* information in equilibrium are aligned to social goals, private *incentives* to acquire that information might be misaligned, leading to inefficiency.

We demonstrate the conditions under which such a wedge arises in 3 benchmark environments, studied extensively by the literature on dispersed information referred to earlier. A feature that all three environments have in common is that agents care not only about fundamentals but also about the actions taken by other agents. These payoff linkages are a source of strategic com-

[1] An inexhaustive reading list will include Woodford (2003), Moscarini (2004), Angeletos and Pavan (2007), Angeletos and La'O (2008, 2009), Nimark (2008), Hellwig (2008b, 2008a), Lorenzoni (2009, 2010), and Hellwig and Venkateswaran (2009). The large literature on noisy rational expectations models in asset pricing, including seminal work by Hellwig (1980) and Diamond and Verrecchia (1981), mostly falls under this category, as does the recent work on global games, following Morris and Shin (1998, 2002).

[2] For example, Mackowiak and Wiederholt (2009, 2011a) consider a setting where agents face a constraint on their ability to process information, while Hellwig and Veldkamp (2009), Gorodnichenko (2008) and Reis (2006) introduce explicit costs of planning or acquiring information. In the asset pricing context, Grossman and Stiglitz (1980), Ganguli and Yang (2009), Barlevy and Veronesi (2000) and Veldkamp (2006b, 2006a) all consider environments where information is chosen endogenously. Myatt and Wallace (2010) study a beauty contest setting where agents choose what signals to pay attention to.

plementarity/substitutability in agents choices.

We start with a general beauty-contest model after Morris and Shin (1998, 2002), modified to allow for endogenous information choice. The environment, though stylized, allows us to deliver the basic intuition and demonstrate the connection between the inefficiency in information use studied by earlier work and the suboptimality of information acquisition. In particular, we find that the inefficiency of responses to information spills over into information choice, but the latter can be suboptimal even in the absence of the former. This occurs because what matters for information use is only the *relative* importance of the various components of the payoffs. Specifically, if aligning with others' actions has the same relative weight in the private payoff as it does in the social welfare criterion, then information is used optimally.

The optimal choice of information, on the other hand, is determined by equating the marginal cost to the marginal value of information, which depends also on the *absolute* level of the payoff function. Thus, any level differences between the social welfare criterion and private payoffs map directly into a wedge between social and private marginal values of additional information, leading to the inefficiency. A natural source for these level differences are payoff externalities (e.g. as in Morris and Shin (2002) or Angeletos and Pavan (2007)).

We then study a micro-founded business cycle model, where agents on informationally-separate 'islands' choose to acquire information about aggregate technology shocks before participating in local labor markets. The environment features a rich set of payoff linkages, arising through general equilibrium interactions. In contrast to the beauty contest model, the information structure here not only affects the response of the economy to shocks, but also has implications for the average *level* of activity. Our analytical characterization of equilibrium allows us to characterize these implications quite sharply. We show that agents respond to information in a socially optimal fashion[3], but the equilibrium features a wedge between the social and private value of additional information, causing agents to invest an inefficiently low amount in information acquisition. This wedge can be traced to 'demand externalities', arising from a standard imperfect substitutability assumption about the various goods produced in the economy. It disappears only in the limiting case where all goods are perfect substitutes.

It is important to note that this result is obtained both under price-taking, i.e. when firms act competitively ex-post or under monopoly pricing[4]. One implication of our results is that pol-

[3]See Angeletos and La'O (2009) for the same finding in a very similar environment.

[4]Market power introduces a constant distortion to the average level of economic activity. Because this takes the form of a constant markup in a CES framework, it is invariant to the information structure and therefore, does not affect the response to information. However, it does affect the wedge between private and social value of information,

icy intervention is called for even in a price-taking environment - in particular, a profit subsidy which aligns the private value of information with the social value. We emphasize that such an implication emerges only when information is chosen endogenously.

Our final environment studies the information acquisition problem in a general equilibrium model of a monetary economy, where firms post nominal prices under imperfect information about aggregate nominal shocks. In the same context, but with exogenous information, Hellwig (2005) demonstrates the inefficiency of equilibrium responses to private signals. As in the beauty contest model, this inefficiency also has an effect on ex-ante incentives to acquire information, but the payoff linkages also have an independent effect on the value of information, as evidenced by the fact that information choice is inefficient even when firms set prices according to the socially optimal response function. The net effect of these forces on information choice is ambiguous in sign, but we are able to characterize the region of the parameter space where the equilibrium features over(under)-investment in information.

Our analytical framework also allows us to explore other interesting questions related to information choice. We use the price-setting application to demonstrate two such extensions. First, we examine the optimal information choice under the assumption that firms are able to coordinate their ex-ante investments in information. In our environment, this leads to a striking result - the collusive optimum features no learning ! This occurs because, in equilibrium, information acquisition is subject to a negative externality - an individual firm's expected profits decline when all other firms in the economy become more informed. This exactly offsets the benefits to those firms and so, when this effect is internalized, information has no value and therefore, will not be acquired at all. Next, we explore the role of strategic considerations in the information acquisition decision. In particular, we characterize how an individual firm's incentives to acquire its own information are affected by the amount of information acquired by other firms. We find that, in the empirically plausible regions of the parameter space, information acquisition is a strategic complement i.e. the better information the overall economy, the greater is the incentive for a firm to become better informed.

Our results are obtained under minimal assumptions on the technology of acquiring information. In particular, we use a general cost function, which encompasses several commonly used specifications (e.g. rational inattention, costly signals). Also, while we focus on private signals for

but the conclusion that firms invest too little in information still holds. Interestingly, policies which offset the effect of monopoly pricing have the additional benefit of removing this wedge, i.e. aligning private and social incentives to *acquire* information.

much of our analysis, it is fairly straightforward to extend our analysis to endogenous acquisition of public information[5].

The primary difference between our analysis and earlier work on endogenous information choice, such as the work on price-setting models (Mackowiak and Wiederholt (2009), Hellwig and Veldkamp (2009), Reis (2006), Myatt and Wallace (2008), Colombo and Femminis (2008, 2011)), or business cycles (Mackowiak and Wiederholt (2011a)) on financial markets (for example, the seminal work of Grossman and Stiglitz (1980) or the recent work of Ganguli and Yang (2009), Barlevy and Veronesi (2000) and Veldkamp (2006b, 2006a)) is the nature of questions we ask. Most of these papers focus on equilibrium implications while we are concerned with considerations of efficiency.

There are several important exceptions. Colombo and Femminis (2008, 2011) investigate the welfare implications of public information provision on incentives to acquire private information. This paper, on the other hand, looks at the role of payoff linkages and externalities in the acquisition of private information. In a recent paper, Mackowiak and Wiederholt (2011b) study the efficiency properties of attention allocation decision of agents to 'rare' events. As in our paper, inefficiencies arise in their environment because of payoff linkages. But, they restrict attention to a general but abstract specification of payoffs whereas we derive our results in fully-specified general equilibrium environments. The richness of payoff linkages in such a setting leads to novel externalities[6], directly interpretable in terms of underlying model primitives. Finally, in an unpublished working-paper version, Hellwig and Veldkamp (2009) show that information acquisition is ex-ante efficient in a beauty-contest model without externalities.

In our applications to the business cycles and price-setting environments, we also depart from the quadratic specification for payoffs often used in the literature referred to above and demonstrate our results in a setting where we can derive explicit expressions for the objects of interest and link efficiency to underlying structural parameters. This allows us to draw more robust conclusions about welfare and set the stage for a quantitative evaluation.

Our work also complements the recent literature on the efficiency properties of economies with informational frictions. Angeletos and Pavan (2007) and Hellwig (2005) show that information can be used inefficiently in equilibrium if private incentives to coordinate are different from the social value of aligning actions. These papers highlight a tight connection between the social value of in-

[5]In Section 2.5, we demonstrate this for the beauty contest model.

[6]For example, in their paper, efficiency under full information and in information use together imply efficiency in information choice. All our applications are efficient under full information, but can feature ex-ante inefficiencies in information choice despite ex-post efficiency in its use.

formation and inefficiencies in the use of information. As mentioned earlier, the main contribution of this paper is to highlight a distinct source of inefficiency - one that arises only when information is endogenous. Equilibrium information choices equate the marginal private value of information to its marginal cost. In general, this may not correspond to the social value of information, leading to over- or under-acquisition of information compared to a socially optimal benchmark. Amador and Weill (2010) analyze a situation where endogeneity of signals (i.e. signals contain information about endogenous variables) leads to an inefficiency in the equilibrium responses of agents, which in turn, implies that information aggregation can be inefficiently low. In our setup, we abstract from this channel by focusing only on the case where signals are exogenous.

The rest of the paper is organized as follows. Section 2 analyzes a standard beauty contest model. This is followed by Section 3, which embeds information acquisition in a general equilibrium real business cycle model with productivity shocks. Section 4 presents the second application - a nominal price-setting model with monetary shocks and Section 5 contains a brief conclusion. Proofs are collected in the Appendix.

2 A General Beauty Contest Model

In this section, we study a general beauty contest model, in the spirit of the global games literature, see Morris and Shin (1998, 2002). Though rather stylized, this setup will allow us to both demonstrate our main results in a relatively simple way as well as to draw connections to earlier work on the efficiency properties of economies with dispersed information. We show that social and private value of information are, in general, different and so information acquisition in equilibrium is typically inefficient relative to a socially optimal benchmark. This inefficiency is partly due to the suboptimal *use* of information, but it persists even when information is used efficiently. Only for a knife-edge combination of parameters do we obtain efficiency in both information use and acquisition. The following two sections will then apply this basic idea to more micro-founded environments and characterize informational inefficiencies in terms of underlying structural parameters.

2.1 Payoffs and Information

There is a continuum of agents, indexed by $i \in [0, 1]$. The game in played in two stages. In stage I, agents choose how much private information (measured by the precision of a private signal about an aggregate fundamental) to acquire subject to a cost function. In stage II, signals are realized and agent i chooses an action $a_i \in \mathbb{R}$ to maximize expected the following *private* payoff function:

$$\Pi_i = \max_{a_i} \; - \mathbb{E}_i(\phi\,(a_i - \theta)^2 + \psi\,(a_i - \bar{a})^2),$$

where $\bar{a} \equiv \int_0^1 a_i di$ is the average action of all agents, θ is the underlying aggregate state and $\mathbb{E}_i\,(\cdot) \equiv \mathbb{E}\,(\cdot|\mathcal{I}_i)$ is the expectation operator conditional on agent i's information set $\mathcal{I}_i$. The random variable θ represents an aggregate state, which is normally distributed with mean zero and variance σ_θ^2.

The payoff function for agent i has two components. The first component is linked to the (squared) deviation between the underlying state θ and agent i's action a_i. The second part is the squared distance between i's action and the average action of all the other agents in the economy, denoted $\bar{a}$. The two components are meant to capture the idea that an agent's payoff depends not only on fundamentals but also on actions of other agents. As we will see in Sections 3 and 4, in many standard macroeconomic environments, this latter feature emerges naturally through general equilibrium interactions. The parameters ϕ and ψ measure the weights attached to these components.

Before choosing a_i, each agent has access to a private signal s_i about the aggregate fundamental:

$$s_i = \theta + e_i \;,$$

where $e_i \sim N\left(0, \hat{\sigma}_e^2\right)$. This variance $\hat{\sigma}_e^2$ is the result of choices made in stage I by the agent. The noise term e_i is independent of θ and independent across the population, i.e. $\mathbb{E}\,(\epsilon_i\,\epsilon_j\,) = 0$ for $i \neq j$. The agent's information set consists only of the common prior and this private signal.

Let $\hat{\Pi}(\cdot)$ denote the expected payoff in stage II (prior to the realization of the signals s_i):

$$\hat{\Pi}_i\left(\hat{\sigma}_e^2, \sigma_e^2\right) \;\equiv\; \mathbb{E}\,(\Pi_i)\,,$$

where $\mathbb{E}\,(\cdot)$ is the expectation operator prior to the realization of signals, $\hat{\sigma}_e^2$ is the variance of the agent's own private signal and σ_e^2 is the variance of the signals of all the other agents in the economy[7]. The problem of the agent in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \;\; \hat{\Pi}_i\left(\hat{\sigma}_e^2, \sigma_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \;,$$

where $\upsilon\,(\cdot)$ is the cost of information as a function of the noise in the signal. Our focus in this paper is on the value of information, so we wish to impose as little structure as possible on the technology

[7] We restrict attention to symmetric equilibria, where all agents make the same information acquisition choices.

for acquiring information. In particular, rather than specify a functional form[8] for $\upsilon(\cdot)$, we will directly make assumptions about its derivatives, in order to ensure that there is an interior solution to the equilibrium and planner's problem of information choice. These assumptions will made explicit in the characterization of the equilibrium and planning problem. For now, we simply assume that $\upsilon'(\cdot) < 0, \upsilon''(\cdot) > 0$.

2.2 Equilibrium

We start with the equilibrium in stage II. The agent's maximization problem in stage II yields the following first order condition:

$$a_i = \frac{\phi}{\phi+\psi}\mathbb{E}_i(\theta) + \frac{\psi}{\phi+\psi}\mathbb{E}_i(\bar{a}) \quad .$$

We conjecture (and verify) that, in a symmetric equilibrium, the average action is linked to the realization of the fundamental θ according to this linear relationship:

$$\bar{a} = \lambda\theta \quad .$$

Given this conjecture,

$$\begin{aligned} a_i &= \frac{\phi+\psi\lambda}{\phi+\psi}\mathbb{E}_i(\theta) \ , \\ a_i &= \frac{\phi+\psi\lambda}{\phi+\psi}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\hat{\sigma}_e^2}\right)(\theta+e_i) = \hat{\lambda}(\theta+e_i) \quad . \end{aligned} \tag{1}$$

The conjecture is verified when

$$\hat{\lambda} = \lambda \ ,$$

which leads to the following result.

Proposition 1 *The unique symmetric equilibrium is given by $a_i = \lambda^{eq}\, s_i$, where λ^{eq} is given by*

$$\lambda^{eq} = \frac{\phi\sigma_\theta^2}{\phi\sigma_\theta^2 + (\phi+\psi)\sigma_e^2} \ . \tag{2}$$

Information Acquisition: Next, we turn to the ex-ante information acquisition decision in stage I. Recall that each agent chooses the precision of her private signals, subject to a cost function $\upsilon(\cdot)$. At the optimum, each agent equates the marginal value of more information to its cost.

[8]For example, under the rational inattention paradigm, as in Sims (2003), this would be the cost of information processing capacity. Alternatively, in an environment with costly iid signals, this could be the cost of acquiring a basket of signals with the same informational content as a single signal with precision σ_e^2.

In a symmetric equilibrium, the envelope theorem implies that this *private* marginal value of information is the same for all agents and is given by:

$$\frac{\partial \hat{\Pi}_i}{\partial \hat{\sigma}_\epsilon^2} = -\left(\phi+\psi\right)\left(\lambda^{eq}\right)^2 \ .$$

Under the assumption of an interior optimum[9], the optimality condition in stage I becomes:

$$-\left(\phi+\psi\right)\left(\lambda^{eq}\right)^2 = \upsilon'(\sigma_e^2) \ .$$

Noting that λ^{eq} is in turn a function of the (symmetric) information choice, the above condition is a fixed point in σ_e^2 and completes the characterization of the equilibrium with endogenous information acquisition.

2.3 Planner's Problem

Now, we study the efficiency properties of the equilibrium characterized in the previous subsection. Towards this end, we define a socially optimal benchmark as the solution to the problem of a planner, who is interested in maximizing the following objective:

$$-\mathbb{E}\left(\phi^* \int_0^1 \left(a_i-\theta\right)^2 di + \psi^* \int_0^1 \left(a_i-\bar{a}\right)^2 di\right) - \int \upsilon\left(\sigma_{e,i}^2\right) di \ .$$

Note that the planner's objective depends on the cross-sectional averages of the same deviations that entered the individual agent's payoff, albeit with possibly different weights. When ϕ^* and ψ^* are both positive, the planner wants to minimize the average deviations from the fundamental θ and the cross-sectional dispersion in actions. The general specification above can nest various externalities, e.g. of the kind studied by Morris and Shin (2002). Such payoff-linked effects will play a crucial role in our analysis, both in this section and the more detailed environments of the following 2 sections.

Importantly, we assume that the planner is information-constrained, i.e. she cannot transfer information across agents. The *efficient* use of information is defined as the response function that maximizes the planner's objective, subject to the constraint that information cannot be pooled across agents. We restrict attention to linear response functions of the form:

$$a_i = \lambda s_i \ .$$

[9] We assume that $\upsilon(\cdot)$ is such that the optimum is reached at an interior point.

Given this form for the response function, the efficient use of information is the solution to

$$\begin{aligned}\mathbb{U} &= \max_{\lambda} \quad -\mathbb{E}\left(\phi^*(\lambda-1)^2\theta^2 + (\phi^*+\psi^*)\lambda^2\sigma_e^2\right) \\ &= \max_{\lambda} \quad -(\phi^*(\lambda-1)^2\sigma_\theta^2 + (\phi^*+\psi^*)\lambda^2\sigma_e^2) \ .\end{aligned}$$

The first order condition[10] of the above problem is:

$$\frac{\partial \mathbb{U}}{\partial \lambda} : \phi^*(\lambda-1)\sigma_\theta^2 + (\phi^*+\psi^*)\lambda\sigma_e^2 = 0 \ . \tag{3}$$

Re-arranging, we derive the following result:

Proposition 2 *The efficient linear response coefficient, denoted λ^* is*

$$\lambda^* = \frac{\phi^*\sigma_\theta^2}{\phi^*\sigma_\theta^2 + (\phi^*+\psi^*)\sigma_e^2}. \tag{4}$$

Comparing the two response coefficients, λ^{eq} and λ^*, we see that the response of agents under dispersed information is efficient from the planner's perspective if, and only if, the agents attach the same relative weight to the two types of deviations in their private payoffs as the planner does. Formally,

Proposition 3 *For a given σ_e^2, equilibrium response is efficient if, and only if, the relative weights of the two components are equal in the private and social payoff function, i.e.*

$$\lambda^{eq} = \lambda^{sp} \qquad \Leftrightarrow \qquad \frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$$

This finding is an application of the insight in earlier work (e.g. Angeletos and Pavan (2007)) - differences between the social and private costs of dispersion and volatility can lead to information being used in a socially sub-optimal manner.

Next, we turn to deriving the socially optimal level of information acquisition. Analogous to the equilibrium information choice, this is obtained by equating the social marginal value of information, which, by the envelope condition, is given by

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = -\left(\phi^*+\psi^*\right)\left(\lambda^*\right)^2 \ , \tag{5}$$

to the marginal cost, i.e.

$$-\left(\phi^*+\psi^*\right)\left(\lambda^*\right)^2 = \upsilon'(\sigma_e^2) \ .$$

[10]The second-order condition requires that $\phi^*\sigma_\theta^2 + (\phi^*+\psi^*)(\sigma_\theta^2+\sigma_e^2) \geq 0$.

This fixed point relationship in σ_e^2 characterizes the information-constrained optimum in this economy. The comparison of the expressions for the marginal values of information for the agent and the planner reveals 2 sources of inefficiency. The first is linked to the suboptimality in information use referred to earlier, i.e. to the fact that λ^{eq} may not be equal to λ^*. In particular, if agents in equilibrium respond less to their own information than the planner would like them to (i.e. $|\lambda^{eq}| < |\lambda^*|$), then, ceteris paribus, their incentives to acquire that information are also reduced. However, even if the equilibrium information use is efficient, i.e. $\lambda^{eq} = \lambda^*$, the private marginal value of information can still diverge from the socially optimal level because of a level effect, i.e. the difference between $\phi^* + \psi^*$ and $\phi + \psi$. To show this more clearly, we rewrite the social value as follows:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \underbrace{\frac{\partial \hat{\Pi}_i}{\partial \hat{\sigma}_e^2}}_{\text{Private value}} - \underbrace{\left[(\phi^* + \psi^*) - (\phi + \psi)\right](\lambda^*)^2 - (\psi + \phi)[(\lambda^*)^2 - (\lambda^{eq})^2]}_{\text{'Externalities'}} .$$

The above expression decomposes the planner's value of learning into a private value component and a term which captures all other effects from additional information. These effects arise either because private payoffs do not fully reflect social value, i.e. $\phi^* + \psi^* \neq \phi + \psi$ or because private incentives to use information are not aligned with social ones, i.e. $\lambda^* \neq \lambda^{eq}$. In the two environments we examine in Sections 3 and 4, these 'externalities' arise because of general equilibrium consequences of information acquisition, which are not internalized by agents. The two environments will also serve to highlight the different sources of inefficiency. In the competitive real business cycle economy in section 3 firms tend to under-invest in learning even though information use is efficient. The monetary economy in section 4, on the other hand, features both sources of inefficiency - agents not only overweight their private information in their pricing decisions relative to a socially optimal benchmark but also acquire an inefficient level of information.

2.4 Implementing the Social Optimum

In this subsection, we consider the nature of interventions that are necessary to correct the information-related inefficiencies in the equilibrium characterized above. Given a precision of signals, σ_e^2, we will show that efficiency in information use can be restored through a 'tax', which aligns the social and private weights attached to the two payoff components. However, in line with the general intuition behind the findings in the previous subsection, we will show that this, by itself, is not sufficient to align private incentives to acquire information with the social ones.

We start with the sub-optimal nature of information *use*. Formally, we consider a tax, τ, of the following form[11]

$$\Pi_i = \max_{a_i} \; - \mathbb{E}_i(\phi\tau\,(a_i - \theta)^2 + \psi\,(a_i - \bar{a})^2)\;.$$

For a given tax τ, the equilibrium response coefficient is:

$$\lambda^\tau = \frac{\phi\tau\sigma_\theta^2}{\phi\tau\sigma_\theta^2 + (\phi\tau + \psi)\sigma_e^2}\;.$$

Any response coefficient λ can be implemented by setting the tax appropriately, i.e. by solving the following equation for τ,

$$\lambda = \frac{\phi\tau\sigma_\theta^2}{\phi\tau\sigma_\theta^2 + (\phi\tau + \psi)\sigma_e^2}\;.$$

In particular, to implement λ^*, the socially optimal response, the tax is simply

$$\tau^* = \frac{\phi^*}{\psi^*}\;\frac{\psi}{\phi}\;.$$

The expression for the optimal tax rate is intuitive - it corrects the inefficiency in information use by aligning the relative weights of the two components in the private and social payoff functions. However, this correction by itself is not enough to align the private incentives to acquire information with those of the planner. In particular, note that the marginal value of information to the agent with payoffs distorted according to τ^*, is given by:

$$\frac{\partial\hat{\Pi}_i}{\partial\hat{\sigma}_e^2} = -\left(\phi\tau^* + \psi\right)(\lambda^*)^2 = -\frac{\psi}{\psi^*}(\phi^* + \psi^*)(\lambda^*)^2\;.$$

Thus, the private marginal value of information is equal to the social marginal value if, and only if[12], $\psi = \psi^*$. In other words, even if payoffs are distorted to achieve efficiency in the *use* of information, the *acquisition* of information still remains inefficient. In general, in order to restore efficiency in both the use and acquisition of information, we need 2 distinct forms of intervention - one which aligns the relative weights in private and social payoffs and another which corrects the level distortions. Here, we propose one such implementation. In addition to the τ policy discussed earlier, we employ another 'tax', denoted κ, which affects total payoffs. In particular, the private payoff is now given by:

$$\Pi_i = \max_{a_i} \; - \kappa\;\; \mathbb{E}_i(\phi\tau\,(a_i - \theta)^2 + \psi\,(a_i - \bar{a})^2)\;.$$

[11]This formulation is not the only way to restore efficiency in use of information. The key point, however, is that correcting the inefficiency in information use is not sufficient to get the economy to the information-constrained optimum.

[12]Note that this condition depends on the nature of tax that was introduced. If, for example, the distortion was a tax to the second component of the payoff function, then we need $\phi^* = \phi$ for the optimal tax to ensure efficiency in information acquisition as well, we need $\phi = \phi^*$..

From the preceding discussion, it is easy to see that the optimal policy is to set

$$\tau = \tau^* = \frac{\phi^*}{\psi^*}\,\frac{\psi}{\phi} \qquad \kappa = \frac{\psi^*}{\psi}$$

2.5 Public Signals

While the analysis in this paper will focus on the acquisition of private information, it can be easily extended to the case with both public and private signals. We demonstrate this for the general beauty contest model studied in this section. The payoff structure is the same as before but the information structure is modified to include the following additional signal in the agent i's information set:

$$S_i = \theta + \eta_i\ \epsilon \quad ,$$

where $e \sim N\left(0, \hat{\sigma}_\epsilon^2\right)$ is a common noise term, while η_i reflects the extent to which agent i's signal is affected by that noise term. As $\eta_i \to \infty$, this signal essentially becomes worthless from the perspective of forecasting θ. In the other direction, as $\eta_i \to 0$, this becomes an arbitrarily precise signal of the fundamental. We will interpret the agent's choice of η as a choice of the extent of common components in her information.

The information cost is now a function of both the public and private information choices, i.e $\upsilon(\sigma_e^2, \eta^2)$. As in the baseline model, we impose very little structure on this cost function, beyond monotonicity and curvature assumptions needed to ensure interior solutions to the optimization problems of the agent and the planner. The following proposition characterizes the equilibrium and efficient response functions and confirms that efficiency in information use is obtained if the relative weights are the same in private and social payoffs.

Proposition 4 1. *There exist constants λ_1^{eq} and λ_2^{eq} such that, in a symmetric equilibrium, optimal actions are given by*

$$a_i = \lambda_1^{eq} s_i + \lambda_2^{eq} S_i$$

2. *There exist constants λ_1^* and λ_2^* such that, the symmetric socially optimal response function is*

$$a_i = \lambda_1^* s_i + \lambda_2^* S_i$$

3. *Given a symmetric information structure, i.e. with the same (σ_e^2, η) for all agents in the economy, the two sets of response coefficients are equal if, and only if, $\frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$.*

The marginal values of private information are exactly the same in the previous subsection and so the earlier discussion on the efficiency properties of that decision is applicable here as well. Turning to the public signal, the private and social marginal values of increasing the precision with which it is observed are given by

$$\frac{\partial \Pi}{\partial \hat{\eta}^2} = -\phi \left(\lambda_2^{eq}\right)^2 \qquad \frac{\partial \mathbb{U}}{\partial \eta^2} = -\phi^* \left(\lambda_2^*\right)^2 \ .$$

As with the private information, we see that the value of public information to the agent (and therefore, the incentives to acquire it) can diverge from the societal value either because such information is used sub-optimally (i.e. $\lambda_2^{eq} \neq \lambda_2^*$) or because of a level effect (i.e. $\phi \neq \phi^*$).

3 A Model of Quantity Choice and Productivity Shocks

In this section, we lay out our first application - a micro-founded business cycle model with dispersed information about aggregate productivity shocks. The setup closely follows that of Angeletos and La'O (2009). On informationally-separate islands, firms and households trade labor services, the only input in a decreasing returns to scale technology. Importantly, the labor market operates under imperfect information about the productivity shock. As in the previous section, the information structure is endogenous and in equilibrium, reflects private incentives to learn. We assume that each firm is specialized in the production of an intermediate input which is imperfectly substitutable with other inputs in the production of the final good. Firms act in a monopolistically competitive fashion. Our assumptions on preferences and technologies are fairly standard, but we make a few simplifying assumptions (e.g. no capital) in the interest of analytical tractability.

The nature of general equilibrium linkages between firms and households implies that the extent of information available has two kinds of effects on economic activity. The first exerts its influence on the average level of economic activity. The second acts through the sensitivity of economic activity to the realization of the aggregate productivity shock. As we will see, in this economy, only the first channel is a source of inefficiency[13].

Our main result is that the equilibrium in this economy does not attain the information-constrained optimum. In particular, there is a distortion in the private value of information relative to the social one which induces firms to acquire a suboptimal low level of information in equilibrium. Only in a limiting case, as goods become perfect substitutes, does the equilibrium achieve efficiency. We also discuss optimal policies that restores efficiency, which take the form of a constant subsidy.

[13]The price-setting economy with monetary shocks in Section 4, on the other hand, features inefficiencies in both.

3.1 Preferences, Technology and Information

Time is discrete, $t = 0, 1, 2....$ The economy has a single representative household, which consists of 4 types of agents - a single consumer, a continuum of entrepreneurs, a continuum of workers and a final good producer. The entrepreneurs each have access to a technology, which transforms labor into a differentiated intermediate good according to an identical decreasing returns to scale production function. These technologies, or firms as we will refer to them in our exposition, are located on a continuum of islands, with one firm per island. Every period, the household sends one of its workers to each island. The firm and the worker on an island trade labor services after observing all the available information on that island. Then, production takes place and the firms sell their output in a monopolistic competitive fashion to the final good producer, pays its workers and pays dividends. The only source of uncertainty in the model is an aggregate technology shock, which affects the productivity of all the firms in the economy.

At the beginning of each period, every entrepreneur decides how much information (about the realization of the aggregate shock) to acquire. This information takes the form of the precision of a signal, that is made available on her island. Importantly, it becomes available to the worker on that island before the labor market opens[14]. In other words, wages and labor input on each island are determined under imperfect information about aggregate conditions. After the labor market shuts down, the aggregate shock becomes commonly known, production takes place. The worker and entrepreneur return to the household with their respective shares of output one each and deliver them to the consumer. Figure 1 shows the timing of events in each period.

We now make explicit assumptions about preferences and technologies in this economy.

The Household: The lifetime utility of the household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \int_0^1 N_{it} di - \int_0^1 \upsilon(\sigma_i^2) di \right) \qquad 0 < \gamma < \infty,$$

where C_t is denoted consumption, N_{it} is the labor input on island i, σ_i^2 is the variance of the noise term in the island-specific signal (to be described later) and the last term is the entrepreneur's cost of information. Parameter β is the discount factor and γ represents the degree of risk aversion of households.

[14]An alternative interpretation of our model is that of worker-entrepreneurs operating on informationally-separate islands. Under this interpretation, there are no local labor markets - each entrepreneur chooses the level of his labor input to maximize the household's expected utility.

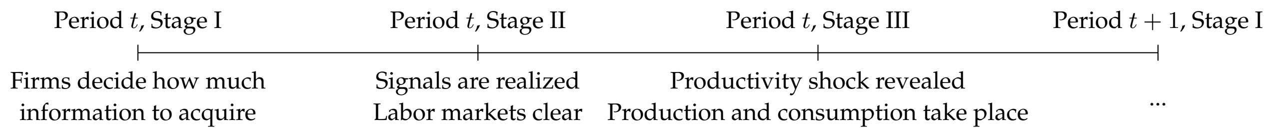

Figure 1: Timeline of Events

Households maximize utility subject to a budget constraint[15]:

$$C_t \leq \int_0^1 W_{it} N_{it} di + \int_0^1 \Pi_{it} di \ ,$$

where W_{it} denotes island-specific wages. In addition to labor income, the household receives the sum of all profits from intermediate producers, denoted by $\int \Pi_{it} di$,.

Final good producer: The single final good is produced using a continuum $[0, 1]$ of intermediate inputs Y_{it}. The production function is a Dixit-Stiglitz aggregator with constant returns to scale. The competitive firm producing the final good solves the following static problem:

$$\max \ \ Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \quad ,$$

where P_{it} is the price of intermediate good i.

Note that intermediate inputs are imperfectly substitutable in production. Imperfect substitutability disappears as θ goes to infinity. Parameter θ also indexes the strength of aggregate demand externalities (or the sensitivity of optimal firm profits to aggregate output), see Angeletos and Pavan (2007). Throughout the analysis we assume that $\theta > 1$.

Intermediate producers: There is a continuum of intermediate good producers indexed $i \in [0, 1]$. The production function is a standard decreasing returns to scale with labor as the sole input.

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} \quad ,$$

where $\delta > 1$ and A_t is the aggregate productivity which is assumed to be log-normal, i.e. $\log A_t \equiv a_t \sim N\left(0, \sigma_a^2\right)$. For expositional simplicity, we focus on the case where this is an i.i.d shock, but

[15] The households also have access to markets in Arrow-Debreu securities. Crucially, these markets are assumed to operate only in the last stage, i.e. are unavailable to firms and workers on the islands. Since we work with a representative household, we keep the exposition simple by omitting the relevant terms from the budget constraint.

our results go through for more general stochastic processes as well[16]. This is the only source of fundamental uncertainty in the model.

Information structure: Before labor markets open, the firm and worker on each island see a signal s_{it} about the current productivity shock:

$$s_{it} = a_t + e_{it} \ ,$$

where $e_{it} \sim N\left(0, \hat{\sigma}^2\right)$ and $\hat{\sigma}_e^2$ is the variance chosen in stage I by the firm.

Labor markets: Firms and workers on an island take the island-specific wage as given and choose labor demand and supply to maximize expected profits and utility respectively. Formally, a firm chooses labor to maximize the expected value of profits:

$$\Pi_{it} = \max_{N_{it}} \ \mathbb{E}_{it} Q_t \left(P_{it} Y_{it} - W_{it} N_{it}\right) \ ,$$

where Q_t is the household's stochastic discount factor (defined below). The operator $\mathbb{E}_{it}\left(\cdot\right)$ represent the expectation conditional on firm i's information $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}\left(\cdot\right) \equiv \mathbb{E}_t\left(\cdot \,|\mathcal{I}_{it}\right)$. Under monopolistic competition, firms take into account the effect of their labor choice on their price P_{it} (defined below).

Similarly, the worker on island i solves

$$\max_{N_{it}} \qquad \mathbb{E}_{it} Q_t W_{it} N_{it} - N_{it} \ .$$

Information acquisition: In the first stage of each period, firms choose the amount of information, taking as given information choices of other firms. Expected profits prior to the realization of the signal and the aggregate state is defined by:

$$\hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) \equiv \mathbb{E}_{t-1}\Pi_{it} \ ,$$

where $\mathbb{E}_{t-1}$ is the expectation conditional on information available at the time of the first stage decision i.e. the (commonly known) history until $t-1$.

The problem of the firm in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \quad \hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where $\upsilon\left(\cdot\right)$ is the cost of information as a function of the noise in the signal. We will assume that $\upsilon'(\cdot) < 0, \ \upsilon''(\cdot) > 0$. We will make additional assumptions on $\upsilon(\cdot)$ to ensure an interior solution. We discuss these in the equilibrium characterization.

[16] For example, if a_t is an $AR(1)$ process, our results go through exactly with the aggregate shock now interpreted as the current innovation to the aggregate productivity level.

3.2 Optimality

We solve the model backwards starting from the last stage.

Stage III: Complete information: In the last stage of each period, there is perfect information of the aggregate state. Optimization by households and the representative final good producer, combined with market clearing, implies the following set of equilibrium conditions:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} , \tag{6}$$

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} , \tag{7}$$

$$P_{it} = Y_t^{\frac{1}{\theta}} Y_{it}^{-\frac{1}{\theta}} , \tag{8}$$

$$Q_t = C_t^{-\gamma} , \tag{9}$$

$$C_t = Y_t . \tag{10}$$

Stage II: Labor markets: The first order condition of the firm and worker take the form,

$$\mathbb{E}_{it} Q_t \left(\frac{\theta - 1}{\delta\theta} Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}} N_{it}^{\frac{\theta-1-\theta\delta}{\delta\theta}} - W_{it} \right) = 0 \tag{11}$$

$$\mathbb{E}_{it} Q_t W_{it} = 1 \tag{12}$$

Substituting for W_{it} from the worker's optimality condition and for Q_t from stage III, we derive the following expression for labor input on island i.[17]

$$N_{it}^{\frac{1+\theta\delta-\theta}{\delta\theta}} = \frac{\theta - 1}{\delta\theta} \left(\mathbb{E}_{it} Y_t^{\frac{1}{\theta}-\gamma} A_t^{\frac{\theta-1}{\theta}} \right) \tag{13}$$

Stage I: Information acquisition Under the assumption of an interior solution, the firm's optimal information choice is characterized by the following optimality condition:

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}_e^2} = 0 ,$$

3.3 Equilibrium

A stationary equilibrium is (i) a set of information choices for each firm (ii) island-specific wages and labor input as functions of the signal on the island (iii) aggregate consumption and output as functions of the aggregate state such that: (a) the labor input is optimal for the worker and the

[17]In our framework, market power and strategic linkages due to imperfect substitutability are controlled by the same parameter θ. One can easily extend this framework where one parameter controls the elasticity of substitution and another the strategic linkages. Importantly, the basic results presented here apply to that case as well.

firm, given island-specific information and wages and the functions in (iii) above, (b) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem, (c) markets clear and (d) the functions in (iii) are correct, i.e. consistent with choices of firms and workers.

We focus on symmetric stationary equilibria, where all firms acquire the same amount of information in stage I and follow the same labor hiring strategies in stage II. The characterization of the equilibrium in stage II essentially follows the same procedure as in Angeletos and La'O (2009). We begin with a conjecture about the aggregate labor input:

$$N_t \equiv \int_0^1 N_{it} di = A_t^{\alpha} K_2 \ ,$$

or, in logs[18]

$$n_t = k_2 + \alpha a_t \ , \tag{14}$$

where α and k_2 are constants to be determined in equilibrium. The former determines the sensitivity of aggregate labor to productivity shocks whereas the latter affects the level of aggregate labor input (and therefore, of economic activity). Both these coefficients will play an important role in our analysis. In a symmetric equilibrium, we can show that (14) implies the following about aggregate output:

$$y_t = \frac{1}{\delta} k_2 - \frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\alpha^2}{\delta}\sigma_e^2 + \left(\frac{\delta+\alpha}{\delta}\right) a_t \ . \tag{15}$$

Recall, from (13), that the labor input on island i is characterized by:

$$n_{it} = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\log\left[\mathbb{E}_{it}\left(Y_t^{\frac{1}{\theta}-\gamma} A_t^{\frac{\theta-1}{\theta}}\right)\right] \ .$$

We substitute for y_t using (15) and, under the assumption that aggregate variables are conditional log-normally distributed[19], derive

$$n_{it} = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\alpha^2}{\delta}\sigma_e^2\right) \tag{16}$$
$$+ \phi_1 \mathbb{E}_{it}(a_t) + \phi_2 \mathbb{V}_{it} \ ,$$

where:

$$\phi_1 \equiv \frac{\theta\delta}{1+\theta\delta-\theta}\left(\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right) \ ,$$
$$\phi_2 \equiv \frac{\theta\delta}{1+\theta\delta-\theta}\frac{\left[\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right]^2}{2} > 0 \ ,$$

[18]Hereafter, variables in small cases denote variables in logs, i.e. $x \equiv \log(X)$

[19]This will be verified later.

and $\mathbb{E}_{it}$ and $\mathbb{V}_{it}$ are the mean and variance of the distribution of a_t, conditional on the information in island i. Using standard results for Bayesian updating, these are given by:

$$\mathbb{E}_{it}\left(a_t\right) = \frac{\sigma_a^2}{\sigma_a^2+\hat{\sigma}_e^2} s_{it} \ ,$$
$$\mathbb{V}_{it} = \frac{\sigma_a^2\hat{\sigma}_e^2}{\sigma_a^2+\hat{\sigma}_e^2} \ ,$$

where σ_e^2 is the variance of the error term in the firm's signal.

Plugging the optimal labor into the firm's profit function, we get the following expression for maximized profit

$$\Pi_{it} = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}} \exp\left(\phi_1 \mathbb{E}_{it}\left(a_t\right) + \phi_2 \mathbb{V}_{it}\left(a_t\right) - \left(\frac{1}{\theta}-\gamma\right)\frac{1}{2}\frac{\alpha^2}{\delta}\sigma_e^2\right) \ , \tag{17}$$

where

$$K_1 \equiv \frac{1+\theta\delta-\theta}{\theta-1}\left(\frac{\theta-1}{\delta\theta}\right)^{\frac{\delta\theta}{1+\theta\delta-\theta}} > 0 \ .$$

Notice that the conjectured behavior of the aggregate labor (14) affects the firm's payoff in two ways. First, the level coefficient, k_2, affects positively the level of profits in the second stage. Second, the labor elasticity to the aggregate shock, i.e. α, enters into the coefficients ϕ_1 and ϕ_2 and has an additional level effect through the last term in the exponent.

In a symmetric equilibrium, where all firms choose the same amount of information and follow the same hiring rule, the cross-sectional distribution of labor is log-normal. Then, by definition,

$$n_t = \bar{\mathbb{E}}\left(n_{it}\right) + \frac{1}{2}\mathbb{D} \ ,$$

where $\bar{\mathbb{E}}\left(\cdot\right)$ and $\mathbb{D}$ denote the cross-sectional mean and variance of labor inputs on the islands:

$$\bar{\mathbb{E}}\left(n_{it}\right) = \int_0^1 n_{it} di \ ,$$
$$\mathbb{D} = \int_0^1 \left(n_{it} - \bar{\mathbb{E}}\left(n_{it}\right)\right)^2 \ di \ .$$

Next, we derive these cross-sectional moments. First, using the expression for n_{it},

$$\bar{\mathbb{E}}\left(n_{it}\right) = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right) + \phi_1\int \mathbb{E}_{it}(a_t)di + \phi_2\mathbb{V} \ . \tag{18}$$

Substituting the Bayesian updating formulae into (18), we get

$$\bar{\mathbb{E}}\left(n_{it}\right) = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2 - \frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right) + \phi_1\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}a_t + \phi_2\mathbb{V} \ ,$$

where we invoke the law of large numbers to show that

$$\int_0^1 s_{it} di = \int_0^1 (a_t + e_{it})\, di = a_t + \int_0^1 e_{it}\, di = a_t \ .$$

Similarly,

$$n_{it} - \bar{\mathbb{E}}(n_{it}) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2} \phi_1 e_{it} \ ,$$

$$\Rightarrow \quad \mathbb{D} = \left(\frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2} \right)^2 \phi_1^2 \sigma_e^2 \ .$$

The next result completes the guess-and-verify procedure and characterizes the response coefficients.

Proposition 5 *In a symmetric equilibrium, aggregate labor input is given by (14), with*

$$\alpha = \frac{\delta\theta\,(1-\gamma)\,\sigma_a^2}{[(\delta-1)\,\theta + \gamma\theta]\,\sigma_a^2 + (1+\delta\theta-\theta)\,\sigma_e^2} \ , \tag{19}$$

$$k_2 = \frac{\theta\delta}{1+\theta\delta-\theta} \log\left(\frac{\theta-1}{\delta\theta}\right) + \frac{(1+\theta\delta-\theta)\,(1-\gamma)}{\theta\,(\delta-1+\gamma)} \frac{\alpha\sigma_e^2}{2} + \frac{1+\theta\delta-\theta}{\theta\,(\delta-1+\gamma)} \frac{\alpha^2\sigma_e^2}{2}. \tag{20}$$

where σ_e^2 is the variance of the error in the signals.

3.4 Information acquisition

Next, we examine the information acquisition decision in stage I. Consider the maximized stage II profit function, equation (17), which we reproduce here

$$\Pi_{it} = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}} \exp\left(\phi_1 \mathbb{E}_{it}\,(a_t) + \phi_2 \mathbb{V}_{it}\,(a_t) - \left(\frac{1}{\theta} - \gamma\right) \frac{1}{2} \frac{\alpha^2}{\delta} \sigma_e^2 \right) \ .$$

In stage I, the firm takes as given the information choices of other firms, or equivalently, the aggregate coefficients, α and k_2. Expected profits, conditional on a choice of individual error variance $\hat{\sigma}_e^2$, are given by taking expectations over the realization of the random variable $\mathbb{E}_{it}(a_t)$. Exploiting log-normality (and dropping the time subscript), this ex-ante expected profit is:

$$\hat{\Pi}(\hat{\sigma}_e^2, \alpha, k_2) = K_1 K_2^{\frac{1-\theta\gamma}{1+\theta\delta-\theta}} \exp\left(\frac{\phi_1^2}{2} \frac{\left(\sigma_a^2\right)^2}{\sigma_a^2 + \hat{\sigma}_e^2} + \phi_2 \frac{\sigma_a^2 \hat{\sigma}_e^2}{\sigma_a^2 + \hat{\sigma}_e^2} - \left(\frac{1}{\theta} - \gamma\right) \frac{1}{2} \frac{\alpha^2}{\delta} \sigma_e^2 \right) \ . \tag{21}$$

Note that expected profits is a function of the firm's own variance, $\hat{\sigma}_e^2$ as well as the aggregate coefficients, which in turn are determined by the information choices of all firms in the economy.

It is straightforward to show that expected profits are decreasing (and convex) in the variance of the error in firm's own signal i.e.[20]

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} = -\frac{\theta - 1}{\delta\theta}\hat{\Pi}\frac{\alpha^2}{2} < 0 \quad \forall\, \hat{\sigma}_e^2 \in \mathbb{R}^+ \ , \tag{22}$$

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} = -\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\left\{\frac{2}{(\sigma_u^2 + \hat{\sigma}_e^2)} - \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right\} > 0 \ .$$

The problem of the firm in stage I is thus:

$$\max_{\hat{\sigma}_e^2} \ \hat{\Pi}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ , \tag{23}$$

where $\upsilon\left(\hat{\sigma}_e^2\right)$ is the cost function.

As discussed earlier, the main focus of this paper is on private versus social value of information, so we impose very little structure on the cost of information. Therefore, instead of specifying a functional form for $\upsilon(\cdot)$, we directly assume that the cost function is such that the solution to the firm's (and later in the analysis, the social planner's) problem will always lead to an interior solution. This requires assuming that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} < 0 \ .$$

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \text{argmax}_{\hat{\sigma}_e^2} \ \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where α and k_2 are functions of σ_e^2 as given by (19)-(20).

3.5 Efficiency in Information Use

We now turn to the efficiency properties of the equilibrium characterized above. First, we repeat the exercise in Angeletos and La'O (2009) about the optimality of information use. In particular, we compare the equilibrium coefficients α and k_2 to those chosen by a social planner, who is interested in maximizing household utility. Importantly, the planner is information-constrained, i.e. cannot pool information across islands. We show that, for a given σ_e^2, the equilibrium response

[20]Recall that the information acquired in equilibrium affects individual profits through k_2 and α., which are taken as given by the firm when choosing its own investment in information. Thus, the effect of overall information σ_e^2, on these coefficients is the basic source of the externality in information choice. We study this in the Appendix A.2.

is inefficient due to a constant distortion of the average level of employment (captured by k_2) but the sensitivity to the shock (the coefficient α) coincides with the choice of the planner one.

To characterize the planner's optimum, we assume that, in stage II, all firms follow a linear labor-hiring rule of the form:

$$n_{it} = \tilde{k_2} - \frac{1}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}\ s_{it}\ . \tag{24}$$

It is straightforward to show that the aggregate employment and consumption are then given by,

$$\begin{aligned} N_t &= \int_0^1 N_{it}di = \widetilde{K}_2 A_t^{\widetilde{\alpha}}\ , \\ C_t &= \widetilde{K}_2^{\frac{1}{\delta}} A_t^{\frac{\delta+\tilde{\alpha}}{\delta}} \exp\left(-\frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\widetilde{\alpha}^2}{\delta}\sigma_e^2\right)\ . \end{aligned}$$

The next step is to express the utility of the household in equilibrium as a function of the amount of information. Using the relationships derived above, the period utility of the household is

$$\begin{aligned} \mathbb{U} &= \frac{1}{1-\gamma}\exp\left(\frac{1-\gamma}{\delta}\tilde{k_2} + \left((1-\gamma)\left(\frac{\delta+\tilde{\alpha}}{\delta}\right)\right)^2\frac{\sigma_a^2}{2} - \frac{1-\gamma}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\tilde{\alpha}^2}{\delta}\sigma_e^2\right) \\ &\quad - \exp\left(\tilde{k_2} + \frac{\tilde{\alpha}^2}{2}\sigma_a^2\right)\ . \end{aligned} \tag{25}$$

The efficient use of information is then defined by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k_2},\tilde{\alpha}}\quad \mathbb{U}(\tilde{k_2},\tilde{\alpha})\ .$$

The next result shows that the response coefficients in equilibrium are optimal.

Proposition 6 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\alpha^* = \alpha\ , \tag{26}$$

$$k_2^* = k_2 + \frac{\delta}{\delta-1+\gamma}\log\left(\frac{\theta}{\theta-1}\right)\ . \tag{27}$$

where α and k_2 are as defined in Proposition 5.

3.6 Efficiency of Information Choice

In this subsection, we compare the level of information acquired in equilibrium to a socially optimum level. We find that, despite the fact that information use is efficient, the equilibrium features a suboptimal level of information.

We restrict attention to the region of the parameter space where utility, net of information acquisition costs, is maximized at an interior level of information choice. In other words, the solution to the following problem

$$\max_{\sigma_e^2} \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) \ , \tag{28}$$

is characterized by the usual first-order condition:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \frac{\partial \upsilon}{\partial \sigma_e^2} \ , \tag{29}$$

where $\mathbb{U}$ is given by (25).

As with the equilibrium information choice, we also need to assume that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \mathbb{U}}{\partial \sigma_e^2 \partial \sigma_e^2} - \frac{\partial^2 \upsilon}{\partial \sigma_e^2 \partial \sigma_e^2} < 0 \ .$$

Now, conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial \mathbb{U}/\partial \sigma_e^2$, versus the private value to the firm, $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$.

The next result shows that information acquisition is typically inefficient. In particular, in any symmetric equilibrium, there is a constant wedge between the private value of information by firms and its value to the planner.

Proposition 7 *In a symmetric equilibrium, the private value of information is always less than its social value, i.e.*

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \left(1 + \frac{\delta}{(\theta-1)(\delta-1+\gamma)}\right)\left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\sigma_e^2 = \hat{\sigma}_e^2} \ < \ 0 \qquad \forall\, \sigma_e^2 \in \mathbb{R}^+$$

Therefore, the level of information acquired in equilibrium is inefficiently low.

Note that this under-acquisition is a consequence of the imperfect substitutability. When this disappears, i.e. $\theta \to \infty$, the gap between the social value and the private value to the firm also vanishes. However, in general, an equilibrium with endogenous information is inefficient (even though it is efficient both under full information and exogenous information). This inefficiency has consequences for both the level of economic activity and the volatility of real business cycles. That is, with endogenous information choice, equilibrium employment no longer coincides with the choice of a planner - whether in terms of the average level or the elasticity to the aggregate shock (because the amount of information is different). This channel is absent in models with exogenous information (e.g. Angeletos and LaO (2009)).

3.7 Optimal Policy

Next, we describe the optimal policy that restores efficiency in this environment. We show that the policy prescription takes the form of a a constant revenue subsidy, which corrects the monopoly power distortion in employment, also aligns the private and social values of information, leading to efficient information acquisition.

Under this policy, the problem of the firm becomes:[21]

$$\Pi_{it} = \max_{N_{it}} \mathbb{E}_i Q_t \left[(1+\tau_R) \, P_{it} Y_{it} - W_{it} N_{it} \right] \ ,$$

The first order condition for labor is:

$$N_{it} = \left(\frac{1}{\delta} \frac{\theta - 1}{\theta} \left(1+\tau_R\right) \left(\mathbb{E}_{it} Q_t Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}} \right) \right)^{\frac{\delta\theta}{1+\theta\delta-\theta}} \ . \tag{30}$$

It is straightforward to show that the level distortion in the response function is removed, i.e. k_2 equals k_2^*, if the subsidy satisfies

$$1+\tau_R = \frac{\theta}{\theta-1} > 1 \ .$$

What about information acquisition ? Recall from Proposition 7 that the private value of information was less than the social value. By subsidizing firms' revenues, the policy described above also raises the private marginal value of information. Remarkably, it brings the private value exactly in line with the planner's valuation, as the following result shows.

Proposition 8 *A symmetric equilibrium with a constant revenue subsidy $\tau_R = 1/\theta$ is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

4 A Price-Setting Model

In this section, we study information acquisition in a standard micro-founded model of nominal price-setting under dispersed information about monetary shocks. The model environment closely follows Hellwig (2005). In line with the results in that paper, we find that payoff externalities lead to information being used inefficiently in equilibrium. In particular, firms pay too much attention to private signals about innovations to money supply. This inefficiency also causes the incentives of firms to acquire information to diverge from social incentives, but, as we show, these incentives remain distorted even when the inefficiency in use is not present.

[21] In addition, a lump sum transfer $\tau_R \int P_{it} Y_{it} di$ is subtracted from the income side of the household's budget constraint.

4.1 Preferences, Technology and Information

Time is discrete, $t = 0, 1, 2....$ The economy is populated by 3 types of agents - a representative household, a continuum of intermediate producers and a final goods producer.

Household: The household solves the following problem:

$$\max_{\{C_t,N_t,M_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t - N_t) \ ,$$

subject to a cash in advance constraint

$$P_t C_t \leq M_{t-1} + T_t \ ,$$

and a budget constraint

$$P_t C_t + M_t \leq W_t N_t + \Pi_t + (M_{t-1} - P_{t-1} C_{t-1}) + T_t \ .$$

Government: The government's budget constraint is given by

$$M_t = M_{t-1} + T_t \ ,$$

where M_t is the stock of money supply. The (exogenous) law of motion for M_t is

$$M_t = M_{t-1} U_t \ .$$

In other words, money supply is assumed to follow a random walk in logs[22]

$$m_t = m_{t-1} + u_t \ ,$$

where $u_t \sim N\left(0, \sigma_u^2\right)$. This shock to the stock of money is the only source of aggregate uncertainty in the model.

Final good producer: The single final good is produced using a continuum $[0, 1]$ of intermediate inputs Y_{it}. The production function is a Dixit-Stiglitz aggregator with constant returns to scale. The final good producing firm solves the following static problem:

$$\max \quad P_t Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

subject to

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

[22] Hereafter, lower case variables are in logs, e.g. $x_t \equiv \ln X_t$.

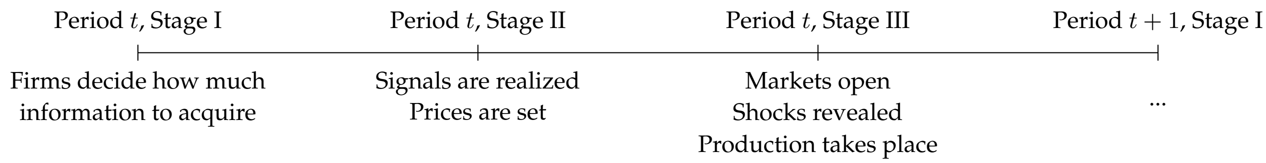

Figure 2: Timeline of Events

where θ is the elasticity of substitution, $\theta > 1$.

Intermediate producers: There is a continuum of intermediate good producers indexed $i \in [0,1]$. These firms make decisions in different stages in every period. In the first stage of the period, each firm chooses the variance of the error term in its private signal about the aggregate state subject to a cost function, $v(\hat{\sigma}_e^2)$. The properties of this function will be specified later. In the second stage, the firm observes its signal and sets prices to maximize expected profits, conditional on its information set. After this, in stage III, markets for goods and labor open, wages are determined and production takes place. Figure 2 shows the timing of events in each period.

The production function is a standard decreasing returns to scale technology with labor as the sole input.

$$Y_{it} = (\delta N_{it})^{\frac{1}{\delta}} \ ,$$

where $\delta > 1$.

In stage II, an intermediate producer sets a nominal price to maximize the expected value of profits (weighted by the household's stochastic discount factor):

$$\Pi_{it} = \max_{P_{it}} \ \mathbb{E}_{it} Q_t \left[P_{it} Y_{it} - W_t N_{it} \right] \ ,$$

where $\mathbb{E}_{it}(\cdot)$ represent the expectation conditional on firm's i information set $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}(\cdot) \equiv \mathbb{E}_t(\cdot \,|\mathcal{I}_{it})$. Note that, by setting a price, the firm commits to delivering any quantity at that price when markets open in stage III.

Information and signal structure: Before setting prices in stage II, each firm has access to a private signal s_{it} about the current innovation to money supply:

$$s_{it} = u_t + e_{it} \ ,$$

where $e_{it} \sim N\left(0, \hat{\sigma}_e^2\right)$ and $\hat{\sigma}_e^2$ is the variance chosen in stage I by the firm. In stage III, i.e. after prices are set, markets open and the aggregate state becomes commonly known. Therefore, at the time of setting prices in period t, the firm's information set consists of the aggregate state (money supply) at the end of the previous period and its private signal about the current innovation i.e. $\mathcal{I}_{it}$ consists of $\{M_{t-1}, s_{it}\}$.

Information acquisition problem: In stage I of the period, intermediate firms choose the amount of information of their private signal, taking as given information choices of other firms. Expected profits prior to the realization of the signal and the aggregate state is defined by:

$$\hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) \equiv \mathbb{E}_{t-1}\Pi_{it} \ ,$$

where $\mathbb{E}_{t-1}$ is the expectation conditional on information available at the time of the first stage decision i.e. the (commonly known) history until $t-1$.

The problem of the firm in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \quad \hat{\Pi}_{it}\left(\hat{\sigma}_e^2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \ ,$$

where $\upsilon\left(\cdot\right)$ is the cost of information as a function of the noise in the signal. We assume that $\upsilon'(\cdot) < 0, \upsilon''(\cdot) > 0$.

4.2 Optimality

As before, we solve the model backwards starting from the last stage.

Stage III: Complete information - In the last stage of each period, both household and firms have perfect information of the aggregate state. Optimization by households and the final goods producer, combined with market clearing, implies the following set of equilibrium conditions:

$$P_t C_t = M_t \ , \tag{31}$$

$$W_t = \gamma M_t \quad \text{where} \quad \gamma = \beta e^{\frac{\sigma_m^2}{2}} \ , \tag{32}$$

$$Q_t = \frac{1}{W_t} \ , \tag{33}$$

$$P_t = \left(\int_0^1 P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}} \ , \tag{34}$$

$$N_t = \int_0^1 N_{it} di \ , \tag{35}$$

$$Y_t = C_t \ . \tag{36}$$

The production decisions of firm i are pinned down by the demand of the final goods producer (given the prices set in stage II):

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} .$$

Stage II: Price-setting - Firms set prices to maximize expected profits, taking into account the nature of equilibrium allocations in stage III:

$$\max_{P_{it}} \ \mathbb{E}_{it} Q_t \left[P_{it} Y_{it} - W_t N_{it} \right] ,$$

subject to:

$$\begin{aligned} Y_{it} &= Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} , \\ Y_{it} &= (\delta N_{it})^{\frac{1}{\delta}} . \end{aligned}$$

Plugging these constraints and equilibrium conditions from Stage III, this profit maximization problem can be written as:

$$\max_{P_{it}} P_{it}^{1-\theta} \mathbb{E}_{it} \left(P_t^{\theta-1} \right) - \frac{\gamma}{\delta} P_{it}^{-\theta\delta} \mathbb{E}_{it} \left(M_t^{\delta} P_t^{\delta(\theta-1)} \right) .$$

The first order condition is:

$$P_{it} = \left(\frac{\gamma\theta}{\theta - 1} \right)^{\frac{1}{1+\theta\delta-\theta}} \left(\frac{\mathbb{E}_{it} \left(M_t^{\delta} P_t^{\delta(\theta-1)} \right)}{\mathbb{E}_{it} \left(P_t^{\theta-1} \right)} \right)^{\frac{1}{1+\theta\delta-\theta}} . \tag{37}$$

Stage I: Information acquisition Under the assumption of an interior solution, the firm's optimal information choice is characterized by the following optimality condition:

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}_e^2} = 0 .$$

4.3 Equilibrium

A stationary equilibrium is (i) a set of information choices in Stage I for each firm (ii) a set of pricing rules (iii) aggregate variables C_t, N_t, W_t, Y_t and P_t as functions of the aggregate history (iv) intermediate production Y_{it} and labor input N_{it} such that (a) taking W_t and P_t as given, the household choices C_t and N_t solve the household's maximization problem (b) taking P_t and P_{it} as given, the choices of Y_t and Y_{it} solve the final goods producer's problem (d) taking the functions in (iii) as given, the pricing rules in (ii) maximize expected profits of the intermediate goods producer, conditional on its information (e) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem (f) Markets clear i.e. $N_t = \int N_{it}\, di$, $Y_{it} = \delta N_{it}^{\frac{1}{\delta}}$ and $Y_t = C_t$.

We focus on symmetric stationary equilibria, where all intermediate producers acquire the same amount of information in Stage I and follow the same pricing strategies in Stage II. We start the characterization of such an equilibrium with a conjecture about the aggregate price level:

$$P_t = M_{t-1} U_t^{\alpha} K_2 \ ,$$

or, in logs

$$p_t = m_{t-1} + \alpha u_t + k_2 \ , \tag{38}$$

where $\alpha \in [0,1]$ and k_2 are constants to be determined in equilibrium. The former determines the sensitivity of aggregate prices to monetary shocks whereas the latter affects the level of aggregate prices. Both these coefficients will play an important role in our analysis.

Intermediate producers We substitute the equilibrium conjecture (38) for aggregate prices into the first order condition (37) and assuming[23] conditional log-normality, take logs :

$$\begin{aligned} p_{it} =& m_{t-1} + \frac{(1-r)}{\delta} \ln\left(\frac{\gamma\theta}{\theta-1}\right) + r k_2 \\ &+ (1 - r + \alpha r)\, \mathbb{E}_{it}(u_t) \\ &+ \frac{1}{2}(1 - r + \alpha r)\left(\delta + (\delta+1)\,\alpha\,(\theta-1)\right) \mathbb{V}_{it}(u_t) \ , \end{aligned}$$

where

$$r \equiv \frac{(\delta-1)(\theta-1)}{1 + \theta\delta - \theta} \ \in [0,1] \ ,$$

and $\mathbb{E}_{it}(u_t)$ and $\mathbb{V}_{it}(u_t)$ are the posterior mean and variance, respectively, conditional on the firm's information set $\mathcal{I}_{it}$.

The firm's optimal price thus has 3 components. The first is a constant term, consisting of the (commonly known) level of last period's money supply and the level coefficients in aggregate prices. The second represents the firm's optimal response to the expected innovation in money supply. The parameter r controls the nature of strategic interactions. The greater the value of r, the more the firm's optimal reaction depends on α, the sensitivity of aggregate prices to the current shock. The third term is an adjustment to the price to account for the fact that the firm is uncertain about the realization of the shock. This 'precautionary' term emerges from the asymmetric nature of the firm's profit function. If the firm's relative price is higher than the optimum, it loses market

[23] This will be verified later.

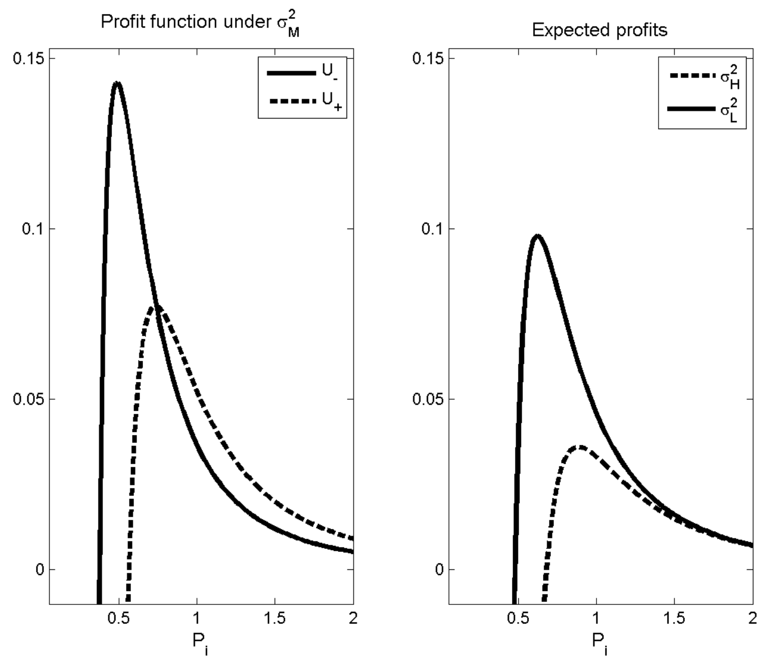

Figure 3: Effect of Uncertainty

share. A low relative price leads to higher quantities sold but due to diminishing returns, these additional units are produced at an increased marginal cost. Given the specific forms of demand and production functions, the latter is a much more costly phenomenon i.e. profits decline much more sharply with a low relative price than a high one. As a result, when the firm is uncertain about the position of its demand curve, the optimal price is a little higher than the expected value of the target price. Figure 3 illustrates this feature with a simple example where the aggregate shock is assumed to take only 1 of 2 possible values - low U_- and high U_+, $(U_+ > U_-)$. The left panel depicts the profit function as a function of P_i under the two realizations of the monetary shock. Notice that for a particular realization of the shock, the profit function is steeper for prices that are below the profit maximizing price. That is, charging prices that are too low is a costlier mistake than charging prices that are too high. In the right panel of Figure 3, we show the *expected* profit function as a function of price P_i and varying degrees of uncertainty about the aggregate shock. The two lines keep the expected value of the shock constant, but vary the levels of variance (low σ_L^2 and high σ_H^2). Expected-profits are strictly decreasing in uncertainty. Also, as uncertainty increases, the optimal price (i.e. the one which maximizes expected profit) increases, reflecting the asymmetric nature of the penalty for charging sub-optimal prices.

Using standard results for Bayesian updating, the firm's posterior expectation and variance about the state of the economy are:

$$\mathbb{E}_{it}(u_t) = \frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} s_{it} , \tag{39}$$

$$\mathbb{V}(u_t) = \frac{\sigma_u^2 \hat{\sigma}_e^2}{\sigma_u^2 + \hat{\sigma}_e^2} , \tag{40}$$

where $\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2}$ is the signal to noise ratio.

Plugging the optimal price into the firm's profit function, we get the following expression for maximized profit

$$\Pi_{it} = e^{\phi_1 \mathbb{E}_{it}(u_t) + \phi_2 \mathbb{V}_{it}(u_t)} \; K_2^{(\theta-1)(1-r)} \; K_1 , \tag{41}$$

where

$$\phi_1 \equiv (1-\theta)(1-r)(1-\alpha) < 0 ,$$

$$\phi_2 \equiv \frac{1}{2}(1-\theta)(1-r)\left(\delta\left(1+\alpha(\theta-1)\right)^2 - \alpha^2\theta(\theta-1)\right) < 0 ,$$

$$K_1 \equiv \left(\frac{\theta-1}{\gamma\theta}\right)^{\frac{\theta\delta}{1+\theta\delta-\theta}} \left(\frac{1}{(\theta-1)(1-r)}\right) > 0 .$$

Notice that the conjectured behavior of the aggregate price (38) affects the firm's payoff in two ways. First, the level effect, k_2, affects positively the level of profits in the second stage. Second, the price elasticity to the aggregate shock, i.e. α, enters into the coefficients ϕ_1 and ϕ_2. It is easy to show that

$$\frac{\partial \phi_1}{\partial \alpha} > 0 , \qquad \frac{\partial \phi_2}{\partial \alpha} < 0 .$$

In other words, the more responsive aggregate prices are to the nominal shock, the lower is the sensitivity of firm's profits to the expected nominal shock, but the greater is the cost of uncertainty.

Equilibrium in Stage II In a symmetric equilibrium, where all firms choose the same amount of information and follow the same pricing rule, the cross-sectional distribution of prices is log-normal. Therefore, taking logs on both sides of the aggregate price expression (34) yields:

$$p_t = \bar{\mathbb{E}}(p_{it}) + \frac{(1-\theta)}{2}\mathbb{D} ,$$

where $\bar{\mathbb{E}}(\cdot)$ and $\mathbb{D}$ denote the cross-sectional mean and dispersion in prices:

$$\bar{\mathbb{E}}(p_{it}) = \int_0^1 p_{it} di ,$$

$$\mathbb{D} = \int_0^1 \left(p_{it} - \bar{\mathbb{E}}(p_{it})\right)^2 \; di .$$

Next, we derive these cross-sectional moments. First, using the expression for p_{it}, the cross-sectional mean is given by

$$\bar{\mathbb{E}}\left(p_{it}\right) = m_{t-1} + \frac{1-r}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + rk_2 + (1-r+\alpha r)\int \mathbb{E}_{it}(u_t)\, di + g(\alpha)\mathbb{V}\ , \tag{42}$$

where

$$g(\alpha) \equiv \frac{1}{2}\left(1-r+\alpha r\right)\left(\delta + \left(\delta+1\right)\alpha\left(\theta-1\right)\right)\ .$$

Substituting the Bayesian updating formulae into (42), we get

$$\bar{\mathbb{E}}\left(p_{it}\right) = m_{t-1} + \frac{1-r}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + rk_2 + (1-r+\alpha r)\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}u_t + g(\alpha)\mathbb{V}\ ,$$

where we invoke the law of large numbers to note that

$$\int_0^1 s_{it}di = \int_0^1 \left(u_t + e_{it}\right) di = u_t + \int_0^1 e_{it}\, di = u_t\ .$$

Similarly,

$$p_{it} - \bar{E}(p_{it}) = (1-r+r\alpha)\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}e_{it}\ ,$$

$$\Rightarrow \quad \mathbb{D} = (1-r+r\alpha)^2\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}\right)^2\sigma_e^2\ .$$

The next result completes the guess-and-verify procedure and characterizes the response coefficients.

Proposition 9 *In a symmetric equilibrium, the aggregate price level is given by (38), with*

$$\alpha = \frac{\left(1-r\right)\sigma_u^2}{\left(1-r\right)\sigma_u^2+\sigma_e^2} \quad \in [0\ 1]\ , \tag{43}$$

$$k_2 = \frac{1}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{g(\alpha)}{1-r}\mathbb{V} + \frac{(1-\theta)}{2\left(1-r\right)}\mathbb{D} \quad > 0\ , \tag{44}$$

where, as defined earlier,

$$g \equiv \frac{\left(1-r+\alpha r\right)\left(\delta+\left(\delta+1\right)\alpha\left(\theta-1\right)\right)}{2}\ ,$$

$$\mathbb{V} \equiv \frac{\sigma_u^2\sigma_e^2}{\sigma_u^2+\sigma_e^2}\ ,$$

$$\mathbb{D} \equiv (1-r+r\alpha)^2\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}\right)^2\sigma_e^2 = \alpha^2\sigma_e^2\ .$$

The expression for α has an intuitive interpretation. It takes the form of a signal-to-noise ratio, except that the variance of the fundamental (in this case, the nominal shock) is adjusted to account for the degree of complementarity. Greater the complementarity, i.e. higher the r, the lower the weight on private signals and higher the reliance on commonly known information, which in this case is just the money stock in the previous period.

4.4 Information acquisition

Next, we examine the information acquisition decision in Stage I. Consider the maximized stage II profit function, equation (41),

$$\Pi_{it} = e^{\phi_1 \mathbb{E}_{it}(u_t) + \phi_2 \mathbb{V}_{it}(u_t)} \; K_2^{(\theta-1)(1-r)} \; K_1 \; . \tag{45}$$

In Stage I, the firm takes as given the information choices of other firms, or equivalently, the aggregate coefficients, ϕ_1, ϕ_2 and k_2. Expected profits, conditional on a choice of individual error variance $\hat{\sigma}_e^2$, are given by taking expectations over the realization of the random variable $\mathbb{E}_{it}(u_t)$. Exploiting log-normality (and dropping the time subscript), this ex-ante expected profit is:

$$\hat{\Pi}(\hat{\sigma}_e^2, \alpha, k_2) = e^{\left(\frac{\phi_1^2}{2} \frac{\left(\sigma_u^2\right)^2}{\sigma_u^2 + \hat{\sigma}_e^2} + \phi_2 \frac{\sigma_u^2 \hat{\sigma}_e^2}{\sigma_u^2 + \hat{\sigma}_e^2}\right)} K_2^{(\theta-1)(1-r)} K_1 \; . \tag{46}$$

Note that expected profits is a function of the firm's own variance, $\hat{\sigma}_e^2$ as well as the aggregate coefficients, which in turn are determined by the information choices of all firms in the economy.

It is straightforward to show that expected profits are decreasing (and convex) in the variance of the error in firm's own signal i.e.

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} = \hat{\Pi} \left(-\frac{\phi_1^2}{2} + \phi_2 \right) \left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} \right)^2 < 0 \quad \forall \; \hat{\sigma}_e^2 \in \mathbb{R}^+ \; , \tag{47}$$

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} = -\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \left\{ \frac{2}{(\sigma_u^2 + \hat{\sigma}_e^2)} - \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \right\} > 0 \; .$$

The problem of the firm in Stage I is:

$$\max_{\hat{\sigma}_e^2} \;\; \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \; , \tag{48}$$

where $\upsilon\left(\hat{\sigma}_e^2\right)$ is the cost function. As before, we assume that $\upsilon(\cdot)$ is sufficiently convex so that the solution to this problem is an interior one and is characterized by the first-order condition.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2

$$\sigma_e^2 = \text{argmax}_{\hat{\sigma}_e^2} \;\; \hat{\Pi}\left(\hat{\sigma}_e^2, \alpha, k_2\right) - \upsilon\left(\hat{\sigma}_e^2\right) \; ,$$

where α and k_2 are functions of σ_e^2 as given by (43)-(44).

4.5 Efficiency in Information Use

As we did for the RBC model, we begin our exploration of the efficiency properties[24] of information use in equilibrium. We show that firms place too much reliance on private signals, relative to an information-constrained social planner, who is interested in maximizing household utility.

To characterize the socially optimal use, we assume that, in Stage II, all firms follow a linear pricing rule of the form:

$$p_{it} = m_{t-1} + \tilde{k_2} - \frac{1-\theta}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}\ s_{it}\ . \tag{49}$$

It is straight forward to see that the aggregate price is then given by

$$p_t = m_{t-1} + \tilde{k_2} + \tilde{\alpha}\ u_t\ ,$$

and life-time utility of the household is

$$\mathbb{U} = -\tilde{k_2} - \frac{1}{\delta}\exp\left\{-\delta\tilde{k_2} + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\,\sigma_u^2\right]\right\}\ .$$

The efficient use of information is then defined by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k_2},\tilde{\alpha}}\quad \mathbb{U}(\tilde{k_2},\tilde{\alpha})\ .$$

The next proposition lays out the optimal response coefficients and shows equilibrium prices that are suboptimally higher on average and too sensitive to nominal shocks.

Proposition 10 *1. The coefficients that maximize the life-time utility of the household are*

$$\alpha^* = \frac{(1-r)\,\sigma_u^2}{(1-r)\,\sigma_u^2 + \theta\sigma_e^2}\ , \tag{50}$$

$$k_2^* = \frac{\delta\theta}{2\,(1-r)}\alpha^*\sigma_e^2\ . \tag{51}$$

2. For a given σ_e^2, these coefficients are lower than the equilibrium ones

$$\alpha^* < \alpha \qquad \textit{and} \qquad k_2^* < k_2\ .$$

4.6 Efficiency of Information Acquisition

In this subsection, we compare the level of information acquired in equilibrium to a socially optimum level. In order to disentangle the effect of the inefficiency in information use identified

[24]The results in this subsection basically repeat the findings in Hellwig (2005).

earlier, we start by comparing the equilibrium information acquisition to the choice of a planner who is also subject to the same inefficiency, i.e. who takes the equilibrium in Stage II as given. In other words, we study the problem of a planner who gets to choose only the amount of information acquired ex-ante, but cannot affect the equilibrium responses. Not surprisingly, the equilibrium features a suboptimal level of information acquisition. We then revisit the optimality of information choice under the assumption of efficient use and find that private incentives to acquire information are still not aligned to social ones.

The first step is to express the utility of the household in equilibrium as a function of the amount of information. Using the equilibrium relationships derived in the previous section, we have:

$$\begin{aligned} C_t &= \frac{U_t^{(1-\alpha)}}{K_2} , \\ N_t &= \frac{1}{\delta} (C_t)^{\delta} \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\theta\delta} di . \end{aligned}$$

More algebra yields the following expression for utility:

$$\mathbb{U}\left(\sigma_e^2\right) = \frac{1}{\delta}\left(\ln\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\theta-1}{\gamma\theta}\right) - \frac{\delta}{2}\frac{\sigma_u^2\sigma_e^2\left(\left(1-r\right)\theta\sigma_u^2+\sigma_e^2\right)}{\left(\left(1-r\right)\sigma_u^2+\sigma_e^2\right)^2} . \tag{52}$$

The next result replicates the findings in Hellwig (2005) that equilibrium welfare is not monotonically increasing in the precision of the private signal.

Proposition 11 1. *Suppose $\theta \leq 2$. Then, welfare decreases with the error in firms' signals i.e.* $\frac{d\mathbb{U}}{d\sigma_e^2} < 0 \ \forall \sigma_e^2$.

2. *If $\theta > 2$, welfare is decreasing in σ_e^2 only if the σ_e^2 is sufficiently small, i.e.*

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} < 0 \qquad \textit{if} \qquad \sigma_e^2 < \frac{\theta}{(\theta-2)}\sigma_u^2(1-r) .$$

The above result shows that the inefficiency in equilibrium information use can be so extreme that more information actually reduces welfare. To see the intuition behind the dependence on θ, recall that the difference between the equilibrium response coefficient and the socially optimal one was increasing in θ.

Again, we restrict attention to the case where utility, net of information acquisition costs, is maximized at an interior point. In other words, the solution to the following problem

$$\max_{\sigma_e^2} \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) , \tag{53}$$

is characterized by the usual first-order condition:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} - \frac{\partial \upsilon}{\partial \sigma_e^2} = 0 \ .$$

Now, conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial \mathbb{U}/\partial \sigma_e^2$, versus the private value to the firm, $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$.

The next proposition shows that information acquisition is typically inefficient. It characterizes the regions of the parameter space where there is over-acquisition of information in equilibrium. For brevity, we only present results for the case where $\theta > 2$ (which is the case for most calibrations of macroeconomic models) but similar results can be obtained for the other case as well.

Proposition 12 *Suppose $\theta > 2$ and $\sigma_e^2 < \frac{\theta}{(\theta-2)}(1-r)\,\sigma_u^2$, so the marginal value of information to the planner is positive. Then,*

1. *If $\gamma > \frac{\theta-1}{\theta}$, there is over-acquisition of information in equilibrium if the following condition holds:*

$$\sigma_e^2 \geq \left(\frac{\theta\gamma - (\theta-1)}{(\theta-1) + \gamma(\theta-2)} \right) (1-r)\,\sigma_u^2 \ .$$

2. *If $\gamma < \frac{\theta-1}{\theta}$, there is over-acquisition of information in equilibrium.*

Since a full-fledged numerical investigation is beyond the scope of this paper, we simply note that common calibrations of incomplete information monetary models satisfy the condition in the first statement, i.e. the empirically relevant region is one where the equilibrium information acquisition is more than the social optimal level. To see this, let $\theta = 4, \delta = 1.5$. Then, clearly $\gamma \approx \beta > \frac{\theta-1}{\theta} = \frac{3}{4}$. The condition on σ_e^2 is equivalent to the condition that prices are not too responsive to contemporaneous nominal shocks, i.e. $\alpha < 0.84$. In other words, so long as heterogeneity in information generates even modest real effects from nominal shocks, there will be over-acquisition of information in equilibrium.

Next, we show that, even when the use of information is efficient, firms do not fully internalize all the effects of their information choice. To show this, we again compare information acquired in equilibrium to the social planner's choice, under the assumption that prices are set using the socially efficient response coefficients α^* and k_2^*. The next result confirms that the inefficiency in information acquisition persists even in this case, though the exact conditions governing over/under-acquisition are different.

Proposition 13 *Suppose firms follow the pricing rule (49), with $\tilde{\alpha} = \alpha^*$ and $\tilde{k}_2 = k_2^*$. Then, if $\gamma > 1$, there is over-acquisition of information in equilibrium. Otherwise, there is under-acquisition of information in equilibrium.*

4.7 The Collusive Optimum

In this subsection and the next, we characterize how an individual firm's profits and its incentives to acquire information are affected by the information choice of other firms. We start with the effects on the level of profits. The main finding is a negative externality - a firm's expected profits decline when other firms in the economy are better informed. Recall that the average amount of information in the economy enters the firm's profits through the aggregate price level, specifically through the (endogenous) coefficients α and k_2. The expression for the ex-ante profit (46) leads to the following observation

Lemma 1 *The firm's ex-ante profit is decreasing in the elasticity of aggregate prices to the nominal shock and increasing in the level of aggregate prices i.e. $\frac{\partial \hat{\Pi}}{\partial \alpha} < 0$ and $\frac{\partial \hat{\Pi}}{\partial k_2} > 0$.*

Thus, an individual firm's profits are decreasing in the elasticity of prices to nominal shocks, but increasing in the overall level of prices. Next, from (43), it is easy to see that α depends negatively on σ_e^2, i.e.

$$\frac{\partial \alpha}{\partial \sigma_e^2} = -\frac{\alpha}{\left((1-r)\,\sigma_u^2 + \sigma_e^2\right)} \quad < 0 \; .$$

In other words, the more accurate are the signals of other firms, the more responsive is the aggregate price level to nominal shocks. In the limit, as $\sigma_e^2 = 0$, aggregate prices will fully adjust to shocks, i.e. $\alpha = 1$.

The relationship of the level coefficient k_2 with information is less clear. The expression for k_2, equation (44) comprises a term which is linear in price dispersion as well as a term where the posterior variance, $\mathbb{V}$, is multiplied by a function of α. Price dispersion is non-monotonic in the precision of firms' private information. As σ_e^2 increases, the dispersion of the firms' signals increases, but firms place less weight on them. If the former effect dominates[25], dispersion increases with σ_e^2, otherwise it decreases. Posterior variance, on the other, always increases with the variance of the error term. However, the sensitivity of the price level to the posterior variance, $g(\alpha)$, is an decreasing function of σ_e^2. The intuition for this stems from our earlier discussion on why firms set higher prices in response to greater uncertainty. A higher σ_e^2 implies a lower α. This

[25]This happens as long as $(1-r)\sigma_u^2 > \sigma_e^2$.

implies that the aggregate price level comoves less positively with the nominal shock, effectively reducing the uncertainty about its target. The combination of these two forces makes this term also non-monotonic with respect to information.

As a result of these distinct forces, the overall effect of σ_e^2 on k_2 is in general ambiguous. The next result provides a complete characterization:

Lemma 2 *1. Suppose $\theta \leq 2$. Then, the aggregate price is, on average, increasing in the variance of the firms' signals i.e. $\frac{dk_2}{d\sigma_e^2} > 0 \;\; \forall \;\; \sigma_e^2$.*

2. Suppose $\theta > 2$. Then, the aggregate price is, on average, increasing in the variance of the firms' signals only if the variance is sufficiently small i.e. $\frac{dk_2}{d\sigma_e^2} > 0 \;\;$ if $\;\; \sigma_e^2 < \frac{\theta(1-r)}{(\theta-2)}\sigma_u^2$.

Thus, the overall amount of information in the economy affects the aggregate price level (and through it, profits) in complicated ways. However, it turns out that, in a symmetric equilibrium, we can characterize the net effect of information on firm profits quite sharply. In particular, there is a negative externality in any symmetric equilibrium - i.e. others' information choices have negative effects on the firm's profits. Formally,

Proposition 14 *In a symmetric equilibrium, a firm's expected profits increase with σ_e^2, the variance of the error term in the signals of other firms, i.e.*

$$\frac{\partial \hat{\Pi}}{\partial \sigma_e^2} > 0 \; .$$

Recall that a firm's profits are always increasing in its own information. As in the simple example, this negative externality will make equilibrium information acquisition suboptimal from the perspective of maximizing total profit. To make this point more formally, we compare the equilibrium information choice to a natural benchmark. The *team profit*, denoted $\hat{\Pi}^T$ is the combined expected profit earned by all firms in the Stage II equilibrium. The next proposition shows that the externality is quite powerful and when internalized, undoes all the beneficial effects of greater information.

Proposition 15 *In a symmetric equilibrium, i.e. where the error in all firms' signals has the same variance σ_e^2, the expected team profit is independent of that variance i.e.*

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = 0 \; .$$

Therefore, the symmetric information choice that maximizes the collective profit is no information i.e. $\sigma_e^2 = \infty$.

In other words, the increase in an individual firm's profits by improving the quality of its own information is exactly offset by the negative effect it has on others' profits. A direct implication of this striking result is that if firms were somehow able to collude on their information acquisition decision and information was costly, the unique symmetric outcome would be to acquire no information at all. Relative to this collusive benchmark, there is too much information acquired in equilibrium, precisely because firms do not internalize the negative effects of their own decisions on others profits.

Next, we explore the implications of efficient information use for the collusive team profits. The next result show that the negative externality and its implications for team profits apply even when information is used in a socially efficient manner, i.e. the response coefficients are α^* and k_2^*.

Proposition 16 *When the response function is the socially optimal one, i.e. characterized by α^* and k_2^*, a firm's expected profits increase with σ_e^2, the variance of the error term in the signals of other firms, i.e.*

$$\frac{\partial \hat{\Pi}}{\partial \sigma_e^2} > 0 \ .$$

Further, the expected team profit is independent of that variance of the error in the firms' signals, i.e.

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = 0 \ .$$

Therefore, the symmetric information choice that maximizes the collective profit is no information i.e. $\sigma_e^2 = \infty$.

4.8 Strategic Motives in Information Acquisition

Next, we use our analytical framework to take a closer look at the role of strategic considerations in the incentives to acquire information. Hellwig and Veldkamp (2009) show that, with a quadratic objective function, the information choice decision inherits the strategic nature of agents actions - if actions are strategic complements, information choices become subject to complementarity as well. This subsection investigates the applicability of their finding to the environment of this section. We show that the basic intuition in Hellwig and Veldkamp (2009) is at work here as well, but there are other forces at work - in particular, those acting through the *level* of aggregate price. Once these additional effects are taken into account, the nature of strategic interaction in information choice is in general ambiguous, even though actions are unambiguously strategic complements.

Recall that, in any interior solution to the firm's Stage I problem (48), the firm equalizes the marginal value of information to the marginal cost i.e.

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} = \upsilon'(\hat{\sigma}_e^2) \ .$$

Our focus is the effect of other firms' information on the term on the left hand side. We say information acquisition decisions are *strategic complements* if

$$\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} > 0$$

and *strategic substitutes* otherwise. In other words, if the firm's marginal value of information is increasing in others information, information choices are complements. As before, we start by examining the effect of the slope and level coefficients α and k_2.

Lemma 3 *The marginal value of information to a firm is increasing in the elasticity of aggregate prices to nominal shocks and the overall level of prices i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \alpha \partial \hat{\sigma}_e^2} < 0$ *and* $\frac{\partial^2 \hat{\Pi}}{\partial k_2 \partial \hat{\sigma}_e^2} < 0$*.*

Thus, more responsive aggregate prices not only affect the level of the firm's profits (Lemma 1), but also increase the sensitivity of the firm's profit to its own information. This is intuitive - when prices comove more with the nominal shocks, the firm's target price becomes more volatile and therefore, acquiring information becomes more attractive.

Combining the first part of the lemma with the definition of α in (43) leads to our next result:

Proposition 17 *Suppose* k_2 *is fixed. Then, an increase in the overall amount of information in the economy increases the marginal value of information for a firm.*

The above result is essentially the insight in Hellwig and Veldkamp (2009). Without any level effects, the complementarity in firm's pricing decisions (as parameterized by r) enters the information choice as well.

This finding does not generally hold once level effects are explicitly taken into account. The non-monotonicity of k_2 with overall information (as demonstrated in Lemma 2) spills over into the effect of overall information on a firm's marginal value of information. The next proposition divides the parameter space into regions according to the sign of the overall effect:

Proposition 18

1. *Suppose* $\sigma_u^2 < \frac{4(\delta-1)}{\delta^2 \theta}$*. Then, information acquisition decisions are strategic complements i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} > 0$ *.*
2. *Suppose* $\frac{4(\delta-1)}{(1-r)\delta^2 \theta} < \sigma_u^2$*. Then, decisions are strategic substitutes i.e.* $\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} < 0$ *.*

3. *If* $\frac{4(\delta-1)}{\delta^2\theta} < \sigma_u^2 < \frac{4(\delta-1)}{(1-r)\delta^2\theta}$*, then this relationship is ambiguous and depends on the value of* $\hat{\sigma}_e^2$*.*

Parameterizations commonly used in macro models are in the region where information choices are strategic complements. For example, with $\theta = 4, \delta = 1.5$, there is strategic complementarity in information acquisition so long as the innovations to money supply have a variance less than 0.22, which is consistent with standard calibrations (e.g. see Golosov and Lucas (2007), who use $(0.0062)^2$ in an annual model). Even with $\theta = 20, \delta = 1.1$, the cutoff level of the variance is 0.02.

5 Conclusion

We have shown that models with dispersed information and payoff externalities typically feature an inefficient level of information being acquired in equilibrium. Importantly, this inefficiency persists even in situations where that information is used optimally ex-post, as in the competitive environment of Section 3. When information use in inefficient, as in the price-setting application of Section 4, this introduces another source of distortion between social and private values of information.

There are several directions for future work. With a view to maintaining analytical tractability, we have made several simplifying assumptions. Relaxing them might make it necessary to use numerical methods to solve the model, but will allow for a quantitative evaluation of the inefficiency and the policy interventions necessary to correct it. On the theoretical side, exploring the connections between the inefficiencies arising from payoff-linkages highlighted in this paper with others identified by the literature (e.g. the inefficiency in information aggregation as in Amador and Weill (2010)) is an interesting and important direction.

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Appendix A Proofs of Results

A.1 A General Beauty Contest Model

Proof of Proposition 1 Follows directly by setting $\hat{\lambda} = \lambda$ in (1) and solving.

Proof of Proposition 2 We solve (3) for λ^*.

Proof of Proposition 3 Follows from the comparison of the expressions for λ and λ^*.

Proof of Proposition 4 As before, we start with a conjecture about the average action,

$$\bar{a} = \lambda_1 \theta + \lambda_2 S \, .$$

Then, the optimality condition of the agent implies

$$a_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \lambda_1 \right) \mathbb{E}_i(\theta) + \frac{\psi}{\phi + \psi} \lambda_2 S \, .$$

Integrating over i,

$$a_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \lambda_1 \right) \bar{\mathbb{E}}(\theta) + \frac{\psi}{\phi + \psi} \lambda_2 S \, .$$

Next, note that

$$\mathbb{E}_i(\theta) = \delta_1 s_i + \delta_2 S \; ,$$

where $\delta_1 = \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\eta^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$ and $\delta_2 = \frac{\frac{1}{\eta^2 \sigma_\epsilon^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\eta^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$. This implies that $\bar{\mathbb{E}}(\theta) = \delta_1 \theta + \delta_2 S$. Substituting in the expression for $\bar{a}$ yields a system of linear equations. The solution is

$$\lambda_1^{eq} = \frac{\phi \delta_1}{\phi + \psi(1 - \delta_1)} \qquad \lambda_2^{eq} = \frac{\phi + \psi}{\phi} \frac{\delta_2}{\delta_1} \lambda_1^{eq} \, .$$

The planner's optimality conditions for λ_1^* and λ_2^* are

$$\phi^*(\lambda_1^* + \lambda_2^* - 1)\sigma_\theta^2 + (\phi^* + \psi^*)\lambda_1 \sigma_e^2 = 0 \, ,$$
$$\phi^*(\lambda_1^* + \lambda_2^* - 1)\sigma_\theta^2 + \phi^* \lambda_2^* \eta^2 \sigma_\epsilon^2 = 0 \, .$$

Solving yields

$$\lambda_1^* = \frac{\phi^* \delta_1}{\phi^* + \psi^*(1 - \delta_1)} \, , \qquad \lambda_2^* = \frac{\phi^* + \psi^*}{\phi^*} \frac{\delta_2}{\delta_1} \lambda_1^* \, .$$

Comparing the two sets of coefficients yields the last part of the proposition.

A.2 A Quantity Choice Model

Proofs for 3.3 This outlines the main steps in the derivation of the equilibrium. We start with a guess about the law of motion for n_t. This guess is verified through the following steps.
Start from a conjecture for firm i labor (in logs):

$$n_{it} = \hat{k}_2 + \hat{\alpha} s_{it} \tag{54}$$

Define aggregate employment:

$$N_t = \int_0^1 N_{it} di$$

By the Central Limit Theorem,

$$n_t = \bar{\mathbb{E}}(n_{it}) + \frac{1}{2}\mathbb{D}$$

where $\bar{\mathbb{E}}(\cdot)$ and $\mathbb{D}$ denote the cross-sectional mean and variance of labor inputs on the islands:

$$\begin{aligned} \bar{\mathbb{E}}(n_{it}) &= \int_0^1 n_{it} di \ , \\ \mathbb{D} &= \int_0^1 \left(n_{it} - \bar{\mathbb{E}}(n_{it})\right)^2 \ di \ . \end{aligned}$$

From (54):

$$n_t = \hat{k}_2 + \hat{\alpha} a_t + \frac{1}{2}\hat{\alpha}^2 \sigma_e^2$$

Thus, given (14) we get the following equivalencies:

$$\begin{aligned} \alpha &= \hat{\alpha}, \\ k_2 &= \hat{k}_2 + \frac{1}{2}\hat{\alpha}^2\sigma_e^2, \end{aligned}$$

where α and k_2 are unknown parameters. Plugging (7)and (54) into (6), get (15). Combine equations (11) and (12) to get:

$$N_{it} = \left(\frac{\theta - 1}{\delta\theta}\left(\mathbb{E}_{it} Q_t Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}}\right)\right)^{\frac{\delta\theta}{1+\theta\delta-\theta}}$$

Substituting the equilibrium conditions (6)-(10) and (15) into the last equation and using log-normality yields (16). The rest consist on computing $\bar{\mathbb{E}}(n_{it})$ and $\mathbb{D}$ from above. Using the definition of n_t, delivers two fixed points for the unknown coefficients α and k_2,

$$\begin{aligned} \alpha &= \frac{\theta\delta}{1+\theta\delta-\theta}\left(\left(\frac{\delta+\alpha}{\delta}\right)\left(\frac{1}{\theta}-\gamma\right)+\frac{\theta-1}{\theta}\right)\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2} \\ k_2 &= \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(\frac{\theta-1}{\delta\theta}\right)+\frac{\theta\delta}{1+\theta\delta-\theta}\left(\frac{1}{\theta}-\gamma\right)\left(\frac{1}{\delta}k_2-\frac{1}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2\right) \\ &\quad + \left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_e^2}\right)^2\phi_1^2\sigma_e^2+\phi_2\frac{\sigma_a^2\sigma_e^2}{\sigma_a^2+\sigma_e^2} \end{aligned}$$

where ϕ_1 and ϕ_2 are given in the main text. After some algebra, these two fixed points can be rewritten as (19) and (20).

Proofs for 3.4 Substituting (16) back into profits yields (17). Expectations at Stage I, i.e. $\mathbb{E}_{t-1}$, of (17) delivers (21). Compute the first and second derivatives of (17) with respect to the firm's own noise, $\hat{\sigma}_e^2$. After some steps, we get (22). It is straightforward to show that profits are decreasing and convex with respect to $\hat{\sigma}_e^2$.

Proofs for 3.5 Start with a conjecture for individual employment like (24), where $\widetilde{\alpha}$ and $\widetilde{k}_2$ are unknown coefficients to be solved next. Define the household's period utility as:

$$\mathbb{U} \equiv (1-\beta)\,\mathbb{E}_{t-1}\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\int_0^1 N_{it}di\right) \tag{55}$$

Using the same steps as in section (3.3), it is possible to show that (24) implies

$$\begin{aligned} N_t &= \int_0^1 N_{it}di = \widetilde{K}_2 A_t^{\widetilde{\alpha}} \ , \\ C_t &= \widetilde{K}_2^{\frac{1}{\delta}} A_t^{\frac{\delta+\widetilde{\alpha}}{\delta}} \exp\left(-\frac{1}{2}\left(\frac{1+\theta\delta-\theta}{\theta\delta}\right)\frac{\widetilde{\alpha}^2}{\delta}\sigma_e^2\right) \ . \end{aligned}$$

Using the relationships derived above, the period utility of the household can be written as (25). Define the coefficients α^* and k_2^* as $(\alpha^*, k_2^*) = \arg\max_{\widetilde{k}_2,\tilde{\alpha}} \quad \mathbb{U}(\widetilde{k}_2, \tilde{\alpha})$. Then, from (25) we get following set of first order conditions,[26]

$$\log\frac{1}{\delta}+\frac{1-\gamma}{\delta}k_2+\left((1-\gamma)\left(\frac{\delta+\alpha}{\delta}\right)\right)^2\frac{\sigma_a^2}{2}-\frac{1-\gamma}{2}\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^2}{\delta}\sigma_e^2 = k_2+\frac{\alpha^2}{2}\sigma_a \ ,$$

$$(1-\gamma)\left(\frac{\delta+\alpha}{\delta}\right)\sigma_a^2-\frac{1+\theta\delta-\theta}{\theta\delta}\alpha\sigma_e^2 = \alpha\sigma_a^2 \ .$$

After simplifying these two first order conditions equations we get (26) and (27).

Proofs for 3.6 The following are useful results. First note that the first order condition for $\widetilde{k}_2$ implies:

$$\mathbb{E}_{t-1}\left(C_t\left(k_2^*, \alpha^*\right)\right)^{1-\gamma} = \delta\mathbb{E}_{t-1}N_t\left(k_2^*, \alpha^*\right) \tag{56}$$

[26]It is straightforward to prove that the second order conditions holds.

where $C_t(k_2^*, \alpha^*)$ and $N_t(k_2^*, \alpha^*)$ are the levels of consumption and employment under (k_2^*, α^*). Second, given the formula in Proposition 6 it follows that

$$N_t(k_2, \alpha) = \left(\frac{\theta - 1}{\theta}\right)^{\frac{\delta}{(\delta-1+\gamma)}} N_t(k_2^*, \alpha^*) \ , \tag{57}$$

$$(C_t(k_2, \alpha))^{1-\gamma} = \left(\frac{\theta - 1}{\theta}\right)^{\frac{1-\gamma}{(\delta-1+\gamma)}} (C_t(k_2^*, \alpha^*))^{1-\gamma} \ , \tag{58}$$

which hold state by state.Third, combining (56) with (57)-(58) we have:

$$\mathbb{E}_{t-1}(C_t(k_2, \alpha))^{1-\gamma} = \delta\left(\frac{\theta}{\theta - 1}\right)\mathbb{E}_{t-1}N_t(k_2, \alpha) \tag{59}$$

Note that ex-ante profits are proportional to aggregate employment. Plugging equation (13) back into the profits, we get:

$$\Pi_{it} = \frac{1 + \theta\delta - 1}{\theta - 1}N_{it}$$

Expected value at Stage I, i.e. $\mathbb{E}_{t-1}$ delivers the formula for ex-ante profits,

$$\hat{\Pi}(k_2, \alpha) \equiv \mathbb{E}_{t-1}\Pi_{it} = \frac{1 + \theta\delta - 1}{\theta - 1}N_t(k_2, \alpha)$$

where $N_t(k_2, \alpha)$ is given by (15). Computing the derivative with respect to noise, we get:

$$\frac{\partial\hat{\Pi}(k_2, \alpha)}{\partial\sigma_e^2} = -\frac{(1 + \theta\delta - \theta)}{\theta\delta}\frac{(1 - \gamma)}{\delta - 1 + \gamma}\hat{\Pi}(k_2, \alpha)\frac{\alpha^2}{2}$$

The latter is the overall effect of information of profits. Note that, in symmetric equilibrium $\sigma_e^2 = \hat{\sigma}_e^2$, The effect of the firm own noise on profits has the functional form of (22). Thus, the overall effect does not coincide with the value of private information. Plugging (59) in the definition of $\mathbb{U}$ given in (55), we have:

$$\mathbb{U}(k_2, \alpha) = \frac{\delta\theta - (1 - \gamma)(\theta - 1)}{(1 - \gamma)(\theta - 1)}\mathbb{E}_{t-1}N_t(k_2, \alpha),$$

which using the previous result about ex-ante profits can be written as:

$$\mathbb{U}(k_2, \alpha) = \frac{(\delta - 1)\theta + \gamma(\theta - 1) + 1}{(1 - \gamma)(1 + \theta\delta - \theta)}\hat{\Pi}(k_2, \alpha)$$

It follows that, after using (22) in a symmetric equilibrium,

$$\frac{\partial\mathbb{U}(k_2, \alpha)}{\partial\sigma_e^2} = \left(1 + \frac{\delta}{(\theta - 1)(\delta - 1 + \gamma)}\right)\left(\frac{\partial\hat{\Pi}(k_2, \alpha)}{\partial\hat{\sigma}_e^2}\right)_{\sigma_e^2 = \hat{\sigma}_e^2} < 0 \ \forall \ \sigma_e^2 \in \mathbb{R}^+$$

The following lemma helps to determine the direction of inefficiency.

Lemma 4 *If $\partial \mathbb{U}/\partial \sigma_e^2$ is greater (smaller) than $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$, then there is over-acquisition (under-acquisition) of information. If $\partial \mathbb{U}/\partial \sigma_e^2$ equals $\partial \hat{\Pi}/\partial \hat{\sigma}_e^2$, then information acquired in equilibrium is socially optimal.*

Proof. This proof focuses on the region where both the equilibrium and social planner information acquisition problems have an interior optimum. Define σ_{eq}^2 as the information acquired in a symmetric equilibrium

$$\sigma_{eq}^2 \equiv \arg\max_{\hat{\sigma}^2} \hat{\Pi}\left(\sigma^2, \hat{\sigma}^2\right) - \upsilon\left(\hat{\sigma}^2\right) \ .$$

where the subscript "e" in the variance term has been erased for exposition. Recall the first and second order conditions:

$$\begin{aligned} \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}^2} &= 0 \ , \\ \frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} &< 0 \ . \end{aligned}$$

Notice that the second order condition indicates that $\partial \upsilon/\partial \hat{\sigma}^2$ crosses $\partial \hat{\Pi}/\partial \hat{\sigma}^2$ from below. In other words, for $\sigma^2 < \sigma_{eq}^2$, $\partial \upsilon/\partial \hat{\sigma}^2 < \partial \hat{\Pi}/\partial \hat{\sigma}^2$ and for $\sigma^2 > \sigma_{eq}^2$, $\partial \upsilon/\partial \hat{\sigma}^2 > \partial \hat{\Pi}/\partial \hat{\sigma}^2$. Define σ_{sp}^2 as the information acquired by the social planner:

$$\sigma_{sp}^2 \equiv \arg\max_{\sigma_e^2} \mathbb{U}\left(\sigma^2\right) - \upsilon\left(\sigma_e^2\right) \ .$$

Recall the first and second order conditions:

$$\begin{aligned} \frac{\partial \mathbb{U}}{\partial \hat{\sigma}^2} - \frac{\partial \upsilon}{\partial \hat{\sigma}^2} &= 0 \ , \\ \frac{\partial^2 \mathbb{U}}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}^2 \partial \hat{\sigma}^2} &< 0 \ . \end{aligned}$$

Note that, because $\partial \upsilon/\partial \hat{\sigma}_e^2 < 0$, the first condition for an interior solution requires $\partial \mathbb{U}/\partial \sigma^2 < 0$. Also notice that the second order condition indicates that $\partial \upsilon/\partial \sigma^2$ crosses $\partial \mathbb{U}/\partial \hat{\sigma}^2$ from below. In other words, for $\sigma^2 < \sigma_{sp}^2$, $\partial \upsilon/\partial \sigma^2 < \partial \mathbb{U}/\partial \sigma^2$ and for $\sigma^2 > \sigma_{sp}^2$, $\partial \upsilon/\partial \sigma^2 > \partial \mathbb{U}/\partial \sigma^2$. Given all these properties, the following is true

$$\frac{\partial \hat{\Pi}\left(\sigma^2, \hat{\sigma}^2\right)}{\partial \hat{\sigma}^2}|_{\hat{\sigma}^2 = \sigma^2 = \sigma_{eq}^2} < \frac{\partial \mathbb{U}\left(\sigma^2\right)}{\partial \sigma^2}|_{\sigma^2 = \sigma_{eq}^2} \Rightarrow \sigma_{eq}^2 < \sigma_{sp}^2 \ .$$

To prove this, suppose to the contrary that $\sigma_{eq}^2 > \sigma_{sp}^2$. Then, by definition of σ_{eq}^2

$$\frac{\partial \hat{\Pi}\left(\sigma^2, \hat{\sigma}^2\right)}{\partial \hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma_{eq}^2} = \frac{\partial \upsilon\left(\hat{\sigma}^2\right)}{\partial \hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma_{eq}^2} .$$

From the social planner problem, if $\sigma_{eq}^2 > \sigma_{sp}^2$

$$\frac{\partial \upsilon\left(\hat{\sigma}^2\right)}{\partial \hat{\sigma}^2}|_{\hat{\sigma}^2=\sigma^2=\sigma_{eq}^2} > \frac{\partial \mathbb{U}\left(\sigma^2\right)}{\partial \sigma^2}|_{\sigma^2=\sigma_{eq}^2} .$$

Contradiction. ■

Proofs for 3.7 Following the same steps as in 3.3, the law of motion of total employment under τ_R is,

$$n_t = \frac{\theta\delta}{1+\theta\delta-\theta}\log\left(1+\tau_R\right) + k_2 + \alpha a_t.$$

To implement $n_t\left(k_2^*, \alpha^*\right)$, it follows that $\tau_R = 1/\theta$.

Next we compute the social value of information under τ_R. Since τ_R is such that the response coefficients are $\left(k_2^*, \alpha^*\right)$ the social value information can be computed directly from (25) evaluated at the optimum, i.e. $\widetilde{k}_2 = k_2^*$ and $\widetilde{\alpha} = \alpha^*$. The envelope condition implies, after using (56)

$$\frac{\partial \mathbb{U}\left(k_2^*, \alpha^*\right)}{\partial \sigma_e^2} = -\frac{1+\theta\delta-\theta}{\theta\delta}\frac{\alpha^{*2}}{2}\mathbb{E}_{t-1}N_t\left(k_2^*, \alpha^*\right)$$

Plugging equations (30) back into profits and use $\tau_R = 1/\theta$, we get:

$$\Pi_{it} = \frac{1+\theta\delta-1}{\theta-1}N_{it}$$

Expected value at Stage I, i.e. $\mathbb{E}_{t-1}$ delivers the formula for ex-ante profits,

$$\hat{\Pi}\left(k_2^*, \alpha^*\right) \equiv \mathbb{E}_{t-1}\Pi_{it} = \frac{1+\theta\delta-1}{\theta-1}N_t\left(k_2^*, \alpha^*\right) ,$$

which holds state by state.

These results imply that:

$$\frac{\partial \mathbb{U}\left(k_2^*, \alpha^*\right)}{\partial \sigma_e^2} = -\frac{\theta-1}{\theta\delta}\frac{\alpha^{*2}}{2}\hat{\Pi}\left(k_2^*, \alpha^*\right),$$

which exactly the functional form of the private value of information given in (22), evaluated at $\left(k_2^*, \alpha^*\right)$.

Externalities in the quantity choice model Note that a firm's expected profits are decreasing in its own noise $\hat{\sigma}_e^2$,

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} < 0 \ .$$

The *team profit*, denoted by $\hat{\Pi}^T$, is the combined expected profit earned by all firms in the stage II equilibrium. As a team, firms can collude in their investment of information. The difference is that under collusion, firms are concerned about the overall effect of information on the team profit. This value can be represented by the total derivative of the team profits with respect to the noise of the signal σ_e^2. In a symmetric outcome this derivative is the sum of two factors

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} \ . \tag{60}$$

The first factor corresponds to the derivative of a firm's expected profits evaluated at the symmetric outcome, i.e. $\hat{\sigma}_e^2 = \sigma_e^2$. The second factor corresponds to the externality.

We have

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \frac{(1+\theta\delta-\theta)}{\theta-1}\frac{(1-\gamma)}{\delta-1+\gamma}\left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2}$$

Thus, if $\gamma < 1$, then $d\hat{\Pi}^T/d\sigma_e^2 < 0$. Moreover,

$$\text{if } \frac{1+\theta\delta-\theta}{\theta-1}\frac{1-\gamma}{\delta-1+\gamma} > 1 \Longrightarrow \frac{d\hat{\Pi}^T}{d\sigma_e^2} < \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2} ,$$

the team's value of information at σ_e^2 is higher than individual firm's value of information.

And,

$$\text{if } \frac{1+\theta\delta-\theta}{\theta-1}\frac{1-\gamma}{\delta-1+\gamma} < 1 \Longrightarrow \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2} < \frac{d\hat{\Pi}^T}{d\sigma_e^2} < 0 \ ,$$

the team's value of information at σ_e^2 is lower than individual firm's value of information.

On ther other hand, if $\gamma > 1$, then $d\hat{\Pi}^T/d\sigma_e^2 > 0$ and the team's value of information at σ_e^2 is zero.

With respect to the externality, from equation (60) we have that,

$$\text{if } \frac{d\hat{\Pi}^T}{d\sigma_e^2} < \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2} \Longrightarrow \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} < 0,$$

and the externality is positive, i.e. more information acquired by others, the higher is payoff to the firm.

On the other hand,

$$\text{if } \frac{d\hat{\Pi}^T}{d\sigma_e^2} > \left(\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}\right)_{\hat{\sigma}_e^2 = \sigma_e^2} \Longrightarrow \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} > 0,$$

and the externality is positive, i.e. more information acquired by others, the higher is payoff to the firm. One special case is when $d\hat{\Pi}^T/d\sigma_e^2 > 0$. In such a case, the externality is strong enough to discourage investment in information completely in the collusive outcome.

A.3 A Price Setting Model

Proof of Lemma 1 From the ex-ante expected profit (46) and given that $\phi_1 < 0$, $d\phi_1/d\alpha > 0$ and $d\phi_2/d\alpha < 0$,

$$\frac{\partial\hat{\Pi}}{\partial\alpha} = \hat{\Pi}\left(\phi_1\frac{d\phi_1}{d\alpha}\sigma_u^2 + \frac{d\phi_2}{d\alpha}\hat{\sigma}_e^2\right)\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2} < 0\ .$$

From the ex-ante expected profit (46),

$$\frac{\partial\hat{\Pi}}{\partial k_2} = (\theta-1)(1-r)\frac{\hat{\Pi}}{K_2} > 0\ .$$

Proof of Lemma 2 After plugging α, equation (43), into k_2, equation (44),

$$k_2 = \frac{1}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta\sigma_u^2}{2(1-r)}\left(\frac{\sigma_e^2\left((1-r)\theta\sigma_u^2+\sigma_e^2\right)}{\left((1-r)\sigma_u^2+\sigma_e^2\right)^2}\right)\ .$$

Hence:

$$\frac{dk_2}{d\sigma_e^2} = \frac{\delta\sigma_u^2}{2}\left(\frac{(1-r)\theta\sigma_u^2+(2-\theta)\sigma_e^2}{\left((1-r)\sigma_u^2+\sigma_e^2\right)^3}\right)\ .$$

If $\theta \leq 2$,

$$\frac{dk_2}{d\sigma_e^2} > 0\ \forall\sigma_e^2\ .$$

If $\theta > 2$, the effect can be non-monotonic, i.e.

$$\frac{dk_2}{d\sigma_e^2} > 0 \qquad \text{if} \qquad \sigma_e^2 < \frac{(1-r)\theta}{(\theta-2)}\sigma_u^2\ .$$

Proof of Lemma 3 From equation (47) and using Lemma 1 and $\phi_1 < 0$, $\phi_2 < 0$, $d\phi_1/d\alpha > 0$ and $d\phi_2/d\alpha < 0$,

$$\begin{aligned}
\frac{\partial^2\hat{\Pi}}{\partial\alpha\partial\hat{\sigma}_e^2} &= \hat{\Pi}\left(-\phi_1\frac{d\phi_1}{d\alpha}+\frac{d\phi_2}{d\alpha}\right)\left(\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2}\right)^2 + \frac{\partial\hat{\Pi}}{\partial\alpha} < 0\ ,\\
\frac{\partial^2\hat{\Pi}}{\partial k_2\partial\hat{\sigma}_e^2} &= \frac{\partial\hat{\Pi}}{\partial k_2}\left(-\frac{\phi_1^2}{2}+\phi_2\right)\left(\frac{\sigma_u^2}{\sigma_u^2+\hat{\sigma}_e^2}\right)^2 < 0\ .
\end{aligned}$$

Proof of Proposition 17

Follows directly from Lemma 3 along with $d\alpha/d\sigma_e^2 < 0$.

Proof of Proposition 10 Suppose firms commit to follow this rule

$$p_{it} = m_{t-1} + \tilde{k_2} - \frac{1-\theta}{2}\tilde{\alpha}^2\sigma_e^2 + \tilde{\alpha}s_{it} \ .$$

Average prices are given by

$$p_t = m_{t-1} + \tilde{k_2} + \tilde{\alpha}u_t \ .$$

The utility of the household can be expressed as

$$\mathbb{U} = -\tilde{k_2} - \frac{1}{\delta}\exp\left\{-\delta\tilde{k_2} + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\sigma_u^2\right]\right\} \ .$$

Then, the optimal use of information is defined by

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k_2},\tilde{\alpha}} \quad \mathbb{U}(\tilde{k_2},\tilde{\alpha}) \ .$$

The focs

$$\begin{aligned} \frac{2\theta}{(1-r)}\tilde{\alpha}\sigma_\epsilon^2 - 2(1-\tilde{\alpha})^2\sigma_u^2 &= 0 \ , \\ -1 - \exp\left\{-\delta\tilde{k_2} + \frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\tilde{\alpha}^2\sigma_\epsilon^2 + (1-\tilde{\alpha})^2\sigma_u^2\right]\right\} &= 0 \ . \end{aligned}$$

It is straightforward to show that the solution to this problem is characterized by

$$\begin{aligned} \alpha^* &= \frac{(1-r)\sigma_u^2}{(1-r)\sigma_u^2 + \theta\sigma_e^2} \ , \\ k_2^* &= \frac{\delta\theta}{2(1-r)}\alpha^*\sigma_e^2 \ . \end{aligned}$$

For a fixed σ_e^2, it follows that α^* is lower than the equilibrium α. Also, for a fixed σ_e^2, the price level under the socially efficient information use of information is lower than the price level under the equilibrium use of information. To prove this, suppose

$$k_2 < k_2^* \ .$$

or, after replacing the values for k_2 and k_2^*

$$\frac{1}{\delta}\ln\left(\frac{\gamma\theta}{\theta-1}\right) + (\delta-1)(1-r)^2\theta\left(\sigma_u^2\right)^2 + (\delta-1)\theta\left(\sigma_e^2\right)^2 + (1-r)\left(\delta\left(\theta^2+1\right)-2\right)\sigma_u^2\sigma_e^2 < 0 \ .$$

which cannot be true. By contradiction, for a fixed σ_e^2, $k_2 > k_2^*$.

Proof of Proposition 11 From equation (52),

$$\frac{\partial\mathbb{U}}{\partial\sigma_e^2} = -\frac{\delta(1-r)\left(\sigma_u^2\right)^2}{2}\left[\frac{(1-r)\theta\sigma_u^2 + (2-\theta)\sigma_e^2}{\left((1-r)\sigma_u^2 + \sigma_e^2\right)^3}\right] \ .$$

Note that if $\theta \leq 2$, then $\partial\mathbb{U}/\partial\sigma_e^2 < 0 \ \forall\sigma_e^2$. If $\theta > 2$ the sign of $\partial\mathbb{U}/\partial\sigma_e^2$ depends on σ_e^2. If $\sigma_e^2 < (1-r)\theta\sigma_u^2/(\theta-2)$, then $\partial\mathbb{U}/\partial\sigma_e^2 < 0$. Otherwise, $\partial\mathbb{U}/\partial\sigma_e^2 > 0$.

Proof of Proposition 12 There is under acquisition of information in equilibrium if

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} < \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} ,$$

or

$$((\theta - 1) + \gamma (\theta - 2)) \sigma_e^2 < (1 - r) (\theta\gamma - (\theta - 1)) \sigma_u^2 . \tag{61}$$

The proof focuses on the case of socially valuable information, i.e. $\frac{d\mathbb{U}}{d\sigma_e^2} < 0$.

- If $\theta > 2$ and $\sigma_e^2 < (1 - r) \theta\sigma_u^2 / (\theta - 2)$, there are two cases.

  Case 1A: Suppose, $\theta\gamma - (\theta - 1) > 0$, then condition (61) requires

  $$\sigma_e^2 < (1 - r) \left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma (\theta - 2)} \right) \sigma_u^2 .$$

  Suppose that

  $$\begin{aligned} \frac{(1 - r)\theta}{(\theta - 2)} \sigma_u^2 &< (1 - r) \left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma (\theta - 2)} \right) \sigma_u^2 \\ &\Rightarrow \theta (\theta - 1) < -(\theta - 2) (\theta - 1) \end{aligned}$$

  which cannot be true.

  There is under acquisition of information if

  $$\sigma_e^2 < (1 - r) \left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma (\theta - 2)} \right) \sigma_u^2 ,$$

  and there over acquisition of information if

  $$\sigma_e^2 \geq (1 - r) \left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma (\theta - 2)} \right) \sigma_u^2 .$$

  Case 1B: Suppose, $\theta\gamma - (\theta - 1) < 0$, then condition (61) information is always over acquired in equilibrium.

- If $\theta < 2$, there are three cases.

  Case 2A: Suppose $\theta\gamma - (\theta - 1) < 0$ and $(\theta - 1) + \gamma (\theta - 2) > 0$ or $\gamma \in (0 , (\theta - 1) / \theta)$, then there is always over acquisition in equilibrium.

  Case 2B: Suppose $\theta\gamma - (\theta - 1) > 0$ and $(\theta - 1) + \gamma (\theta - 2) > 0$ or $\gamma \in ((\theta - 1) / \theta , (\theta - 1) / (2 - \theta))$.

  There is under acquisition of information if

  $$\sigma_e^2 < (1 - r) \left(\frac{\theta\gamma - (\theta - 1)}{(\theta - 1) + \gamma (\theta - 2)} \right) \sigma_u^2 ,$$

and there over acquisition of information if

$$\sigma_e^2 \geq (1-r)\left(\frac{\theta\gamma-(\theta-1)}{(\theta-1)+\gamma(\theta-2)}\right)\sigma_u^2 \,.$$

Case 2C: Suppose $\theta\gamma-(\theta-1)>0$ and $(\theta-1)+\gamma(\theta-2)<0$ or $\gamma\in((\theta-1)/(2-\theta),\infty)$, then there is always under acquisition in equilibrium.

Proof of Proposition 13 Suppose firms use information efficiently at Stage II. Then, a firm profit at Stage I can be written as

$$\hat{\Pi}\left(\hat{\sigma}_e^2,\alpha^*,k_2^*\right)=1-\frac{\gamma}{\delta}\exp\left\{-\delta k_2^*+\frac{\delta^2}{2}\left[\frac{\theta}{(1-r)}\alpha^{*2}\hat{\sigma}_e^2+(1-\alpha^*)^2\sigma_u^2\right]\right\} \,.$$

At the symmetric outcome, the marginal value of information for an individual firm, after replacing k_2^*, is:

$$\frac{\partial\hat{\Pi}}{\partial\hat{\sigma}_e^2}=-\frac{\gamma}{2}\frac{\delta\theta}{(1-r)}\alpha^{*2}<0 \,.$$

Notice that when information is used efficiently, the firm's marginal value of information to its own information choice is constant, i.e. $\partial^2\hat{\Pi}/\partial\hat{\sigma}_e^2\partial\hat{\sigma}_e^2=0$.
The household's lifetime utility when information is used efficiently (after replacing k_2^* and α^*) is

$$\mathbb{U}=-\frac{\delta\theta}{2(1-r)}\alpha^*\sigma_e^2-\frac{1}{\delta} \,.$$

The marginal value of information for the social planner is given by

$$\frac{\partial\mathbb{U}}{\partial\sigma_e^2}=-\frac{\delta}{2}\frac{\theta}{(1-r)}\alpha^{*2}<0$$

Also, note that

$$\frac{\partial\mathbb{U}}{\partial\hat{\sigma}_e^2}=\frac{1}{(1-r)}\frac{\delta\theta^2\alpha^{*2}}{((1-r)\sigma_u^2+\theta\sigma_e^2)^2}>0$$

Using Lemma 4, if the marginal value of information to the social planner is smaller (greater) than the marginal value of information to the firm, then there is over-acquisition (under-acquisition) of information in equilibrium. Over-acquisition of information in equilibrium happens when $\partial\mathbb{U}/\partial\sigma^2$ greater than $\partial\hat{\Pi}/\partial\sigma^2$. It is easy to see that this is true so long as $\gamma>1$. If this condition does not hold, there is under-acquisition of information.

Proof of Proposition 14 Note that a firm's expected profits are decreasing in its own noise $\hat{\sigma}_e^2$,

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} < 0 \ .$$

Compute the derivative of the team profits with respect to the noise of the signal σ_e^2. In a symmetric outcome this derivative is the sum of two factors

$$\frac{d\hat{\Pi}^T}{d\sigma_e^2} = \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} \ . \tag{62}$$

The first factor corresponds to the derivative of a firm's expected profits evaluated at the symmetric outcome, i.e. $\hat{\sigma}_e^2 = \sigma_e^2$. The second factor corresponds to the externality.

Combine Proposition 17 with the expression from equation (62) so that in a symmetric equilibrium:

$$\begin{aligned} \frac{d\hat{\Pi}^T}{d\sigma_e^2} &= \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} + \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} = 0 \quad \forall \sigma_e^2 \\ &\Rightarrow \frac{\partial \hat{\Pi}}{\partial \sigma_e^2} = -\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2}|_{\hat{\sigma}_e^2 = \sigma_e^2} > 0 \ . \end{aligned}$$

Hence, there is a negative externality in any symmetric equilibrium.

Proof of Proposition 15 Plug α from equation (43) into the ex-ante expected profit (46). The team profits are given by

$$\Pi^T = e^{(\theta-1)(1-r)\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right)} K_1 = \frac{1}{\gamma\theta\,(1-r)} \ ,$$

which does not depend on σ^2, i.e.

$$\frac{d\hat{\Pi}^T}{d\sigma_\epsilon^2} = 0 \ .$$

Proof of Proposition 16 The proof is analogous to the proof for Proposition 17. Team profits are obtained after replacing (α^*, k_2^*) into $\hat{\Pi}\left(\hat{\sigma}_e^2, \alpha^*, k_2^*\right)$,

$$\Pi^T = 1 - \frac{\gamma}{\delta} \ ,$$

which requires $\delta > \gamma$ to guarantee positive profits. Hence the marginal value of information is:

$$\frac{d\Pi^T}{d\sigma_e^2} = 0 \ .$$

To prove that sign of the externality, we proceed as in the proof for Proposition 14.

Proof of Proposition 18 From equation (47):

$$\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} = \left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} \right)^2 \left\{ \hat{\Pi} \left(-\phi_1 \frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2} \right) + \frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} \left(-\frac{\phi_1^2}{2} + \phi_2 \right) \right\} .$$

Using Proposition 14, in any symmetric outcome

$$\frac{\partial \hat{\Pi}}{\partial \hat{\sigma}_e^2} = -\hat{\Pi} \left(-\frac{\phi_1^2}{2} + \phi_2 \right) \left(\frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} \right)^2 .$$

Hence

$$\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} = \left(\frac{\sigma_u^2}{\sigma_u^2 + \hat{\sigma}_e^2} \right)^2 \hat{\Pi} \left\{ \left(-\phi_1 \frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2} \right) - \left(-\frac{\phi_1^2}{2} + \phi_2 \right)^2 \left(\frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} \right)^2 \right\} ,$$

where

$$-\phi_1 \frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2} = \frac{(\theta - 1)^2 (1 - r)^3 \theta (\delta - 1) \sigma_u^2 \left(\sigma_e^2 + \sigma_u^2 \right)}{\left((1 - r) \sigma_u^2 + \sigma_e^2 \right)^3} > 0 .$$

Plug the previous expression into $d^2 \hat{\Pi} / d\sigma_e^2 d\hat{\sigma}_e^2$

$$\frac{\partial^2 \hat{\Pi}}{\partial \sigma_e^2 \partial \hat{\sigma}_e^2} = \frac{1}{4} \left(-\phi_1 \frac{d\phi_1}{d\sigma_e^2} + \frac{d\phi_2}{d\sigma_e^2} \right) \left\{ \frac{(1 - r) \left(4 (\delta - 1) - \delta^2 \theta \sigma_u^2 \right) \sigma_u^2 + \left(4 (\delta - 1) - (1 - r) \delta^2 \theta \sigma_u^2 \right) \sigma_e^2}{(\delta - 1) \left((1 - r) \sigma_u^2 + \sigma_e^2 \right)} \right\} .$$

Note that if $\sigma_u^2 < 4 (\delta - 1) / \delta^2 \theta$, then decisions are strategic complements, i.e. $\partial^2 \hat{\Pi} / \partial \sigma_e^2 \partial \hat{\sigma}_e^2 > 0$. On the other hand, if $4 (\delta - 1) / (1 - r) \delta^2 \theta < \sigma_u^2$ then decisions are strategic substitutes, i.e. $\partial^2 \hat{\Pi} / \partial \sigma_e^2 \partial \hat{\sigma}_e^2 > 0$. Finally, if $4 (\delta - 1) / \delta^2 \theta < \sigma_u^2 < 4 (\delta - 1) / (1 - r) \delta^2 \theta$, then the sign of $\partial^2 \hat{\Pi} / \partial \sigma_e^2 \partial \hat{\sigma}_e^2$ depends on the value of σ_e^2.