Learning in a large, non-linear model. Size matters.

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Preliminary version, June 2008

Abstract

The applied literature on adaptive learning has mostly focused on small, linear models, where the minimum state variable (MSV) solution of the rational expectations equilibrium is used as the agents' perceived law of motion (PLM), which enables to preserve consistency of beliefs, one key aspect of rational expectations. In non-linear models a closed-form MSV solution does not exist and if the model is medium or large-sized there is no univocal linear approximation that is eligible as a candidate PLM. Accordingly, the temporary equilibrium of the learning process no longer converges to the REE, but rather it tends to an asymptotic limit which depends on the specific form of the expectations equations. The objective of this paper is to assess whether in such a model economy the optimal monetary policy exhibits properties that are simular to those found in the literature for small, linear models (e.g. Orphanides and Williams 2007). Besides, given the potential role that central bank communication can play as a coordination device for expectations, it studies whether more transparent policies can be welfare-enhancing. The main results are the following: (1) contrary to previous findings, the monetary policymaker has no incentive to adopt more inflation-averse policies to keep expectations anchored to targets: too strong a reaction to price shocks increases both inflation and output volatility and tends to make the model unstable and non-learnable; (2) more transparent policies seem to be welfare-enhancing, mostly because they reduce the slope of the term structure and the variability of long-term interest rates. A higher degree of transparency calls for higher policy inertia and stronger inflation aversion, so partially recovering the findings by Orphanides and Williams.

KEYWORDS: Bounded rationality, generalised stochastic gradient learning, transparency.

JEL classification: E52,E31,D84.

1 Introduction

The vast literature that is now available on adaptive learning is overwhelmingly focused on small linear models. Issues like the stability of the equilibrium, the speed of convergence or the dynamics of the learning process are studied only in models whose dimension is no larger than a handful of equations. The implications for monetary policymaking as well are analysed in this very restricted setting, where the linearity assumption and the small number of state variables limit the extent of the possible model uses. The complications that arise when studying stochastic recursive algorithms in high-dimensional, non-linear systems are largely responsible for this lack of attention, but this is unfortunate, because several issues that are relevant only in the context of large-scale models are not paid due attention.

In most of the literature on adaptive learning, it is assumed that the perceived law of motion (PLM) coincides with the minimum state variable (MSV) solution of the corresponding rational expectations equilibrium (REE). This is a convenient simplification, avoiding the complexities of dealing with a potential vast multitude of alternative PLMs and allowing a straightforward analysis of the asymptotic properties of the learning algorithm. With non-linear models this is no longer possible, since a closed-form MSV solution does not exist; if moreover the model is medium or large-sized, a univocal linear approximation is unavailable either, given the large number of state variables that in principle could be included in the PLM. If agents act like econometricians, who look at the data to choose the correct specification of a regression equation, the larger the size of the model economy, the larger the set of options among which to select a PLM: in a self-referential model, the lack of a univocal PLM implies that the equilibrium may not be unique.

Additional problems arise if multi-step ahead expectations are needed, because the explanatory variables entering the PLM must be forecast themselves. Consequently, the difficulty of choosing a proper PLM is compounded by the need to select many of them. Last but not least, since the PLM is misspecified and underparameterised, it nests no longer the REE and the learning algorithm converges to a limit point which is indeterminate and depends on the specific form of the expectations equations. With an unknown limit point, the issue of convergence of the learning algorithm becomes blurred.

If one drops the simplifying assumptions of linearity and small size, a Pandora's box opens up, making most of the findings of the recent literature on adaptive learning of dubious generality. Not only analytic issues, but also policy prescriptions get intricate. For monetary policymaking, Orphanides and Williams (2007) have shown that when agents learn adaptively, the incentives and constraints facing monetary authorities change substantially: compared with the rational expectations case, imperfect knowledge[1] reduces the scope for stabilisation of the real economy, demands for more inflation-averse policies and increases the inertia in interest rate setting. The problems are compounded by unobserved structural changes (e.g. in natural rates), which call for quasi-difference rules in a quest for policy robustness. According to Orphanides and Williams, it is the non-linear nature of the learning process that dictates the policy response. Upon reestimation of the inflation forecasting model, a pos-

[1] Since imperfect knowledge is a precondition for bounded rationality and learning, that expression is used here and henceforth as a synonym of learning and as an antonym of rational expectations, following the usage in Orphanides and Williams (2002).

itive price shock passes through to the intercept of the forecasting equation, raising next period expected and hence actual inflation: unless the policy response is prompt and firm, the persistence and volatility of price changes rise and the monetary authority fails to keep a firm grip on the value of money. While introducing learning in an otherwise linear system changes the nature and the equilibrium properties of the model, it is not clear what happens when the model is already non-linear. Several questions arise and call for an answer: does the switch from full rationality to imperfect knowledge amplify the impact of learning or does it just add an additional source of inertia, whose effects on the economy are negligible? how does the speed of learning change because of the more complex structure of the economic environment? can the central bank enhance welfare by providing information to households and firms or should it exploit its private information to generate inflation surprises?

As suggested by the last question, a relevant policy issue is to assess the gains from transparency when private agents are modelled as adaptive learners. In the case of the standard New Keynesian model, Berardi and Duffy (2006) show that when the central bank operates under commitment, the gains from adopting transparent policies are unambiguously positive, while under discretion there are cases when opaqueness may ensure superior outcomes. Eusepi (2005) shows that a sufficient degree of transparency helps make the monetary policy rule robust to expectation errors. These findings are however of limited generality, since in both papers uncertainty is restricted to unknowingness of the inflation objective and of the functional form of the policy rule.[2] In a large-size model, where non-linearities abound, the flow of information from the monetary authority to the private sector is potentially much richer and the role of communication much larger: agents do not know which variables enter the policy rule; whether the central bank targets lagged, current or future variables; which is the degree of policy inertia. Since each of these aspects of the policy strategy affects the transitional dynamics and the steady state growth path of the system, the monetary authority can, to a large extent, decide on the amount of information to provide to the public so as to influence the equilibrium outcomes.

This paper is concerned mostly with two of the issues mentioned in the previous paragraphs. First, it tries to validate the findings of Orphanides and Williams about the influence of imperfect knowledge on the features of the monetary policy rule, using a model less simple than theirs; in this respect, it is vital to be able to identify the channels through which expectation errors affects the persistence and the volatility of output and inflation. Second, it tries to measure the benefits associated with more transparent policies.

This work is original in a few respects. First, it analyses learning in an economy where expectations have so pervasive a role that is unmatched in the literature. The very few papers studying bounded rationality in large non-linear models, introduce learning only in the exchange rate equation; in this study learning affects not only the value of the domestic currency, but also the term

[2] In Berardi and Duffy (2006), uncertainty about the monetary policy strategy means that agents do not know whether the lagged output gap enters the reaction function of the central bank (i.e. whether policy is conducted under discretion or under commitment).

structure of interest rates, the price setting behaviour of firms and monetary policy. Second, it studies the consequences on social welfare of varying degrees of monetary policy transparency. When the REE cannot be achieved, due to the complexity of the economic environment, the disclosure of information from the central bank affects the selection of market participants' PLMs and determines the restricted perceptions equilibrium to which the economy converges. The elaborated description of the channels of monetary policy transmission helps understanding whether the prevailing effect of central bank communication is to reduce the noise or on the contrary to provide distorted incentives to market participants, forcing them to give too much weight to central bank communication and too little to their own information.

One weakness of the paper is that it does not rely upon a microfounded model. The model exhibits behavioural equations that are consistent with maximising agents obeying budget constraints and possesses a steady-state growth path, but it resembles more a gigantic VAR system, estimated by informally imposing tight priors, than a microfounded model. This modelling choice however can be justified on several grounds: first, no microfounded model so large exists and those displaying a comparable size have loose ends; second, imperfect knowledge, heterogeneous expectations and (possibly) changing forecasting equations alter the cross-equations restrictions characterising the rational expectations equilibrium and relax the connections between model coefficients and deep parameters; third, the Orphanides-Williams paper, whose (monetary policy) findings are the benchmark of this work, does not use a microfounded model either, though the authors claim it does.

The paper is organised as follows. Section 2 provides a survey of the literature on adaptive learning, focusing on the monetary policy implications of imperfect knowledge. Section 3 outlines the model used in the simulations and introduces stochastic gradient learning.[3] Section 4 presents some evidence, obtained by means of simulation, on the distortions on monetary policymaking caused by assuming that agents have bounded rationality; the focus is on whether adaptive leaning induces a bias toward conservatism and on whether central bank communication is beneficial. Sensitivity analysis is conducted in order to assess to what extent the simulation evidence depends on the number of replications and on initial conditions. Section 5 concludes.

2 A survey of the literature

There are very few papers focusing on learning in large non-linear models and they deal mainly with the asymptotic convergence of the learning algorithm, disregarding all the monetary policy implications. Garratt and Hall (1997) use the LBS macromodel, adjusted to include adaptive learning schemes to form expectations on the exchange rate, to study whether the choice of the PLM affects the uniqueness and the stability of the equilibrium and whether the volatility of the transition path depends on how agents learn. The issue is to

[3] A more detailed description of the model is presented in appendix A

assess whether adaptive learning is E-stable even when the PLM is overparameterised.[4] According to the authors, E-stability is achieved when the parameters of the expectations rule cease changing. They stress that such an approach is to be preferred to those requiring the computation of the eigenvalues of the ODE associated with the recursive algorithm, being easier to implement and providing more reliable results. Garratt and Hall find that the choice of the PLM modifies the volatility and the speed of convergence, but they obtain less clear-cut evidence on strong E-stability: while the end-point of output seems to be the same regardless of the specific form of the PLM, inflation is clearly affected by it. The authors also seek to establish a range of values for the hyperparameters[5] of the learning rule and conclude that the dynamics and end-value responses of output and inflation are sensitive to the values of the hyperparameters, but much less to variation in the expectations rules examined. The paper is interesting and innovative, but presents a few serious shortcomings: first, the empirical analysis is based on simulations on very short time horizons (less than 10 years); second, the learning process relies upon hyperparameters that are calibrated rather than estimated; third, only exchange rate expectations play a role; finally, policy issues are entirely neglected.

Beeby, Hall and Henry (2001) go one step further and propose three methods to select a "sensible" PLM when an obvious choice is not available. The first option requires to shock each variable of the model and to pick out those ones whose impact on the exchange rate is larger; the second method prescribes to compute the rolling correlation (on a 4-quarter window) between the exchange rate and each potential regressor and to rank the correlations according to their standard deviations, choosing the series with the less volatile correlations; the last procedure is based on identifying the principal components from all the series in the model and on selecting the variables that most closely move with them. The paper investigates also whether allowing for boundedly rational agents makes the properties of a model different from those obtained assuming rational expectations. Concerning the first issue, Beeby, Hall and Henry find that, regardless of the method used, learning algorithms are quite effective in extracting information from any series, so that the exact form of the rule is unimportant. However, while a wide range of adaptive rules give almost identical answers, they all differ substantially from the RE solution, suggesting that even small deviations from the benchmark of full information and full rationality may have a strong impact on model properties. Like Garratt and Hall's paper, the study by Beeby *et al.* uses the Kalman filter to update the parameters of the expectations rule, with hyperparameters exogenously fixed, and completely disregards the policy implications of departing from the rational expectations framework; moreover, adaptive learning is employed only for exchange rate expectations, which tends to downplay the self-referential nature of the learning

[4] A learning process that converges to the REE even when the PLM is overparameterised is said to be strongly E-stable.

[5] For updating the coefficients of the expectations rule, Garratt and Hall use the Kalman filter. The hyperparameters are the diagonal elements of the covariance matrices of the measurement and transition equations.

equilibrium.

The impact on policymaking of assuming boudedly rational agents are the subject of the paper by Orphanides and Williams (2007). The authors examine the performance and robustness of alternative monetary policy rules in an estimated macroeconomic model where private agents and the central bank possess imperfect knowledge about the true structure of the economy. Orphanides and Williams find that policies that appear to be optimal under perfect knowledge can perform very poorly when knowledge is incomplete, partly as a result of the persistent policy errors due to misperceptions of the natural rates and partly as a result of the learning process that agents use to form expectations. Efficient policies that take account of private learning and of unobservability of natural rates have two features: first, they call for more aggressive responses to inflation; second, they exhibit a high degree of inertia in the setting of the monetary policy interest rate. Indeed, difference rules (i.e. rules having on the right-hand-side the lagged interest rate with a coefficient equal to 1), which circumvent the need to rely on uncertain estimates of the natural rates, appear to be robust to potential misspecifications of private sector learning and to the magnitude of variation in natural rates.

The value of communication in monetary policy under imperfect knowledge is studied in several papers, among others Berardi and Duffy (2006) and Eusepi and Preston (2007). Berardi and Duffy link transparency of monetary policy to the specification of the forecast rule adopted by the private sector, unlike the traditional view that equates transparency with more or better information. They adopt the standard cashless, three-equation, New Keynesian model and find that while under commitment central bank communication is unequivocally welfare-enhancing, under discretion the value of transparency relative to opaqueness is ambiguous and depends on target values. Eusepi and Preston find that in a dynamic stochastic general equilibrium model with imperfect knowledge, under no communication the policy rule fails to stabilise macroeconomic dynamics, promoting expectations driven fluctuations. However, by announcing the details of the policy process, stability is restored: communication permits households and firms to construct more accurate forecasts of future macroeconomic conditions. Moreover, Eusepi and Preston find that if the central bank only announces the desired inflation target, economies with persistent shocks will frequently be prone to self-fulfilling expectations.

3 The model

The model used in the paper is a reduced-scale version (a so-called maquette) of the Bank of Italy Quarterly Model.[6] The sample that has been used to estimate the model covers a 30-year horizon, starting in the early 1970s and ending in

[6] A detailed description of the theoretical underpinnings of the Bank of Italy Quarterly Model is in Terlizzese (1994) and Busetti *et al.* (2005). A brief summary of the main features of the model is reported in Appendix A. The technique that has been followed to build the maquette is described in Appendix B.

the late 1990s, before Italy joined the European Monetary Union.

The behavioural equations are consistent with maximising agents, but the model is not *strictu sensu* microfounded, since it does not contain all the cross equations restrictions that hold when agents are fully rational and its structural equations are not tied down exclusively by taste and technology parameters.[7] Like the model in Orphanides and Williams (2007), its main merits are to fit the sample data reasonably well and to provide a quantitative feel for the cost of employing policies designed under perfect knowledge and full rationality, when instead agents are actually learning. The Lucas critique in principle applies and the model would seem inadequate to measure the welfare implications of competing policy rules. In fact the empirical relevance of the Lucas critique is close to nihil[8] and even its theoretical importance has been questioned.[9]

The model has some 90 endogenous 70 exogenous variables; like other old-fashioned models, it describes the behaviour of optimising agents without incorporating all the cross-equation restrictions which are typical of DSGE models. It is Keynesian in the short-run, with the level of economic activity primarily determined by the behaviour of aggregate demand, and neo-classical in the long-run. The dynamics of the model stems from a number of sources, namely (i) the stickiness of prices and wages, (ii) the putty-clay nature of capital and (iii) expectation errors. Beliefs enter the model in several ways: ex-ante real interest rates affect the demand for both consumption and capital goods; next-period expected inflation drives the pricing strategies of firms for the current period and indirectly influences wage claims; beliefs about future price developments affect the policy interest rates[10] and the term structure, which is modelled according

[7] Incidentally, one could convincingly object that a model that assumes imperfect knowledge in expectations formation should **not** feature structural equations that are consistent with full rationality.

[8] Most - if not all - of the papers that have used the concept of superexogeneity to examine the Lucas critique empirically, have found no supporting evidence. See as an example Ericsson and Irons (1995). Estrella and Fuhrer (1997) have shown that backward-looking, rather than forward-looking behaviour, seems to be a better approximation of reality in macroeconomic models: indeed, when there is a change in the monetary policy regime, some forward-looking models turn out to be less stable than their better-fitting backward-looking counterparts, which is inconsistent with the Lucas critique. A more sympathetic attitude is shown in Lindé (2001), who tries to explain the empirical failure of the Lucas critique claiming that superexogeneity tests based on conditional models have low power, because they do not condition on all the shocks that hit the economy and affect parameter stability.

[9] Sims has downgraded the Lucas critique to the rank of a cautionary footnote, on the grounds that true regime changes are rare events: private individuals have seen a sufficient range of policy actions and are able to attach probabilities to these actions recurring. See Sims (1982) and Hoover (1988). Sargent (1999) has shown that within a self-confirming equilibrium, the relevance of some aspects of the Lucas critique vanishes and procedures that violate it - as for instance when agents use forecasting functions like those in rational expectations models, but with coefficients that adapt to fit recent data - ironically may yield better outcomes than those that respect it.

[10] The short-term (policy) interest rate depends on the current unemployment gap and on next period inflation, the latter variable expressed in terms of deviations from target inflation. Some inertia in the policy instruments is allowed by including the lagged interest rate among the arguments of the policy rule.

to the expectations hypothesis;[11] anticipated changes in the nominal exchange rate bear upon competitiveness and the terms of trade. Moreover, beliefs play a direct role in shaping policy decisions, since natural rates are unobservable and the central bank has to estimate them, before deciding on the proper monetary stance.

3.1 The learning mechanism

When agents are assumed to be boundedly rational, expectations are commonly modelled by using recursive least squares (RLS) learning. A convenient alternative to RLS is the stochastic gradient (SG) algorithm, whose main advantage is that it does not rely on information on the second moments of the variables entering the forecasting equation. SG learning, which under standard conditions is consistent but not efficient, has been found to work well in complex environments, suggesting that it has robustness properties that are absent in RLS. The main drawbacks of the method are that (i) it is not invariant with respect to changes in the units of measurement of the variables entering the PLM and that (ii) E-stability does not always imply convergence of SG learning. Recently, Evans *et al.* (2006), have proposed a generalisation of the SG algorithm, called Generalised Stochastic Gradient (GSG) learning, that solve the invariance problem. Moreover, they show that the GSG algorithm has other important justifications: first, it approximates a Bayesian estimator in models where parameters drift; second, it is a maximally robust optimal prediction rule when there is parameter uncertainty; third, though in general conditions for stability of generalised stochastic gradient learning differ from those governing stability under least squares learning, E-stability in most cases remains a necessary condition for asymptotic convergence of GSG learning.

In all the experiments described in this paper, expectations have been modelled by means of SGS algorithms. This means that if the PLM of the variable y_t is $\varphi'_{t-1}x_{t-1}$, then the recursion for φ_t is:

$$\varphi_t = \varphi_{t-1} + \gamma_t \Gamma x_{t-1} \left(y_t - \varphi'_{t-1} x_{t-1} \right) \tag{1}$$

where Γ has been set equal to the inverse of cross-product matrix of the regressors, namely $\Gamma \equiv \left(\frac{1}{T} \sum_{t=1}^{T} x_t x'_t \right)$, with T being the sample size.

3.2 The role of expectations in the model

Expectations play a pervasive role in the model. Besides price and wage setting equations, they affect monetary policy decisions and ensure that the no-arbitrage condition is fulfilled in asset markets.

[11] Long-term interest rates are assumed to be a weighted average of (current and future) short-term ones, with the term spread being constant.

The central bank is assumed to set the policy instrument[12] i_t according to the following reaction function:

$$i_t = \rho i_{t-1} + (1-\rho)\left[r^* + \overline{\pi} + \alpha_\pi\left(\widehat{E}^{CB}_{t-1}\pi_{t+1} - \overline{\pi}\right) - \alpha_u\left(u_t - u^*\right)\right] \qquad (2)$$

where $\widehat{E}^{CB}$ are central bank expectations, r^* and u^* are, respectively, the natural (real) rate of interest and that of unemployment, which are constant in equilibrium. Since both variables are unobservables, the policymaker estimates them by computing the sample average of the corresponding observables. The policymaker's PLMs for π, r^* and u^* are:

$$\begin{cases} \widehat{E}^{CB}_{t-1}\pi_t &=& \pi_{t-1} + \alpha_{1,t-1}\Delta i_{t-1} + \alpha_{2,t-1}\Delta\pi_{t-1} \\ \widehat{E}^{CB}_{t-1}r^*_t &=& r^*_{t-1} + \gamma_t\left(i_{t-1} - \pi_{t-1} - r^*_{t-1}\right) \\ \widehat{E}^{CB}_{t-1}u^*_t &=& u^*_{t-1} + \gamma_t\left(u_{t-1} - u^*_{t-1}\right) \end{cases} \qquad (3)$$

where γ_t is the gain sequence.[13] The PLM for inflation is admittedly simple, but it captures the idea that inflation is sticky and responds to changes in the monetary policy stance. The explanatory variables have been chosen on the grounds that they minimise the standard error of the regression in a two-variable equation; moreover the policy interest rate exhibits the highest and most stable correlation with the inflation rate, which is one of the criteria suggested by Beeby *et al.* to select the regressors to include in the PLM.[14] The specification is in first differences, so that it is consistent with time-varying inflation objectives, reflecting the historical experience of monetary policymaking in Italy in the three decades preceding the introduction of the monetary union.

[12]To keep the structure of the model simple, the policy instrument is assumed to be the short-term interest rate, defined as the weighted average of the yields of 3, 6, and 12-month Treasury bills.

[13]In the decreasing gain case, $\gamma_t = \frac{1}{t}$, while for the perpetual learning case, $\gamma_t = \overline{\gamma}$.

[14]For the equation for expected inflation, in addition to lagged actual inflation, the following variables have been considered as eligible regressors: (1) the output gap; (2) the growth rate of GDP; (3) the unemployment rate; (4) the oil price; (5) the nominal effective exchange rate. Absent a univocal procedure for selecting the regressors, the horse race was based on 4 criteria: (1) the standard error of the regression (i.e. $\sigma = \sqrt{\frac{1}{T-k}\sum\left(\pi_t - \widehat{E}^{CB}_{t-1}\pi_t\right)^2}$, where k is the number of regressors in the PLM); (2) the correlation of $\widehat{E}^{CB}_{t-1}\pi_t$ with survey measures of inflation expectations; (3) the rolling correlation (with a 4-year window) with actual inflation; (4) the co-movement with the 1^{st} and 2^{nd} principal components. The last two criteria are suggested by Beeby *et al.*: (3) helps picking out the variables whose correlation with inflation is high and stable through time; (4) is aimed at selecting regressors that do not overlap in the amount of predictive information. Both measures however have been found by Beeby and his coauthors to be unable to select the best predictors. In principle, the maximisation of the correlation between $\widehat{E}^{CB}_{t-1}\pi_t$ and survey-based inflation expectations is what one should be concerned with when choosing the specification of the PLM; in practice, survey data are not a fully-satisfactory proxy of households' and firms ' anticipations of future price dynamics. Accordingly, both the first and the second criteron have been given priority over the other two. Among the specification featuring only two regressors, that with lagged inflation and the policy interest rate minimises the standard error of the regression; exhibits the second highest correlation with (survey-based) expected inflation; has the highest and most stable correlation with actual inflation; presents regressors moving closely with the first principal component.

According to the chosen PLM, next-period expected inflation is equal to:

$$\widehat{E}^{CB}_{t-1}\pi_{t+1}{=}A_{1,t-1}\pi_{t-1} + A_{2,t-1}\pi_{t-2} + A_{3,t-1}\Delta i_t + A_{4,t-1}\Delta i_{t-1} \quad (4)$$

where $A_{1,t-1} = \frac{1-\alpha^3_{2,t-1}}{1-\alpha_{2,t-1}}$, $A_{2,t-1} = 1{-}A_{1,t-1}$, $A_{3,t-1} = \alpha_{1,t-1}$ and $A_{4,t-1} = (1+\alpha_{2,t-1})\,\alpha_{1,t-1}$. Equations (2) and (4) jointly provide a solution for $\widehat{E}^{CB}_{t-1}\pi_{t+1}$ and i_t.

Private sector expectations affect the value of the domestic currency and bond yields. The PLM for the exchange rate includes, as explanatory variables, the ratio of the net foreign asset position over nominal GDP and the lagged exchange rate, i.e.

$$\widehat{E}^{P}_{t-1}\Delta e_t \quad = \quad \beta_{1,t-1}\Delta\tfrac{FA_{t-1}}{Y_{t-1}} + \beta_{2,t-1}\Delta e_{t-1} \quad (5)$$

Among the set of two-regressor specifications, the selected one minimises the standard error of the regression, exhibits the second higest correlation with survey measures of exchange rate changes, have explanatory variables that move closely with the first two principal components and presents the second largest and most stable correlation with the (change in the) exchange rate.

Along with the UIP, equation (5) determines e_t as a function of its own lags, the interest rate differential and foreign indebtedness.

According to the expectations hypothesis, k-year bond yields are equal to the k-year moving average of the current and the future short-term interest rates plus a constant term premium. Future rates are modelled assuming that agents know the variables entering the central bank policy rule, though they are uncertain about the precise form of the reaction function:[15] the more transparent monetary policy, the more alike the perceived and actual policy rules. In the baseline scenario, it is assumed that the private sector has the following PLM for i_t:

$$\widehat{E}^{P}_{t-1}i_t \quad = \quad \delta_{0,t-1} + \delta_{1,t-1}\pi_{t-1} + \delta_{2,t-1}u_{t-1} + \delta_{3,t-1}i_{t-1} \quad (6)$$

To obtain $\widehat{E}^{P}_{t-1}i_{t+j}$, for $1 \leqslant j \leqslant 5$, j-step ahead projections of the right-hand-side variables are needed: absent the MSV solution or a univocal approximation to be used as a PLM, it is assumed that agents fit simple $AR(1)$ processes, namely $\widehat{E}^{P}_{t-1}x_t \;=\; \psi^x_{0,t-1} + \psi^x_{1,t-1}x_{t-1}$, where x_t is either the inflation rate or the unemployment rate. It follow that

$$\widehat{E}^{P}_{t-1}i_{t+j} \quad = \quad \Delta_{0,t-1} + \Delta_{1,t-1}u_{t-1} + \Delta_{2,t-1}\pi_{t-1} + \Delta_{3,t-1}i_{t-1} \quad (7)$$

where $\Delta_{0,t-1} = \delta_{0,t-1}\frac{1-\delta^{j+1}_{3,t-1}}{1-\delta_{3,t-1}}$, $\Delta_{1,t-1} = \delta_{1,t-1}\left(\sum\limits_{h=0}^{j}\left(\psi^{\pi}_{1,t-1}\right)^h(\delta_{3,t-1})^{j-h}\right)$, $\Delta_{2,t-1} = \delta_{2,t-1}\left(\sum\limits_{h=0}^{j}\left(\psi^{u}_{1,t-1}\right)^h(\delta_{3,t-1})^{j-h}\right)$ and $\Delta_{3,t-1} = \delta^{j+1}_{3,t-1}$. It is further

[15] For instance, they do not know whether the central bank targets current of next period inflation, whether the lagged interest rate enters the policy rule or which is the policymaker's estimate of the natural rates.

assumed that agents do not know the constant term premium and use historical averages as an estimate. To avoid that iterating on expectations induces a systematic bias, the spread between long and short-term interest rates is corrected for the mean difference between expected and actual past policy rates:

$$\widehat{E}^{P}_{t-1} term_t \quad = \quad term_{t-1} + \gamma_t \left[\left(i^L_{t-1} - i_{t-1} \right) + \tfrac{1}{6} \sum_{j=1}^{6} \xi_{t-j} - term_{t-1} \right] \qquad (8)$$

where $\xi_{t-j} \equiv i_{t-j} - \widehat{E}^{P}_{t-1-j} i_{t-j}$ measures the surprise on the policy interest rate.

4 Simulation results

The monetary policy rules are ranked on the basis of their impact on social welfare. Society dislikes both price and output variability, defined as the unconditional variances of inflation and of the growth rate of GDP. The target value of both variables is the steady-state equilibrium value and the two objectives have the same weight in the welfare function, which is equal to:

$$W = - \left[E \left(\pi_t - \overline{\pi} \right)^2 + E \left(\Delta y_t - \overline{\Delta y} \right)^2 \right]$$

Using unconditional variances rather than discounted future losses implicitly favours policies that minimise the overall impact of shocks, penalising those that trade off smaller fluctuations today for larger ones tomorrow. Unlike Orphanides and Williams (2007), output, rather than unemployment, enters the welfare function, but the change is inconsequential, since in all the experiments the ranking of the policy rules is the same regardless of which variable is used as the argument. Interest rate volatility is not included, but it indirectly affects social welfare, since the term structure exerts a powerful influence on GDP.

Model simulations are used to illustrate how the interaction between the expectations formation mechanism and the monetary policy rule affects the equilibrium outcomes. The selection of the optimal policy is implemented through a grid search on the parameters $\{\rho, \alpha_\pi, \alpha_u\}$ of the Taylor-type reaction function: in order to save on computation time, the step-length of the grid search is initially quite large (.1 for ρ; .5 for α_π and α_u), but gradually diminishes, as soon as the region containing the welfare-maximising triplet is located. Each experiment is based on 500 replications and all simulations cover an interval of 490 years (from year 2011 to year 2500). In the first 90 periods, the the main stochastic equations[16] are shocked to test the effectiveness of the monetary policy rule;[17] in the subsequent 400 years, the model is not subjected to any shock

[16] The equations are those modelling (i) household consumption, (ii) exports, (iii) the private sector value added deflator and (iv) the consumption deflator. The shocks accordingly may be interpretated as referring to domestic and foreign household preferences and domestic and foreign mark-ups.

[17] To ensure a fair comparison across policy rules, the same sequences of random draws are used for each triplet $\{\rho, \alpha_\pi, \alpha_u\}$.

and it settles down on the steady-state equilibrium growth path, which allows to assess the convergence properties of the learning algorithm. The GSG algorithm is initialised using OLS estimates on historical data.

4.1 Optimal monetary policy under rational expectations

Under rational expectations the optimal monetary policy exhibits a small degree of interest rate smoothing, a high response to inflationary pressures and a non-negligible concern for changes in the unemployment rate (see Fig.1): the welfare-maximising coefficients are $\rho = 0.4$, $\alpha_\pi = 2$ and $\alpha_u = 1.5$.

The evidence on the performance of alternative rules shows a few noteworthy results. First, the degree of inertia does not seem to matter much: the welfare function is quite flat for values of ρ that are positive and smaller than 0.7. For larger values, both output and inflation variability increase, suggesting that too smooth a path of the policy interest rate fails to stabilise the economy, inducing both variables to cycle around the steady-state growth path with wide and long-lasting fluctuations. For values of ρ equal or greater than 0.9, the system no longer converges, showing that difference rules are not a viable alternative. Second, the equilibrium outcomes are not overly sensitive to the value of α_u, possibly because part of the burden to stabilise the economy is borne by fiscal policy. Close to the local optimum, the welfare function exhibit a hump-shaped response to α_u; away from it, no well-defined relationship is apparent. Third, the policymaker's response to deviations of inflation from target ought to be quite strong: the optimum is achieved when $\alpha_\pi = 2$, while for $\alpha_\pi \leq 1$ the model is not stable, suggesting that the Taylor principle holds. This finding is not trivial, since unlike what happens in small closed-economy models with no government, there exist channels other than monetary policy that help taming inflationary pressures.[18] Social welfare turns out to be very sensitive to changes in α_π, contrary to what happens with α_u or ρ: other things equal, it falls by nearly one sixth when $\alpha_\pi = 3$ and by one third when $\alpha_\pi = 4$. Fourth, mild changes in the relative weights of the objectives of the loss function are inconsequential: the optimal policy stays the same when the weight of output stabilisation is halved and it remains close to optimal when it is doubled.

4.2 Optimal monetary policy under learning

The results of the previous section are based on three partly interrelated hypotheses: (i) the economic environment is *stationary*, since equations do not change over time; (ii) expectations are *rational* and (ii) the central bank is *credibly committed* to an unchanging policy rule. Each assumption has a strong

[18] An increase in inflation deteriorates price competitiveness and reduces the real value of non-indexed financial wealth; the ensuing decline in exports and private-sector spending translates into less employment and more subdued cost developments. Besides, since tax brackets are usually not fully indexed, fiscal revenues rise more than proportionately to taxable income, forcing a weakening of aggregate demand. Since these channels are at work, it may not be necessary that real interest rates have to rise in the face of inflationary shocks to ensure that price stability is preserved.

impact on the properties of the system and on the incentives and constraints facing the monetary authorities. Uncertainty about the structure of the economy is problematic for a policymaker following an interest rate rule, because his/her policy decisions must rely upon either some estimate of the unobserved natural rates (of interest, output and unemployment) or alternatively must eschew natural rates altogether and follow a difference rule. Lack of knowledge on the expectations formation mechanism and imperfect credibility of the central bank affect the ability of the policymaker to steer market expectations and reduce the effectiveness of monetary impulses, drawbacks that can in principle be mitigated by a cleaver use of central bank communication.

In order to assess the impact on the central bank's strategy of the last two of the three above-mentioned assumptions, I consider three sets of simulations. In the benchmark case, labelled "no transparency", I assume that agents learn adaptively, natural rates are unknown and monetary policy is opaque. In the second experiment, I assume that the central bank discloses to the general public next-period value of the policy interest rate, so that $\widehat{E}^{P}_{t-1} i_t = i_t$. This case is dubbed "partial transparency", because the monetary policymaker communicates neither his/her own estimates of the natural rates ($\widehat{E}^{CB}_{t-1} r^*_t$ and $\widehat{E}^{CB}_{t-1} u^*_t$) nor the coefficients of the reaction function (ρ, α_π and α_u). In the final set of simulations, I consider a fully transparent central bank, that provides to private agents all the information it processes in making policy decision.[19] By comparing the first experiment to the rational expectations equilibrium, it is possible to measure the welfare losses due to imperfect knowledge and to see how the optimal monetary policy rule changes to adjust to the new informational setup. By comparing the other two experiments to the benchmark learning case, it is possible to gauge the gains the policymaker can achieve from being transparent.

4.2.1 The benchmark case

Table 1 presents a few summary statistics describing how alternative policy rules work when agents learn adaptively and the central bank does not disclose its strategy. Monetary strategies are appraised according to two indices: the level of welfare and the rejection rate (i.e. the share of non-converging replications). For each combination of parameters of the central bank's reaction function, the table shows the response of (i) output, (ii) inflation, (iii) the exchange rate, (iv) the direct tax rate, (v) the coefficients of the PLM of the policy rate,[20] (vi) interest rates; for each variable, it shows the first and second moment from the steady-state value. Results about the optimal rule are presented in the first row; the other policies are arranged so that only one parameter at a time changes, making easier to see how sensitive the performance of the rule is to changes in the coefficients.

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[19] The expression "full transparency" is somewhat abused here: in fact, the central bank does not communicate everything to the public, since the inflation PLM is not disclosed.

[20] The 7^{th} and 8^{th} columns of Tables 1 to 3 show the first and second moments of $\delta_{0,t-1} - (1-\rho)\left(\widehat{E}^{CB}_{t-1} r^*_t + \overline{\pi} - \alpha_\pi \overline{\pi} + \alpha_u \widehat{E}^{CB}_{t-1} u^*_t\right)$ and, respectively, $\delta_{1,t-1} - (1-\rho)\,\alpha_\pi$.

Comparing the equilibrium outcomes under rational expectations and learning, it seems that neither the uncertainty about the natural rate nor the expectations formation mechanism entail substantial welfare losses: the optimal policy under learning achieves nearly the same welfare level as the optimal policy under rational expectations and even suboptimal ones in most cases perform quite well. What changes considerably instead is the shape of the policy rule: under adaptive learning, the optimal policy requires a weaker response to deviations of inflation from target ($\alpha_\pi = 1.1$ rather than $\alpha_\pi = 2.0$) and a stronger concern for output stabilisation ($\alpha_u = 2.0$ rather than $\alpha_u = 1.5$). The degree of inertia increases ($\rho = 0.7$ rather than $\rho = 0.4$), but it does not seem to play a substantial role: social welfare is to a large extent unaffected by the value of the coefficient of the lagged interest rates and does not change sizably for values of ρ in the range $[0.4, 0.8]$. These results are quite at odds with those shown in Orphanides and Williams (2007): they find that when private agents have imperfect knowledge, the central bank benefits from adopting more inflation averse policies, that are more effective at preventing price expectations from decoupling from target inflation and at stabilising both inflation and output. What justifies this contrasting evidence is the difference in the monetary policy transmission mechanism. The model used in Orphanides and Williams is a plain-vanilla three-equation New-Keynesian model: the policy instrument affects directly aggregate demand and indirectly (through the output gap) inflation; as long as interest rate changes offset inflationary pressures, they stabilise the economy and have negligible spillovers on social welfare.[21] The model used in this paper contains a much richer transmission mechanism, where a prominent role is played by the yield on long-term securities: a strong response to price shocks makes actual short-term interest rates highly erratic and poorly predictable and is responsible for a more volatile and more steeply sloped term structure.[22] Not only the volatility, but also the mean value of the long-term yield is affected by the frequency and size of changes in the central bank's instrument, which creates a trade-off between the benefits of keeping inflation anchored to the target and the costs, in terms of household and firm spending, of increasing the slope of the term structure. Table 1 shows that not only too low (as in Orphanides and Williams), but also too high values of α_π are conducive to higher price volatility: for $\alpha_\pi = 2.5$ the standard deviation of inflation is nearly twice as large as that achieved when $\alpha_\pi = 1.0$. What happens is that by responding too strongly to inflationary pressures, the policymaker induces larger fluctuations in consumption and investment, which in turn cause addi-

[21] Provided of course that interest rate volatility does not have a large weight in the loss function.

[22] The link between learning and long-term interest rates is not a novel feature of this paper. Dewachter and Lyrio (2006) present a macroeconomic model, in which agents learn about the central bank's inflation target and the real interest rate, to explain the joint dynamics of . output, inflation and the term structure of interest rates. Learning generates endogenous stochastic endpoints which act as level factors for the yield curve. Dewachter and Lyrio find that their model improves in terms of fitting upon those based on rational expectations and generates sufficiently volatile endpoints to match the variation in long-maturity yields and in surveys of inflation expectations.

tional pressures on prices: the net effect may well be an amplification of the initial shock, rather than an attenuation.

An additional channel that affects the transmission of monetary impulses is that based on the exchange rate: a tightening of the policy stance determines an appreciation of the value of the domestic currency, that keeps in check price dynamics both directly (through a lower import deflator) and indirectly (through the impact of a deterioration in price competitiveness on economic activity). Simulation results however suggest that this channel play a minor role in shaping the response of the economy to monetary impulses, at least compared with the impact of learning on the term structure.

A few features of the optimal policy rule are worth mentioning. First, compared with the rational expectations optimum, both output and inflation volatility are higher - the latter more than the former - but the reduction in welfare is modest and does not reach 10 percentage points. Second, the Taylor principle hardly matters: the welfare-maximising value of α_π is 1.1 and even for $\alpha_\pi < 1$, the model remains stable and learnable. Third, even if the coefficients of the PLM for i_t are biased - and in the case of the intercept largely biased - i_t^e is an unbiased estimator of i_t, suggesting that errors in forecasting the value of $(1-\rho)\left(\widehat{E}_{t-1}^{CB} r_t^* + \overline{\pi} - \alpha_\pi \overline{\pi} + \alpha_u \widehat{E}_{t-1}^{CB} u_t^*\right)$, $(1-\rho)\,\alpha_\pi$, $(1-\rho)\,\alpha_u$ and ρ tend to cancel one another; moreover, as the low volatility of $i_t^e - i_t$ clearly shows, i_t^e is not only an unbiased, but also quite an efficient estimator. Fourth, even when agents are not fully rational, monetary policy manages aggregate demand mostly through expectations, as witnessed by the response of the long-term interest rate: on average the policy instrument remains below the state-state level, while the long-term rate stays above it and does most of the job of curbing inflationary pressures.

4.2.2 The case of partial transparency

Transparency of monetary policy refers to the absence of information asymmetries between policymakers and the private sector. Perfect transparency in the setup of this paper implies that the central bank discloses to the general public both its estimates of the natural rates and the precise form of the policy rule. Incomplete transparency is instead assumed to represent a situation where the policymaker communicates in advance only the monetary stance (i.e. the value of i_t); accordingly, expectations about future policy rates, which are needed to price long-term securities, are formed with a PLM that differs from the true interest-rate rule, namely:

$$\widehat{E}_{t-1}^{P} i_{t+1+j} \quad = \quad \delta_{0,t-1} + \delta_{1,t-1} \widehat{E}_{t-1}^{P} \pi_{t+j} + \delta_{2,t-1} \widehat{E}_{t-1}^{P} u_{t+j} + \delta_{3,t-1} \widehat{E}_{t-1}^{P} i_{t+j}$$

where $\widehat{E}_{t-1}^{P} i_{t+j} = i_{t+j}$ for $j = 0$.

Table 2 shows the results of the model simulations under partial transparency. There seems to be indeed gains to be reaped by being transparent: the optimal policy achieves a level of welfare that is 10 percentage points higher than the best outcome under opaqueness. When agents know in advance what the central bank is going to do, they behave in a way that is consistent with the monetary stance, fostering the achievement of the policy objectives with milder changes in interest rates: since the central bank has no information advantage, it can reap no benefit from surprising market participants. With transparency, the short-term interest rate is less volatile and the term structure flatter, which allows to pursue price stability with less output volatility. A more stable economic environment is also conducive to more precise appraisals of the unobserved natural rates: Fig.3a shows that the bias in the estimates of the natural unemployment and (real short-term) interest rates (the former overestimated and the latter underestimated) is smaller in the case of partial transparency and so is the standard deviation of $\widehat{E}^{CB}_{t-1}u^*_t$ and $\widehat{E}^{CB}_{t-1}r^*_t$.

Figg.2a and 2b confirm the evidence reported in Table 2. They show the period-by-period average response and standard deviation (across replications) of the monetary policy instrument, the bond yield and the exchange rate. Both pictures clearly show that there is just a negligible difference in the behaviour of the three variables compared with the no-transparency case, though the policy rate is a bit less volatile and the exchange rate a bit more erratic. More dissimilar are instead the properties of the coefficients of the PLM of the policy interest rates: though higher transparency seems to promote a more efficient estimate of the inflation coefficient and of the degree of policy inertia, it does not help in gauging the size of the other parameters (see Fig.4).

The most clear-cut difference with respect to the benchmark case is in the sensitivity of the welfare function to changes in the policy response to output and inflation gaps. High values of α_π are still suboptimal and quite ineffective, but they are no longer a source of instability, as witnessed not only by the level of welfare, but also by the rejection rate, which is for all the combinations of parameters reported in the table equal to zero. When $\alpha_\pi \in [2.0, 2.5]$ the bond yield continues to fluctuate widely and the term structure to be steep, but much less than they do in the no-transparency case. The same finding applies to the case $\alpha_u \in [2.0, 2.5]$.

4.2.3 The case of full transparency

The equilibrium outcomes change substantially when the central bank is fully transparent and discloses all the information it uses to choose the monetary stance. Full trasparency holds when no information asymmetry between the central bank and the general public exists. Since the central bank provides to market participants its own estimates of the natural rates, the coefficients of the policy rule and its inflation objective, expectations about future policy rates are set according to the following equation:

$$\begin{cases} \widehat{E}^P_{t-1} i_{t+j} & = & \rho \widehat{E}^P_{t-1} i_{t-1+j} + (1-\rho)\, i^*_{t+j} \\ i^*_{t+j} & = & \widehat{E}^{CB}_{t-1} r^*_t + \overline{\pi} + \alpha_\pi \left(\widehat{E}^P_{t-1} \pi_{t+1+j} - \overline{\pi} \right) - \alpha_u \left(\widehat{E}^P_{t-1} u_{t+j} - \widehat{E}^{CB}_{t-1} u^*_t \right) \end{cases}$$

for $j > 0$. The only remaining information asymmetry is the one about the PLMs for inflation and the unemployment rate, which differ between the central bank and the private sector. As shown in Table 3, the best performing rule features a much higher degree of inertia, a slightly smaller inflation aversion and a somewhat higher concern for output fluctuations. Moreover, the sensitivity to changes in the value of ρ is extreme: for $\rho = .6$, welfare is nearly 50 p.p. lower than at the optimum; for $\rho \leq .5$ the learning process does not converge and the rejection rate tends to 100%. A low degree of inertia tends to destabilise the exchange rate and raises substantially the cost of financing. This is in stark contrast with the flatness of the welfare function for values of ρ higher than .85 (the optimal value) but lower than 1. What explains so sharp a deterioration in the monetary policy performance? A comparison with the previous two cases provides the answer. Private sector predictions of the short-term interest rate tend to be quite precise, even though the estimates of the coefficients of the PLM are highly biased; this happens because errors, including those regarding anticipations of future inflation and unemployment, tend to offset each other. When the natural rates and the policy parameters are not estimated, being provided by the central bank, there is no automatic error-correction mechanism at work; the result is that expected future policy rates are extremely erratic and the term structure severely biased and volatile. Orphanides and Williams (2007) show that a viable solution to the troubles caused by the uncertainty about natural rates is to adopt a difference rule: the fact that in the full transparency case the optimal policy exhibits a high degree of inertia, could be viewed as a validation of their suggestion, since as $\rho \longrightarrow 1$ the natural rates get more and more irrelevant in setting the policy rate.

Table 3 reports a summary of the simulation results. It is apparent that under full transparency the welfare function is very sensitive to the parameterisation of the policy rule, so that even small deviations from the optimal policy cause heavy welfare losses: in a few experiments, monetary policy even fails to anchor expectations and to stabilise the economy. Regardless of the parameterisation of the policy rule, the exchange rate and the long-term yield fluctuate widely, with negative effects on export dynamics and capital accumulation. One noteworthy feature is that even when the overall performance of the policy rule is poor, the standard deviation of output is smaller that under partial transparency or opaqueness: it is dubious however that this accomplishment is to be attributed to monetary policy, since it seems mostly due to fiscal policy, which is very active.[23]

If one considers just the optimal policy, it seems that full transparency is welfare-enhancing; if one considers also suboptimal regimes, the appraisal is more complex. In most parameterisation, monetary policy turns out to be ineffective; in some, it does not even succeed in stabilising the economy. All in all, it seems that central bank talk has a beneficial impact on agents' expectations and behaviour, but may occasionally lead to poor economic outcomes.

[23] At the optimum point, the standard deviation of the tax rate on disposable income (which is the fiscal policy instrument used to keep the debt-to-GDP ratio close to its target of 0.6) is 4.3 per cent; its mean level is nearly 4 p.p. higher than in the steady-state equilibrium. With a smaller degree of transparency, direct taxes are usually lower than in the steady-state growth path and fluctuate much less.

The explanation of this finding echoes the warning of Amato, Morris and Shin (2002), who note that central bank communication has a dual function: on the on hand, it provides signals about the policymaker's private information; on the other hand, it serves as a coordination device for the beliefs of private agents. They argue that central bank communication may be welfare-reducing if it acts as a focal point and induces agents to do away with their own information.

4.3 Perpetual learning

The canonical justification for adopting gain sequences that remain bounded above zero is that the economy is subject to structural shifts and, accordingly, past observations should be given less weight than recent data in the learning algorithm. There is actually a second rationale for using constant-gain estimators that fits the model in this paper perfectly: the possibility of nonconvergence to the REE. If convergence to the perfect information equilibrium is for whatsoever reason unlikely, then the actual stochastic process followed by the economy may best be modelled - given the PLMs employed by agents - as undergoing structural change over time. The main implication of constant-gain learning is that agents' estimates are always subject to sampling variation and never converge to fixed values; for this reason, some authors name this adaptive scheme "perpetual learning".

Table 4 to 6 report the simulation results under the three alternative communication strategies in the case of perpetual learning.

Under no trasparency, there is hardly any difference with the decreasing-gain case. The best policy is essentially the same, just slightly less reactive to fluctuations in real activity ($\alpha_u = 1.9$ rather than $\alpha_u = 2$). Welfare is apparently not affected by the memory of the learning algorithm: it is either the same or slightly higher, suggesting that observations far away in the past are indeed barely informative. The ranking of suboptimal policies is not altered either: the worst outcomes are achieved when either α_π is too high or α_u is too low.

Similar results are obtained when the central bank discloses the information it uses in making policy decisions. Under partial transparency, the welfare-maximising policy features both a slightly milder response to inflation ($\alpha_\pi = 1.3$ vs. $\alpha_\pi = 1.4$) and a fairly strong aversion to labour market imbalances, though not as strong as in the decreasing learning case ($\alpha_u = 1.7$ vs. $\alpha_u = 1.8$). Under full transparency the opposite happens: the optimum is achieved with a somwhat higher value of α_π and a somewhat lower value of α_u. In general, the simulation evidence confirms that when the monetary policymaker reacts too aggressively to price shocks or too meekly to demand fluctuations, the economy becomes unstable and social welfare plunges.

5 Sensitivity analysis

The simulation results just described are based on several ad-hoc assumptions. On some of them - the number of replications in each experiment or the initial conditions of the learning process - a thorough sensitivity analysis can be conducted; on others - the choice of the PLMs - no fully-satisfactory testing procedure is available: with hundreds of variables, there are too many PLMs that can be picked out, most of them indistinguishable in terms of parsimony or fitting.

To test the generality of the findings described in the previous section, three sensitivity analysis exercises have been conducted: in the first, the model has been simulated with 10,000 replications and the results compared with those obtained in the baseline experiment, to test whether the latter are distorted by the small number of replications; in the second, the initial conditions of the learning algorithm have been changed, rescaling upwards/downwards the (fixed) covariance matrix of the regressors entering the Kalman gain; in the last experiment, to have a feeling of how sensitive the ranking of monetary policies is to the selected PLMs, the inflation forecasting equation of the central bank has been modified, including the (change in) the growth rate of GDP.

5.1 Experiment #1: the number of replications

For each experiment, the number of replications has been chosen so as to guarantee reliable results while keeping the time needed for a full search of the optimal policy at an acceptable level. The model, augmented with the learning recursions, contains nearly 300 equations: when all 500 replications converge, it takes roughly a couple of minutes to complete a simulation; when some of them diverge, it can require two hours of computer time. Since the search for the optimum policy calls for the evaluation of more than 300 combinations of the Taylor-rule coefficients, 500 replications has been viewed as an acceptable compromise.

To assess whether the results shown in Tables 1 to 3 are affected by small sample bias, the equilibrium outcomes of the three communication regimes at the optimum have been compared with the results obtained by running 10,000 replications. Table 4 presents a summary of the findings. Only three variables are compared: social welfare, output growth and inflation; for the latter two, both the first (bias) and the second moment (volatility) from the steady-state equilibrium are considered. Each entry is the ratio between the value computed in 10,000 replications and that obtained in 500 ones; for all ratios, the mean, the median, the maximum and the minimum across replications are shown.

According to the evidence presented in Table 4, the size of the small sample bias is negligible: regardless of the transparency regime, the difference in welfare does not reach 2 percentage points and the discrepancy is even smaller for the volatility of output growth and inflation. The estimates of the biases are less alike and sometimes even change sign, but this is no evidence of the existence of a significant small-sample bias: both the numerator and the denominator of the ratios are close to zero, so that even small differences can lead to high jumps in the ratio. The precision of the estimates based on few replications is confirmed by looking at the ratios between the maxima and minima, which are surprisingly low. The full-transparency regime is an exception, but this depends on the fact that the stability (and E-stability) region is not so large as in the other cases, so that more extreme outcomes are more easily obtained.

5.2 Experiment #2: the size of Γ

A second type of sensitivity analysis has been conducted on initial conditions of the learning algorithm. A critical parameter is Γ, the moment matrix of the regressors entering the (generalised) stochastic gradient learning recursive equations: unlike the coefficients of the PLM, the matrix Γ is not updated, but held fixed at some assigned level. To assess the influence of the value of Γ on the ranking of the policy rules, other simulations have been run, using $k\Gamma$ as the moment matrix of the regressors. Six cases have been considered, corresponding to $k = \left\{\frac{9}{10}, \frac{11}{10}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \right\}$. Table 5 shows the results for the three monetary regimes and the 6 values of k; the entries in the table indicate the rank of each policy rule in terms of social welfare. In the final two rows, the Spearman ρ and the Kendall τ rank correlation coefficients are presented.

The results are reassuring. In the full transparency case, there is no uncertainty about which is the welfare-maximising policy rule: all values of $k\Gamma$ point to the same rule. Something similar happens in the partial transparency case, where only for $k = \frac{3}{2}$ the optimal policy is wrongly identified, while the ranking in the no-transparency regime seems to be somewhat more dependent on initial conditions. The sample values of the rank correlation coefficients - surprisingly high in nearly all cases - confirm that initial conditions are quite irrelevant not only in detecting the optimal policy, but also in ordering suboptimal ones.

5.3 Experiment #3: the forecasting equation for inflation

The impossibility of selecting univocal forecasting equations in large non-linear models makes any choice somewhat arbitrary. The PLMs used in the simulations are sensible because they fit survey expectations well, have a good track record as predictors and are highly correlated with the first or second principal component. There are however other specifications that exhibit similar properties: regarding inflation, if one considers an equation with three explanatory variables, an educated choice of the additional regressor allows to improve both the fitting of actual inflation and the correlation with survey expectations. In particular, while the unemployment rate, the oil price and the exchange rate turn out to be poor regressors, the inclusion the growth rate of GDP (or, to a lesser extent, the output gap) boosts the performance of the forecasting model, reducing by nearly one-half the standard error of the regression and raising to above .9 the correlation with survey inflation expectations.

To assess whether the main findings of the papers survive the change in the specification of the PLMs, a new set of simulations has been run. The forecasting equation for inflation used in the experiment is:

$$\widehat{E}_{t-1}^{CB}\pi_t = \pi_{t-1} + \alpha_{1,t-1}\Delta i_{t-1} + \alpha_{2,t-1}\Delta\pi_{t-1} + \alpha_{3,t-1}\Delta^2 y_{t-1} \quad (9)$$

The PLM for exchange rate changes is instead left unchanged, on the grounds that there is no evidence that adding an additional explanatory variable improves upon the two-variable specification.

The inclusion of GDP growth[24] in (9) turns out to be ineffective. The results under both opaqueness and partial transparency are essentially the same, regardless of which PLM is used: in both cases, output and inflation volatility are hardly different[25] and the ranking of policy rules nearly identical,[26] as witnessed by the Spearman and Kendall rank correlation coefficient.[27] Some uncertainty is present only for rules which are neither too bad nor too good, but this has no impact on the monetary policy implications of the exercise. Results are even closer under full transparency, where the ranking turns out to be exactly the same regardless of the PLM used.

6 Conclusions

This paper has analysed the properties of a large non-linear model endowed with boundedly rational agents, focusing in particular on monetary policymaking and central bank communication. In large, non-linear models agents by necessity adopt PLMs that are misspecified and underparameterised and accordingly the temporary equilibrium no longer converges to the REE, but instead tends to an asymptotic limit which depends on the specific form of the expectations equations. The task of the monetary policymaker becomes therefore that of choosing a high-welfare equilibrium and of helping private agents to coordinate their beliefs. The paper has tried to assess whether in such a model economy the implications for monetary policy are similar to those found in the literature for small, linear systems (e.g. Orphanides and Williams 2007); besides, it has studied whether higher degrees of transparency are welfare-enhancing. The main findings are the following. First, it is no longer true that when agents are boundedly rational, the monetary policymaker has an incentive to adopt more inflation-averse policies; on the contrary, too strong a reaction to price shocks increases both inflation and output volatility and tends to make the model unstable and non-learnable. At first sight, this outcome could seem quite counterintuitive: a central bank that is more committed to tame inflationary

[24] GDP growth enters the specification in first differences, in order to avoid biasing inflation projections upwards (if $\alpha_{3,t-1} > 0$) or downwards (if $\alpha_{3,t-1} < 0$) .

[25] For neither rule, the change in social welfare is above 12% (half as much under partial transparency). The differences are larger for the worst-performing rules and (much) smaller for the best ones.

[26] The ranking of the monetary policy rules has been restricted to the 13 alternatives presented in Tables 1 to 6.

[27] The Spearman ρ exceeds .95 under both transparency regimes, while the Kendall τ is in both cases close to .90.

pressures, should be more credible and more effective in anchoring long-run inflation expectations and bond yields. This connection however is not present in the model and credibility depends on outcomes, not intentions: agents learn from the data and what matters is whether monetary policy makes the economy more or less stable.

Second, more transparent policies seem to be welfare-enhancing, mostly because they reduce the slope of the term structure and the variability of long-term interest rates; besides, the degree of transparency alters the form of the optimal policy rule, by increasing inflation aversion and the smoothness coefficient. Disclosing more information is however not always beneficial: under full transparency, even small departures from the optimum policy may cause very poor economic outcomes and at times can make the system unstable.

7 Appendix A - The model

The model used in the paper is a maquette (i.e. a reduced-scale version) of the Bank of Italy Quarterly Model, of which it reproduces the basic features. It is Keynesian in the short-run, with the level of economic activity primarily determined by the behaviour of aggregate demand, and neo-classical in the long-run. Along a steady-state growth path, the dynamics of the model stems solely from capital accumulation, productivity growth, foreign inflation and demographics; in the short-run, a number of additional features matters, namely (i) the stickiness of prices and wages, (ii) the putty-clay nature of capital and (iii) expectation errors. Beliefs enter the model in several ways: ex-ante real interest rates affect the demand for both consumption and capital goods; next-period expected inflation drives the pricing strategies of firms for the current period and indirectly influences wage claims; beliefs about future price developments affect the policy interest rates and accordingly the whole term structure; anticipated changes in the exchange rate bear upon competitiveness and the terms of trade.

In equilibrium - i.e. when no shocks affect the model, expectations are fulfilled and all adjustment processes are completed - the model describes a full employment economy, in which output, employment and the capital stock are consistent with an aggregate production function, relative prices are constant and inflation equals the exogenous rate of growth of foreign prices. Money is neutral, though not super-neutral.

The supply sector can be thought of as being composed by producers who are price-setters in output markets and price-takers in factor markets. Each producer, being endowed with the same Cobb-Douglas constant-returns-to-scale technology, knows the minimum average cost of his competitors and adopts pricing strategies that keep potential entrants out of business; the demand for capital and labour is set so as to minimise production costs. Life-cycle consumers choose the desired addition to the real stock of total wealth, which is then allocated among foreign assets, physical capital and government debt, and the real exchange rate adjusts so as to balance supply and demand and to clear capital markets. As consumers compute their life-time resources without anticipating the need for the government to satisfy a long-run solvency condition, the stock of public debt is perceived to be part of total wealth and Ricardian equivalence does not hold.

7.1 Production function and factor demands

The supply-side block of the BIQM hinges on a Cobb-Douglas production function and on the assumption that, since capital is non-malleable, the choice of productive factors is limited to the expansion of existing stocks. In each period, given demand expectations, firms set the desired addition to capacity. Cost minimisation yields the optimal capital/output and labour/output ratios and the minimum average cost associated with one additional unit of production. Since it takes time to produce and deliver capital goods, actual and planned investment differ, with the former being a weighted average of the most recent values of the latter. As typical with models assuming that capital is putty-clay, investments react to changes in demand and changes in relative factor prices differently: while in the first instance the shape of the response conforms to the accelerator principle, in the second it is smooth and monotone.

Associated with each vintage of capital is a fixed amount of labour which is needed to operate the new machinery and an efficient quantity of output which can be produced with it. The equations modelling employment and potential output are accordingly derived from the parameters of the investment function and from the sequence of vintages of new capital. The demand for labour is determined in a stepwise procedure: first, as shown above, cost minimisation determines the optimal labour-to-output ratio for the planned addition to capacity ; then the labour requirement of the last vintage of installed rather than desired investment is derived; finally, the demand of labour associated with the overall stock of capital is computed.[28] Frictions and adjustment costs make total employment temporarily differ from the desired value. The labour demand function is then modelled as an error correction mechanism, where the long-run employment is driven by the labour requirement associated with the existing capital stock.

The investment function represents also the building block for modelling potential output. The optimal capital-output ratio may be viewed as a conversion factor mapping desired addition to production capacity into capital accumulation: the level of potential output is obtained by cumulating the net addition to capacity corresponding to each vintage of capital.[29]

[28] When capital is putty-clay, the already installed vintages of capital can either be used or scrapped, if they are no longer profitable at current prices. If one assumes that physical and economic obsolescence coincides, than total employment can be obtained by adding up the labour associated with each vintage of capital.

[29] See Parigi and Siviero (2001) for a reference.

7.2 Domestic demand components

Consumption is modelled consistently with the life-cycle theory and is driven by permanent income, proxied by a weighted average of disposable income and wealth. Disposable income is computed by adjusting for the capital gains/losses on financial assets engendered by changes in the inflation rate; the real interest rate is also included among the explanatory variables. Employment and purchases of goods and services (in nominal terms) in the public-sector are exogenous, implying that (to a first approximation) so is government consumption.

Private sector investment is modelled as described in the previous subsection, while public sector spending on capital goods is assumed to be exogenous (in nominal terms). Inventories, as a share of total demand, depend on the real interest rate and on changes in demand, capturing the undesired accumulation/decumulation of stocks throughout the business cycle.

7.3 Prices and costs

The pricing strategies of firms are described by the equations modelling the private-sector value added deflators (at factor costs). Prices are set as a mark-up over marginal costs. In equilibrium, marginal and minimum average costs coincide and are proportional to unit labour costs. The mark-up is assumed to depend on the prices set by foreign competitors and on cyclical conditions, the former measured by the real exchange rate and the latter by the output gap. Deflators of aggregate demand components are modelled as a function of import and value added deflators.

A Phillips curve relation completes the price-wage block: wage inflation depends with unit elasticity on price changes, measured by a convex combination of expected and actual (past) inflation, and on the rate of unemployment. The bargaining power of unions is proxied by the number of working hours lost due to strikes, while indirect taxes, social contributions and terms of trade have an impact on wages through the tax wedge. In equilibrium, the Phillips curve determines the NAIRU, while the price equation determines the factor shares in income distribution. Compensations for public-sector employees are assumed to be closely related to those bargained for the private-sector.

7.4 The trade equations

The modelling of the demand for imports and exports relies on the assumption of imperfect substitutability between foreign and domestic goods. In accordance with demand theory, imports may be viewed as the solution to the maximisation problem of a representative consumer, who acts taking into account a budget constraint. Separability and homogeneity of the utility function ensure that the saving decision and consumption allocation - in particular the choice between domestic and foreign goods - can be treated separately. Absence of money illusion is imposed by considering relative prices and real income. The scale variable driving imports is a weighted average of aggregate demand components, with exports and investments in machinery having the largest weights; the degree of utilised capacity is included in the specification so as to capture the fact that, because of bottlenecks, sudden changes in domestic demand are initially met more than proportionally by foreign production.

Exports are modelled in a similar way: the scale variable is world imports and changes in domestic demand are used as a proxy for non-price competitiveness. Both imports and exports have unit elasticity with respect to their respective scale variables.

7.5 The monetary and financial sector

The theoretical framework underlying the monetary and financial block is to a large extent the one outlined in Ando-Modigliani (1975) and conforms to the methodology that used to underlie the MPS econometric model. Each market is described by a demand function and an inverted supply equation, in which the endogenous variable is the relevant interest rate. Substitutability among assets is not perfect, since lenders and borrowers have their preferred point in maturity along the term-structure, as suggested by the preferred-habitat theory. The structure of the block is designed so as to provide a detailed account of the channels through which the central bank affects the economy via portfolio adjustment in the monetary and financial markets. The model therefore includes a detailed description of the working of the money market: the central bank controls liquidity conditions through open market operations and hence affects the market for Treasury securities. Term structure equations link the return on fixed-income long-term Treasury bonds to the three-month Treasury bill rate; bank lending rates are modelled as functions of the yields of government securities.

7.6 Fiscal and monetary policy

Some of the variables describing fiscal policy are exogenous (indirect tax rates; government employment, investment and intermediate consumption); others are tightly related to developments in the private sector (compensations). The only instrument that the fiscal policymaker uses to stabilise the economy is the income tax rate, which moves so as to keep the debt-to-GDP ratio close to its target level of 0.6; in order to avoid strongly procyclical policy moves, it is assumed that the tax rate also responds to the unemployment gap, implying a sort of "flexible" targeting of the debt-to-GDP ratio. Finally, the inertia in setting the instrument value is captured by an autoregressive term. All the three parameters have been calibrated so as to ensure both data coherency and good stabilising properties.

Monetary policy has been modelled by means of a Taylor-type rule. The variables entering the reaction function of the central bank are the inflation rate and the unemployment rate, whose target values are those characterising the steady state growth path. The rule exhibits some inertia, reflecting interest rate smoothing motives.

8 Appendix B - The maquette

There are several methods to build a maquette. They differ in many respects, depending on whether (i) the maquette is a structural-form or a reduced-form model;[30] whether (ii) it is intended to reproduce the overall properties of the parent model or only its dynamics;[31] whether (iii) it is estimated on actual or simulated data.[32] The standard procedure for deriving a structural-form system, as described in Masson (1989), can be summarized as follows. The first step is to select a control solution, often coinciding with the steady-state path. The following step is to identify the block of equations of the large model that must be aggregated into a single equation in the maquette. Each right-hand-side variable of the block is then given a unit displacement and the model is simulated over a sufficiently long interval, generating a set of equations of the following form:

$$\delta y_{it} = \sum_{k=0}^{T} J_k^i \left(\delta y_{2t-k}\right) + \sum_{k=0}^{T} H_k^i \left(\delta x_{t-k}\right) \tag{10}$$

The δ in (10) stands for the deviations from the control solution; y_{it} is the vector of endogenous variables being shocked; y_{2t} is the set of endogenous variables appearing as regressors in the *i-th* block; the matrices J_k^i and H_k^i measure the responses of the vector y_{it} to unit changes in the explanatory variables. The next stage of the procedure is to transform by aggregation the vector δy_{it} into a scalar and the vector δx_{it} into a lower dimension vector.[33] The final step consists in regressing the endogenous variables on the "shock-minus-control" values of the RHS variables, in order to generate coefficient estimates. Since the number of periods (identified by the value of T) that are necessary to return to the the equilibrium is in general very large, ratios of polynomials in the lag operator are used to achieve a parsimonious parameterisation.

The procedure which was adopted to generate the maquette of the BIQM is quite different. There are three major points of divergence, justified by the purposes underlying the estimation of a small scale version of the BIQM:

[30] Maciejowski and Vines (1984) generate a linear reduced-form of a larger macroeconometric model, which they then reduce in scale by aggregating variables. Haas and Masson (1986) generate a structural-form two-country maquette, in order to conduct policy experiments under different regimes of expectations formation mechanisms.

[31] Terlizzese (1994) uses a maquette as a device for describing the theoretical underpinnings and the statistical features of the BIQM.

[32] Battenberg, Enzler and Havenner (1975), for instance, created a small version of the MPS model, called MINNIE, by resorting to actual data, while Haas and Masson (1986) used simulation results in order to ensure a closer adherence of their maquette to the MCM of the FED.

[33] While the treatment of the endogenous variables is an easy task, there is not an unambiguous way of aggregating the exogenous ones. The xs should be weighted according to their effects on the endogenous variables, but this is not possible when they do not move toghether or they do not have the same impact on the ys. Therefore, in general, by aggregating exogenous variables, one is creating a maquette that will not replicate exactly the parent model.

- the maquette of the BIQM is intended to provide insights on either the long and the short-run properties of the parent model. The maquette is intended to detect and to provide information on the features of the steady-state growth path; to asses the stability/instability of the economic system in the absence of policy interventions; to describe the response of the main macroeconomic aggregates to exogenous impulses; to shed light on the effects of alternatives expectations formation mechanisms; to allow policy evaluation experiments.[34] In order to achieve these objectives, the maquette must be a structural-form model and the variables must be modelled **in levels**, not **as deviations** from a control solution.

- the specification of the BIQM is aimed at providing a very detailed account of the institutional features of the Italian economy. In particular, items in the balance of the public sector are modelled so as to distinguish expenditure and revenue items on an accrual and on a cash flow basis. Though this modelling strategy contributes to improving forecast accuracy and to answering day-to-day policy issues, it contributes to obscure the theoretical underpinnings of the BIQM. To simplify the structure of the maquette, data were aggregated so that the maquette is estimated on **annual** rather than quarterly **data**.

- the BIQM, like any econometric model, is subject to several sources of uncertainty, the most important being that associated with the residuals. The standard procedure used to estimate the equations of a maquette neglects residual uncertainty, since it is based on a single simulation of the parent model. A different technique has been adopted here: first, a set of 100 stochastic simulations of the BIQM have been run; second, after aggregating the ys and the xs corresponding to each replication, the equations of the maquette have been estimated, obtaining $\beta^{(r)}$, the vector of estimated coefficients; finally, the parameter vector β has been computed by averaging the set of $\beta^{(r)}$.

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[34] The above objectives are strictly intertwined. For instance, finding the steady-state equilibrium path provides terminal conditions for solving a model with rational epectations. Besides, the expectations formation mechanism affects policy strategies, which in turn can alter the dynamic properties of the system.

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Table 1 - Monetary policy effectiveness under no transparency

(decreasing gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods). Agents assume that the monetary policy rule is: $i_t = \delta_0 + \delta_1 \pi_{t-1} + \delta_2 u_{t-1} + \delta_3 i_{t-1}$ (where i is the policy instrument, π inflation and u the unemployment rate), rather than the reaction function shown in th text. For each set of parameters of the Taylor rule, mean *(bias)* and squared differences *(volatility)* with respect to the steady-state values are reported (in p.p.). W stands for welfare (as a ratio to the optimum, reported in row 1) and RR for the rejection rate; Δy is the growth rate of GDP, π is inflation and e the exchange rate (as a ratio to the steady-state value); τ is the tax rate on disposable income; $\Delta\delta_0$ and $\Delta\delta_1$ are the difference between the intercept and inflation coefficient in the PLM and in the true Taylor rule; finally i is the policy rate, $i^e - i$ is the difference between expected and actual short-tem interest rate, *error* is the difference between expected and actual short-term rate up to 5 periods ahead and i^L is the yield on Treasury bonds.

				W	RR	Δy	π	e	τ	$\Delta\delta_0$	$\Delta\delta_1$	i	$i^e - i$	*error*	i^L
$\rho=0.7$	$\alpha_\pi=1.1$	$\alpha_u=2.0$	*vol.*	-0.030	0.0	1.132	1.303	23.391	0.87	0.810	0.199	1.570	0.340	1.380	1.710
			bias			-0.004	0.108	20.265	-0.47	-0.800	0.197	-0.500	0.000	1.090	0.770
$\rho=0.8$			*vol.*	0.929	0.0	1.098	1.408	27.18	1.00	0.640	0.280	1.540	0.320	1.23	1.670
			bias			-0.003	0.136	23.69	-0.68	0.060	0.280	-0.550	0.010	1.06	0.680
$\rho=0.6$			*vol.*	0.987	0.0	1.172	1.283	27.04	0.92	1.860	0.169	1.580	0.450	1.49	1.730
			bias			-0.003	0.103	21.10	-0.53	-1.850	0.165	-0.540	-0.010	1.11	0.760
$\rho=0.5$	$\alpha_\pi=1.0$	$\alpha_u=2.0$	*vol.*	0.962	0.0	1.203	1.287	23.19	0.87	2.700	0.132	1.630	0.630	1.60	1.790
			bias			-0.004	0.088	19.37	-0.39	-2.690	0.123	-0.510	-0.010	1.14	0.820
$\rho=0.4$			*vol.*	0.927	0.0	1.231	1.304	21.85	0.83	3.470	0.113	1.680	0.810	1.70	1.860
			bias			-0.004	0.074	17.53	-0.24	-3.450	0.090	-0.470	-0.020	1.17	0.890
$\rho=0.3$			*vol.*	0.894	0.0	1.253	1.327	20.51	0.82	4.180	0.109	1.750	0.980	1.80	1.920
			bias			-0.005	0.061	15.65	-0.08	-4.150	0.064	-0.430	-0.030	1.19	0.960
	$\alpha_\pi=0.5$		*vol.*	0.767	0.0	1.176	1.573	54.62	2.09	1.520	0.332	1.630	0.380	1.25	1.720
			bias			0.004	0.205	45.46	-1.73	-1.510	0.331	-0.890	0.010	0.97	0.240
	$\alpha_\pi=1.0$		*vol.*	0.997	0.0	1.135	1.304	25.86	0.98	0.940	0.218	1.542	0.320	1.36	1.670
			bias			-0.003	0.119	22.60	-0.63	-0.930	0.217	-0.560	0.000	1.09	0.700
$\rho=0.7$	$\alpha_\pi=1.5$	$\alpha_u=2.0$	*vol.*	0.918	0.0	1.143	1.382	16.76	0.80	0.330	0.128	1.800	0.510	1.51	1.970
			bias			-0.006	0.058	12.98	0.11	-0.230	0.122	-0.280	-0.280	1.07	0.990
	$\alpha_\pi=2.0$		*vol.*	0.647	1.2	1.215	1.714	13.65	1.54	0.660	0.077	2.560	0.860	1.88	2.640
			bias			-0.010	-0.018	4.74	1.03	0.540	0.027	0.130	-0.070	0.96	1.340
	$\alpha_\pi=2.5$		*vol.*	0.422	15.2	1.368	2.227	16.51	3.03	1.370	0.141	3.810	1.370	2.62	3.700
			bias			-0.018	-0.150	-3.63	2.30	1.250	-0.079	0.720	-0.170	0.72	1.780
		$\alpha_u=1.0$	*vol.*	0.673	4.8	1.060	1.780	16.99	0.95	0.800	0.199	1.690	-0.170	1.22	2.010
			bias			-0.009	0.060	9.17	0.57	0.790	0.199	0.050	-0.170	1.03	1.180
		$\alpha_u=1.5$	*vol.*	0.944	0.0	1.088	1.397	19.44	0.68	0.140	0.208	1.460	0.290	1.27	1.720
			bias			-0.005	0.082	15.38	0.09	-0.070	0.208	-0.350	0.000	1.08	0.920
$\rho=0.7$	$\alpha_\pi=1.0$	$\alpha_u=2.0$	*vol.*	0.997	0.0	1.135	1.304	25.86	0.98	0.940	0.218	1.540	1.580	1.36	1.670
			bias			-0.001	0.119	22.60	-0.63	-0.930	0.217	-0.560	-0.560	1.09	0.700
		$\alpha_u=2.5$	*vol.*	0.959	0.0	1.190	1.304	33.19	1.37	1.790	0.229	1.690	0.360	1.47	1.750
			bias			-0.001	0.151	29.40	-1.05	-1.780	0.228	-0.690	0.000	1.06	0.540

Table 2 - Monetary policy effectiveness with partial transparency

(decreasing gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods). Agents know which is the current value of the policy rate and assume that future rates are set according to the PLM: $i_t=\delta_0+\delta_1\pi_{t-1}+\delta_2 u_{t-1}+\delta_3 i_{t-1}$ (where i is the policy instrument, π inflation and u the unemployment rate). For each set of parameters of the Taylor rule, mean (*bias*) and squared differences (*volatility*) with respect to the steady-state values are reported (in p.p.). W stands for welfare (as a ratio to the optimum, reported in row 1) and *RR* for the rejection rate; Δy is the growth rate of GDP, π is inflation and e the exchange rate (as a ratio to the steady-state value); τ is the tax rate on disposable income; $\Delta\delta_0$ $\Delta\delta_1$ are the difference between the intercept and inflation coefficient in the PLM and in the true Taylor rule; i^e-i is the difference between expected and actual short-tem interest rate, *error* is the difference between expected and actual short-term rate up to 5 periods ahead and i^L is the yield on Treasury bonds.

				W	RR	Δy	π	e	τ	$\Delta\delta_0$	$\Delta\delta_1$	i^e,i	error	i^L
ρ=0.7	α_π=1.4	α_u=1.8	vol.	-0.027	0.0	1.146	1.163	17.841	0.730	0.860	0.032	1.380	1.260	1.770
			bias			-0.004	0.084	15.307	-0.190	-0.830	0.006	-0.280	0.940	0.840
ρ=0.8			vol.	0.912	0.0	1.171	1.247	31.486	1.340	0.640	0.135	1.310	1.040	1.700
			bias			0.001	0.164	28.367	-1.050	-0.750	0.122	-0.500	0.830	0.500
ρ=0.6			vol.	0.944	0.0	1.218	1.160	27.149	1.120	2.650	0.042	1.370	1.310	1.690
			bias			-0.001	0.119	23.973	-0.790	-2.650	0.029	-0.510	0.940	0.610
ρ=0.5	α_π=1.0	α_u=2.0	vol.	0.948	0.0	1.220	1.152	24.931	1.010	3.540	0.028	1.410	1.420	1.690
			bias			-0.002	0.101	21.696	-0.650	-3.530	-0.010	-0.500	0.990	0.670
ρ=0.4			vol.	0.951	0.0	1.218	1.152	22.751	0.900	4.380	0.054	1.450	1.510	1.700
			bias			-0.003	0.086	19.430	-0.490	-4.380	-0.044	-0.480	1.030	0.740
ρ=0.3			vol.	0.951	0.0	1.214	1.155	20.619	0.810	5.200	0.087	1.510	1.600	1.720
			bias			-0.004	0.072	17.187	-0.330	-5.190	-0.075	-0.450	1.070	0.810
	α_π=0.5		vol.	0.608	0.0	1.330	1.603	66.927	2.590	2.370	0.171	1.620	1.060	1.910
			bias			0.009	0.226	56.106	-2.180	-2.330	0.158	-0.890	0.680	-0.010
	α_π=1.0		vol.	0.938	0.0	1.204	1.181	29.310	1.240	1.740	0.084	1.332	1.190	1.690
			bias			0.000	0.140	26.182	-0.930	-1.720	0.072	-0.510	0.890	0.550
ρ=0.7	α_π=1.5	α_u=2.0	vol.	0.993	0.0	1.169	1.148	18.683	0.770	1.050	0.031	1.440	1.310	1.790
			bias			-0.004	0.090	16.200	-0.260	-1.030	-0.008	-0.310	0.940	0.820
	α_π=2.0		vol.	0.944	0.0	1.165	1.208	13.456	0.800	0.380	0.094	1.720	1.480	1.990
			bias			-0.006	0.051	10.578	0.200	-0.300	-0.088	-0.160	0.960	0.990
	α_π=2.5		vol.	0.779	0.0	1.204	1.379	10.859	1.240	0.530	0.182	2.220	1.750	2.380
			bias			-0.007	0.002	5.284	0.760	0.430	-0.171	0.060	0.910	1.190
		α_u=1.0	vol.	0.814	0.0	1.076	1.436	16.472	0.690	0.300	0.075	1.310	1.050	1.860
			bias			-0.005	0.074	11.628	0.180	-0.010	0.062	-0.070	0.870	0.990
		α_u=1.5	vol.	0.961	0.0	1.128	1.227	21.351	0.790	0.920	0.078	1.220	1.100	1.690
ρ=0.7	α_π=1.0		bias			-0.003	0.099	18.257	-0.400	-0.880	0.065	-0.310	0.890	0.760
		α_u=2.0	vol.	0.938	0.0	1.204	1.181	29.310	1.240	1.740	0.084	1.330	1.190	1.690
			bias			0.000	0.140	26.180	-0.930	-1.720	0.072	-0.510	0.890	0.550
		α_u=2.5	vol.	0.852	0.0	1.300	1.200	38.752	1.730	2.550	0.092	1.510	1.310	1.780
			bias			0.003	0.179	34.780	-1.400	-2.530	0.082	-0.670	0.860	0.004

Table 3 - Monetary policy effectiveness with full transparency

(decreasing gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods) when agents learn adaptively and the central bank discloses all the relevant information. Agents know which is the current value of the policy rate, the parameters of the Taylor rule and the central bank's estimates of the natural rates; they assume that future rates are set according to a policy rules having the same parameters than the true one. For each set of parameters of the Taylor rule, mean (bias) and squared differences (volatility) with respect to the steady-state values are reported (in p.p.). *W* stands for welfare (as a ratio to the optimum, reported in row 1) and *RR* for the rejection rate; Δy is the growth rate of GDP, π is inflation and e the exchange rate (as a ratio to the steady-state value); τ is the tax rate on disposable income and *deb* the government debt to GDP ratio; finally i^{e} are the expected and actual short-tem interest rate (which by assumption coincide), *error* is the difference between expected and actual short-term rate up to 5 periods ahead AND i^L is the yield on Treasury bonds.

				W	*RR*	Δy	π	*e*	τ	*deb*	i^e, i	*error*	i^L
ρ=0.85	α_π=1.0	α_u=2.2	*vol.*	-0.027	0	1.056	1.236	25.338	4.300	0.860	1.380	2.940	2.860
			bias			-0.021	-0.243	-21.490	3.970	-0.830	-0.740	2.870	2.520
ρ=0.8			*vol.*	0.918	0	1.052	1.332	34.997	6.180	0.640	1.440	3.390	3.550
			bias			-0.023	-0.335	-31.240	5.760	-0.750	-0.560	3.310	3.200
ρ=0.6			*vol.*	0.558	8.8	1.116	1.884	65.187	14.250	2.650	2.100	5.210	6.800
			bias			-0.002	-0.688	-60.402	13.400	-2.650	0.510	5.060	6.300
ρ=0.5	α_π=1.0	α_u=2.0	*vol.*	?	100.0	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
ρ=0.4			*vol.*	?	100.0	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
ρ=0.3			*vol.*	?	100.0	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
	α_π=0.5		*vol.*	0.590	0.0	1.037	1.851	51.387	10.490	2.370	1.840	4.140	5.160
			bias			-0.022	-0.503	-46.457	9.710	-2.330	0.120	4.050	4.750
	α_π=1.0		*vol.*	0.678	0.0	1.081	1.660	53.485	10.920	1.740	1.730	4.440	5.360
			bias			-0.023	-0.537	-49.192	10.240	-1.720	0.000	4.320	4.930
ρ=0.7	α_π=1.5	α_u=2.0	*vol.*	0.659	0.0	1.167	1.637	55.012	11.320	1.050	1.780	4.730	5.640
			bias			-0.021	-0.563	-50.991	10.660	-1.030	-0.120	4.580	5.110
	α_π=2.0		*vol.*	0.618	0.0	1.276	1.640	56.270	11.660	0.380	1.870	5.020	5.930
			bias			-0.018	-0.586	-52.384	11.010	-0.300	-0.250	4.830	5.260
	α_π=2.5		*vol.*	0.568	0.8	1.396	1.659	57.560	12.040	0.530	1.980	5.310	6.250
			bias			-0.015	-0.611	-53.723	11.370	0.430	-0.360	5.060	5.420
		α_u=1.0	*vol.*	0.761	0.0	1.024	1.539	28.454	4.930	0.300	1.430	2.630	3.150
			bias			-0.021	-0.246	-24.839	4.610	-0.010	-0.200	2.550	2.720
		α_u=1.5	*vol.*	0.785	0.0	1.052	1.506	40.824	7.520	0.920	1.510	3.550	4.050
ρ=0.7	α_π=1.0		*bias*			-0.024	-0.385	-37.140	7.060	-0.880	-0.270	3.460	3.680
		α_u=2.0	*vol.*	0.678	0.0	1.081	1.660	53.482	10.920	1.740	1.730	4.440	5.360
			bias			-0.023	-0.537	-49.196	10.240	-1.720	0.000	4.320	4.930
		α_u=2.5	*vol.*	0.605	38.8	1.104	1.789	65.809	14.360	2.550	2.070	5.310	6.840
			bias			0.000	-0.696	-60.741	13.440	-2.530	0.440	5.170	6.350

Table 4 - Monetary policy effectiveness under no transparency

(constant gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods). Agents assume that the monetary policy rule is: $i_t=\delta_0+\delta_1\pi_{t-1}+\delta_2 u_{t-1}+\delta_3 i_{t-1}$ (where i is the policy instrument, π inflation and u the unemployment rate), rather than the reaction function shown in th text. For each set of parameters of the Taylor rule, mean *(bias)* and squared differences *(volatility)* with respect to the steady-state values are reported (in p.p.). W stands for welfare (as a ratio to the optimum, reported in row 1) and RR for the rejection rate; Δy is the growth rate of GDP, π is inflation and e the exchange rate (as a ratio to the steady-state value); τ is the tax rate on disposable income; $\Delta\delta_0$ and $\Delta\delta_1$ are the difference between the intercept and inflation coefficient in the PLM and in the true Taylor rule; finally i is the policy rate, i^e-i is the difference between expected and actual short-tem interest rate, *error* is the difference between expected and actual short-term rate up to 5 periods ahead and i^L is the yield on Treasury bonds.

				W	RR	Δy	π	e	τ	$\Delta\delta_0$	$\Delta\delta_1$	i	i^e-i	*error*	i^L
ρ=0.7	α_π=1.1	α_u=1.9	*vol.*	-0.030	0.0	1.125	1.287	22.172	0.831	0.645	0.193	1.471	0.299	1.321	1.695
			bias			-0.003	0.094	18.971	-0.412	-0.628	0.192	-0.391	0.011	1.060	0.756
ρ=0.8			*vol.*	0.943	0.0	1.098	1.372	26.442	0.996	0.200	0.274	1.440	0.290	1.192	1.650
			bias			-0.002	0.123	22.953	-0.668	0.040	0.273	-0.460	0.020	1.027	0.640
ρ=0.6			*vol.*	0.980	0.0	1.180	1.262	24.765	0.956	1.850	0.167	1.520	0.420	1.444	1.720
			bias			-0.003	0.090	21.094	-0.549	-1.840	0.164	-0.460	0.010	1.076	0.710
ρ=0.5	α_π=1.0	α_u=2.0	*vol.*	0.958	0.0	1.210	1.261	23.678	0.910	2.710	0.134	1.560	0.590	1.551	1.770
			bias			-0.003	0.074	19.658	-0.437	-2.680	0.122	-0.430	0.000	1.096	0.760
ρ=0.4			*vol.*	0.937	0.0	1.231	1.269	22.409	0.872	3.510	0.115	1.610	0.770	1.644	1.820
			bias			-0.004	0.060	17.955	-0.300	-3.470	0.086	-0.390	0.000	1.118	0.830
ρ=0.3			*vol.*	0.917	0.0	1.245	1.281	21.028	0.852	4.270	0.112	1.660	0.930	1.731	1.860
			bias			-0.005	0.048	16.117	-0.149	-4.220	0.054	-0.340	-0.010	0.900	0.900
	α_π=0.5		*vol.*	0.773	0.0	1.184	1.535	51.124	1.999	1.560	0.324	1.560	0.360	1.235	1.750
			bias			-0.001	0.097	40.899	-1.479	-1.550	0.322	-0.760	0.030	0.950	0.280
	α_π=1.0		*vol.*	0.993	0.0	1.140	1.282	25.615	0.990	0.950	0.214	1.470	0.280	1.322	1.670
			bias			-0.002	0.107	22.224	-0.634	-0.940	0.213	-0.470	0.010	1.056	0.670
ρ=0.7	α_π=1.5	α_u=2.0	*vol.*	0.943	0.0	1.147	1.333	17.623	0.773	0.350	0.129	1.710	0.450	1.447	1.940
			bias			-0.005	0.064	14.260	-0.035	-0.200	0.122	-0.240	-0.010	1.044	0.900
	α_π=2.0		*vol.*	0.737	0.0	1.216	1.558	14.131	1.203	0.800	0.088	2.280	0.730	1.725	2.510
			bias			-0.008	-0.006	7.592	0.656	0.610	0.037	0.050	-0.040	0.936	1.160
	α_π=2.5		*vol.*	0.509	10.8	1.382	1.928	15.156	2.327	1.700	0.149	3.240	1.130	2.333	3.490
			bias			-0.017	-0.124	0.441	1.657	1.480	-0.046	0.460	0.100	1.543	1.540
		α_u=1.0	*vol.*	0.675	1.6	1.055	1.753	16.926	0.904	0.800	0.197	1.630	0.280	1.182	1.930
			bias			-0.007	0.058	9.257	0.520	0.790	0.196	0.020	0.010	0.998	1.130
		α_u=1.5	*vol.*	0.955	0.0	1.086	1.366	19.148	0.679	0.150	0.204	1.380	0.270	1.234	1.680
ρ=0.7	α_π=1.0		*bias*			-0.004	0.077	15.148	-0.109	0.080	0.203	-0.270	0.010	1.052	0.880
		α_u=2.0	*vol.*	0.993	0.0	1.140	1.282	25.615	0.990	0.950	0.214	1.470	0.280	1.322	1.670
			bias			-0.002	0.107	22.224	-0.634	-0.940	0.213	-0.470	0.010	1.056	0.670
		α_u=2.5	*vol.*	0.934	0.0	1.209	1.294	33.374	1.402	1.780	0.227	1.640	0.310	1.441	1.800
			bias			-0.001	0.124	29.178	-1.043	-1.770	0.226	-0.610	0.010	1.023	0.500

Table 5 - Monetary policy effectiveness with partial transparency

(constant gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods). Agents know which is the current value of the policy rate and assume that future rates are set according to the PLM: $i_t=\delta_0+\delta_1\pi_{t-1}+\delta_2 u_{t-1}+\delta_3 it\text{-}1$ (where i is the policy instrument, π inflation and u the unemployment rate). For each set of parameters of the Taylor rule, mean (*bias*) and squared differences (*volatility*) with respect to the steady-state values are reported (in p.p.). W stands for welfare (as a ratio to the optimum, reported in row 1) and *RR* for the rejection rate; Δy is the growth rate of GDP, π is inflation and e the exchange rate (as a ratio to the steady-state value); τ is the tax rate on disposable income; $\Delta\delta_0$ $\Delta\delta_1$ are the difference between the intercept and inflation coefficient in the PLM and in the true Taylor rule; i^e-i is the difference between expected and actual short-tem interest rate, *error* is the difference between expected and actual short-term rate up to 5 periods ahead and i^L is the yield on Treasury bonds.

				W	RR	Δy	π	e	τ	$\Delta\delta_0$	$\Delta\delta_1$	i^e,i	error	i^L
$\rho=0.7$	$\alpha_\pi=1.3$	$\alpha_u=1.7$	vol.	-0.027	0.0	1.137	1.165	18.312	0.743	0.838	0.043	1.277	1.184	1.734
			bias			-0.002	0.086	15.681	-0.256	-0.792	0.018	-0.204	0.897	0.785
$\rho=0.8$			vol.	0.928	0.0	1.160	1.229	29.749	1.276	0.900	0.138	1.220	1.027	1.690
			bias			0.001	0.146	26.583	-0.976	-0.770	0.115	-0.400	0.803	0.484
$\rho=0.6$			vol.	0.937	0.0	1.225	1.153	26.621	1.116	2.650	0.045	1.300	1.279	1.692
			bias			-0.001	0.102	23.252	-0.773	-2.640	0.027	-0.416	0.909	0.582
$\rho=0.5$	$\alpha_\pi=1.0$	$\alpha_u=2.0$	vol.	0.934	0.0	1.233	1.149	24.877	1.021	3.530	0.029	1.350	1.376	1.693
			bias			-0.002	0.086	21.344	-0.644	-3.530	-0.011	-0.404	0.952	0.642
$\rho=0.4$			vol.	0.931	0.0	1.234	1.150	23.095	0.931	4.390	0.058	1.401	1.463	1.698
			bias			-0.002	0.071	19.392	-0.507	-4.386	-0.047	-0.382	0.987	0.705
$\rho=0.3$			vol.	0.934	0.0	1.230	1.154	21.245	0.854	5.233	0.094	1.462	1.547	1.708
			bias			-0.003	0.058	17.341	-0.357	-5.225	-0.081	-0.347	1.016	0.774
	$\alpha_\pi=0.5$		vol.	0.629	0.0	1.321	1.562	61.250	2.425	2.427	0.174	1.529	1.077	1.902
			bias			0.001	0.091	49.706	-1.887	-2.377	0.151	-0.781	0.678	0.041
	$\alpha_\pi=1.0$		vol.	0.941	0.0	1.202	1.170	28.298	1.209	1.756	0.088	1.260	1.165	1.691
			bias			0.000	0.122	25.057	-0.889	-1.720	0.069	-0.414	0.860	0.527
$\rho=0.7$	$\alpha_\pi=1.5$	$\alpha_u=2.0$	vol.	0.985	0.0	1.177	1.141	19.150	0.809	1.035	0.034	1.388	1.281	1.787
			bias			-0.002	0.089	16.609	-0.325	-1.005	-0.009	-0.245	0.911	0.760
	$\alpha_\pi=2.0$		vol.	0.931	0.0	1.182	1.197	14.503	0.785	0.380	0.095	1.670	1.433	1.991
			bias			-0.004	0.056	11.672	0.081	-0.258	-0.087	-0.117	0.920	0.914
	$\alpha_\pi=2.5$		vol.	0.828	9.2	1.217	1.304	11.668	1.026	0.643	0.180	2.055	1.631	2.308
			bias			-0.006	0.020	7.523	0.490	0.501	-0.167	0.035	0.885	1.066
		$\alpha_u=1.0$	vol.	0.825	0.0	1.066	1.424	16.135	0.686	0.355	0.080	1.276	1.032	1.838
			bias			-0.003	0.076	11.264	0.166	-0.017	0.060	0.008	0.843	0.953
		$\alpha_u=1.5$	vol.	0.971	0.0	1.119	1.214	20.691	0.781	0.955	0.082	1.163	1.082	1.676
$\rho=0.7$	$\alpha_\pi=1.0$		bias			-0.002	0.093	17.543	-0.380	-0.887	0.062	-0.218	0.863	0.731
		$\alpha_u=2.0$	vol.	0.941	0.0	1.202	1.170	28.298	1.209	1.756	0.088	1.260	1.165	1.691
			bias			0.000	0.122	25.057	-0.889	-1.720	0.069	-0.414	0.860	0.527
		$\alpha_u=2.5$	vol.	0.841	0.0	1.312	1.196	37.633	1.688	2.543	0.096	1.440	1.287	1.804
			bias			0.002	0.146	33.423	-1.346	-2.518	0.080	-0.585	0.831	0.341

Table 6 - Monetary policy effectiveness with full transparency

(constant gain learning)

The table reports a summary of the results (500 replications) of the model simulations (for the initial 140 periods) when agents learn adaptively and the central bank discloses all the relevant information. Agents know which is the current value of the policy rate, the parameters of the Taylor rule and the central bank's estimates of the natural rates; they assume that future rates are set according to a policy rules having the same parameters than the true one. For each set of parameters of the Taylor rule, mean (bias) and squared differences (volatility) with respect to the steady-state values are reported (in p.p.). *W* stands for welfare (as a ratio to the optimum, reported in row 1) and *RR* for the rejection rate; *Δy* is the growth rate of GDP, *π* is inflation and e the exchange rate (as a ratio to the steady-state value); *τ* is the tax rate on disposable income and *deb* the government debt to GDP ratio; finally i^e are the expected and actual short-tem interest rate (which by assumption coincide), *error* is the difference between expected and actual short-term rate up to 5 periods ahead AND i^L is the yield on Treasury bonds.

				W	*RR*	*Δy*	*π*	*e*	*τ*	*deb*	i^e, i	*error*	i^L
ρ=0.85	$\alpha_\pi=1.2$	$\alpha_u=2.1$	*vol.*	*-0.027*	*0.0*	1.054	1.226	22.102	3.714	8.241	1.356	2.834	2.658
			bias			-0.018	-0.209	-18.359	3.439	7.531	-0.720	2.762	2.294
ρ=0.8			*vol.*	*0.937*	*0.0*	1.052	1.052	32.467	5.621	11.759	1.410	3.352	3.333
			bias			-0.022	-0.307	-28.771	5.245	11.130	-0.589	3.268	2.991
ρ=0.6			*vol.*	*0.566*	*0.0*	1.110	1.853	61.926	13.400	26.872	2.020	5.110	6.404
			bias			-0.013	-0.629	-57.382	12.608	25.801	0.403	4.967	5.914
ρ=0.5	$\alpha_\pi=1.0$	$\alpha_u=2.0$	*vol.*	*?*	*100.0*	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
ρ=0.4			*vol.*	*?*	*100.0*	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
ρ=0.3			*vol.*	*?*	*100.0*	?	?	?	?	?	?	?	?
			bias			?	?	?	?	?	?	?	?
	$\alpha_\pi=0.5$		*vol.*	*0.592*	*0.0*	1.038	1.828	49.555	9.965	19.850	1.847	4.091	4.966
			bias			-0.022	-0.486	-44.566	9.197	18.956	0.120	3.998	4.551
	$\alpha_\pi=1.0$		*vol.*	*0.704*	*0.0*	1.075	1.606	50.664	10.085	20.399	1.671	4.372	5.029
			bias			-0.023	-0.496	-46.519	9.463	19.549	-0.084	4.260	4.621
ρ=0.7	$\alpha_\pi=1.5$	$\alpha_u=2.0$	*vol.*	*0.690*	*0.0*	1.154	1.578	51.718	10.326	21.015	1.711	4.651	5.243
			bias			-0.022	-0.510	-47.919	9.745	20.130	-0.239	4.509	4.734
	$\alpha_\pi=2.0$		*vol.*	*0.648*	*0.0*	1.256	1.580	52.767	10.600	21.662	1.805	4.930	5.501
			bias			-0.021	-0.526	-49.144	10.032	20.715	-0.382	4.749	4.854
	$\alpha_\pi=2.5$		*vol.*	*0.596*	*0.8*	1.370	1.596	53.985	10.952	22.438	1.924	5.205	5.809
			bias			-0.019	-0.547	-50.432	10.375	21.412	-0.496	4.973	5.000
		$\alpha_u=1.0$	*vol.*	*0.756*	*0.0*	1.023	1.533	26.705	4.576	9.990	1.413	2.593	3.019
			bias			-0.019	-0.223	-23.054	4.280	9.221	-0.176	2.508	2.586
		$\alpha_u=1.5$	*vol.*	*0.804*	*0.0*	1.049	1.465	38.383	6.916	14.285	1.473	3.508	3.826
ρ=0.7	$\alpha_\pi=1.0$		*bias*			-0.023	-0.352	-34.794	6.501	13.641	-0.296	3.414	3.459
		$\alpha_u=2.0$	*vol.*	*0.704*	*0.0*	1.075	1.606	50.664	10.085	20.399	1.671	4.372	5.029
			bias			-0.023	-0.496	-46.519	9.463	19.549	-0.084	4.260	4.621
		$\alpha_u=2.5$	*vol.*	*0.599*	*0.8*	1.103	1.790	62.871	13.647	27.332	2.020	5.200	6.501
			bias			-0.011	-0.646	-58.007	12.781	26.148	0.385	5.059	6.006

Table 7 - Sensitivity analysis: number of replications

(decreasing gain learning)

Each entry in the table is the ratio between the value of the first or second moment of welfare, output growth and inflation computed on 10,000 and 500 replications. For output growth and inflation not only the mean, but also the maximum, minumum and the median of each set of replications are presented.

NO TRANSPARENCY

Welfare ratio = 0.981

volatility ratios

	max	min	mean	median
Δy	1.057	0.856	1.011	1.018
π	1.379	0.947	1.007	1.011

bias ratios

	max	min	mean	median
Δy	1.118	1.268	1.000	0.981
π	1.169	-0.935	0.973	0.984

PARTIAL TRANSPARENCY

Welfare ratio = 0.985

volatility ratios

	max	min	mean	median
Δy	1.093	0.886	1.010	1.008
π	1.138	0.919	1.004	1.008

bias ratios

	max	min	mean	median
Δy	1.179	1.141	0.974	1.019
π	1.132	-1.174	0.993	1.019

FULL TRANSPARENCY

Welfare ratio = 0.982

volatility ratios

	max	min	mean	median
Δy	1.056	0.927	1.010	1.015
π	1.554	0.904	1.009	1.005

bias ratios

	max	min	mean	median
Δy	2.500	1.080	1.000	0.987
π	0.990	1.324	1.002	0.997

Table 8 - Sensitivity analysis: initial conditions

(constant gain learning)

The table reports the ranking in terms of welfare of the competiting policy rules for a set of values of the Γ matrix (the normalising factor of the generalised stochastic gradient algorithm). The values of Γ considered are (1) the one used in the baseline simulations; (2) Γ scaled up and down by 10 p.p.; (3) Γ multiplied by 1.25 and 0.75; (4) 1.5 and 0.5 times the benchmark value. As in the previous tables, only the initial 140 observations are used in computing the welfare ranking. For each policy rule, the model is simulated 500 times. Each row of the table refers to a policy rule, while the columns are divided into three subgroups, corresponding to the alternative monetary regimes (i.e. no transparency, partial transparency and full transparency). In the last two rows of the table, the Spearman's and Kendall's rank correlation coefficients are shown.

			no transparency							***partial transparency***							***full transparency***						
kΓ where k is:			1	0.9	1.1	3/4	5/4	1/2	3/2	1	0.9	1.1	3/4	5/4	1/2	3/2	1	0.9	1.1	3/4	5/4	1/2	3/2
ρ=0.8			6	7	6	9	5	9	7	9	9	9	9	9	9	9	1	1	1	1	1	1	1
ρ=0.6			2	2	2	1	2	1	2	7	7	6	4	5	3	4	10	10	10	10	9	10	9
ρ=0.5	*α_π=1.0*	*α_u=2.0*	3	3	5	3	6	2	3	5	6	5	6	4	4	3	?	?	?	?	?	?	?
ρ=0.4			7	5	8	4	8	4	5	4	4	2	7	2	6	2	?	?	?	?	?	?	?
ρ=0.3			9	9	9	6	9	5	6	3	3	3	8	3	8	1	?	?	?	?	?	?	?
	α_π=0.5		10	10	10	10	10	10	10	13	13	13	13	13	13	13	8	7	8	8	8	9	8
	α_π=1.0		1	1	1	2	1	3	1	8	8	8	3	8	2	7	4	4	4	4	4	4	4
ρ=0.7	*α_π=1.5*	*α_u=2.0*	8	8	7	8	7	8	9	1	1	1	1	1	1	5	5	5	5	5	5	5	5
	α_π=2.0		12	12	12	11	12	12	12	6	5	7	5	7	5	10	6	6	6	6	7	6	7
	α_π=2.5		13	13	13	13	13	13	13	12	12	12	11	12	11	12	9	9	9	9	10	8	10
		α_u=1.0	11	11	11	12	11	11	11	11	11	11	12	11	12	11	3	3	3	3	3	3	3
ρ=0.7	*α_π=1.0*	*α_u=1.5*	5	6	4	7	4	7	8	2	2	4	2	6	7	6	2	2	2	2	2	2	2
		α_u=2.5	4	4	3	5	3	6	4	10	10	10	10	10	10	8	7	8	7	7	6	7	6
Spearman *ρ (%)*				98	98	90	96	87	93		99	97	80	93	70	80		98	100	100	98	99	98
Kendall *τ (%)*				72	64	49	59	36	72		85	62	72	74	56	51		69	100	100	60	82	60

Fig.1 − Social welfare under rational expectations

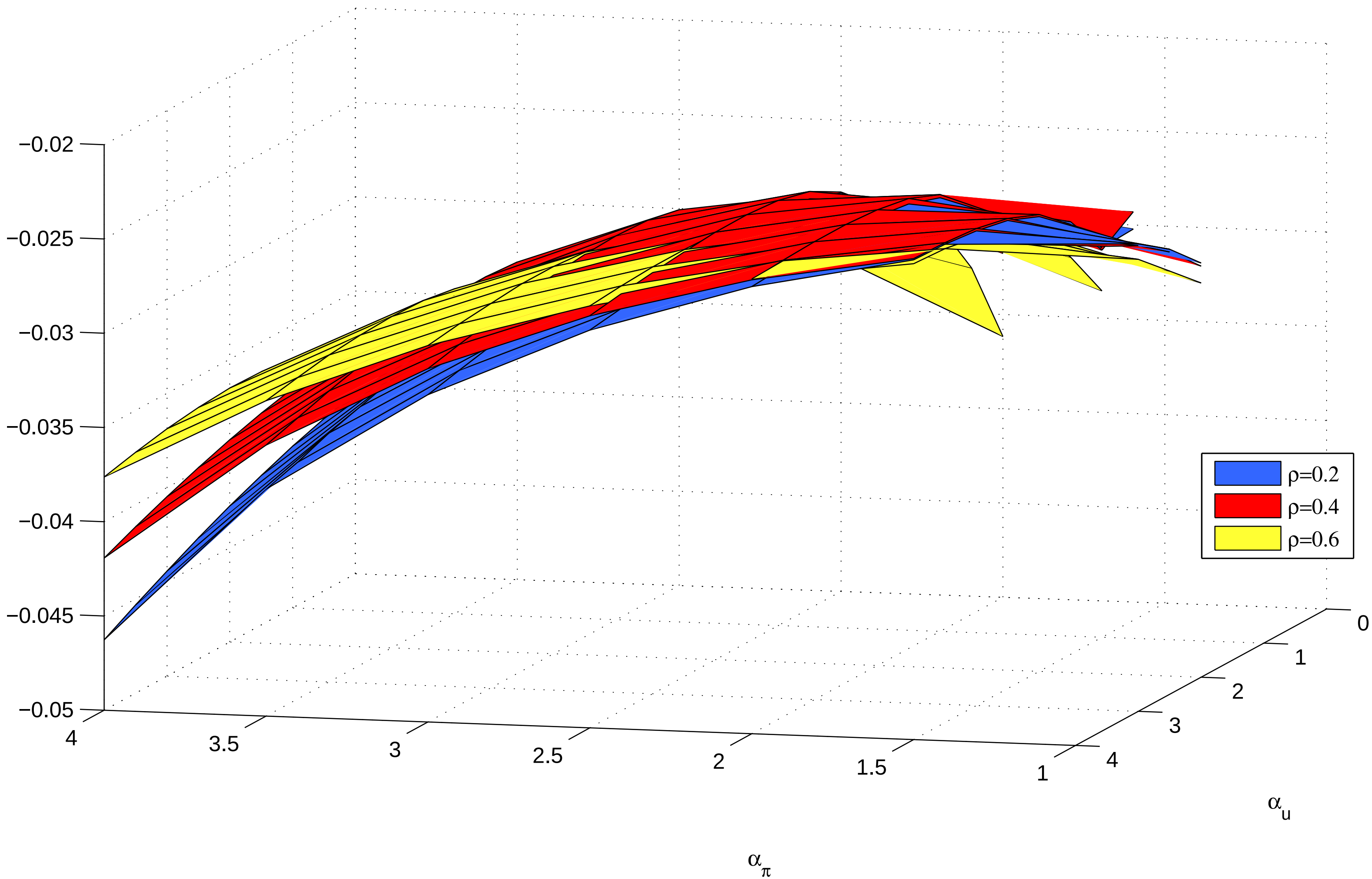

Fig.2a - Interest and exchange rates under no, partial and full transparency (averages)

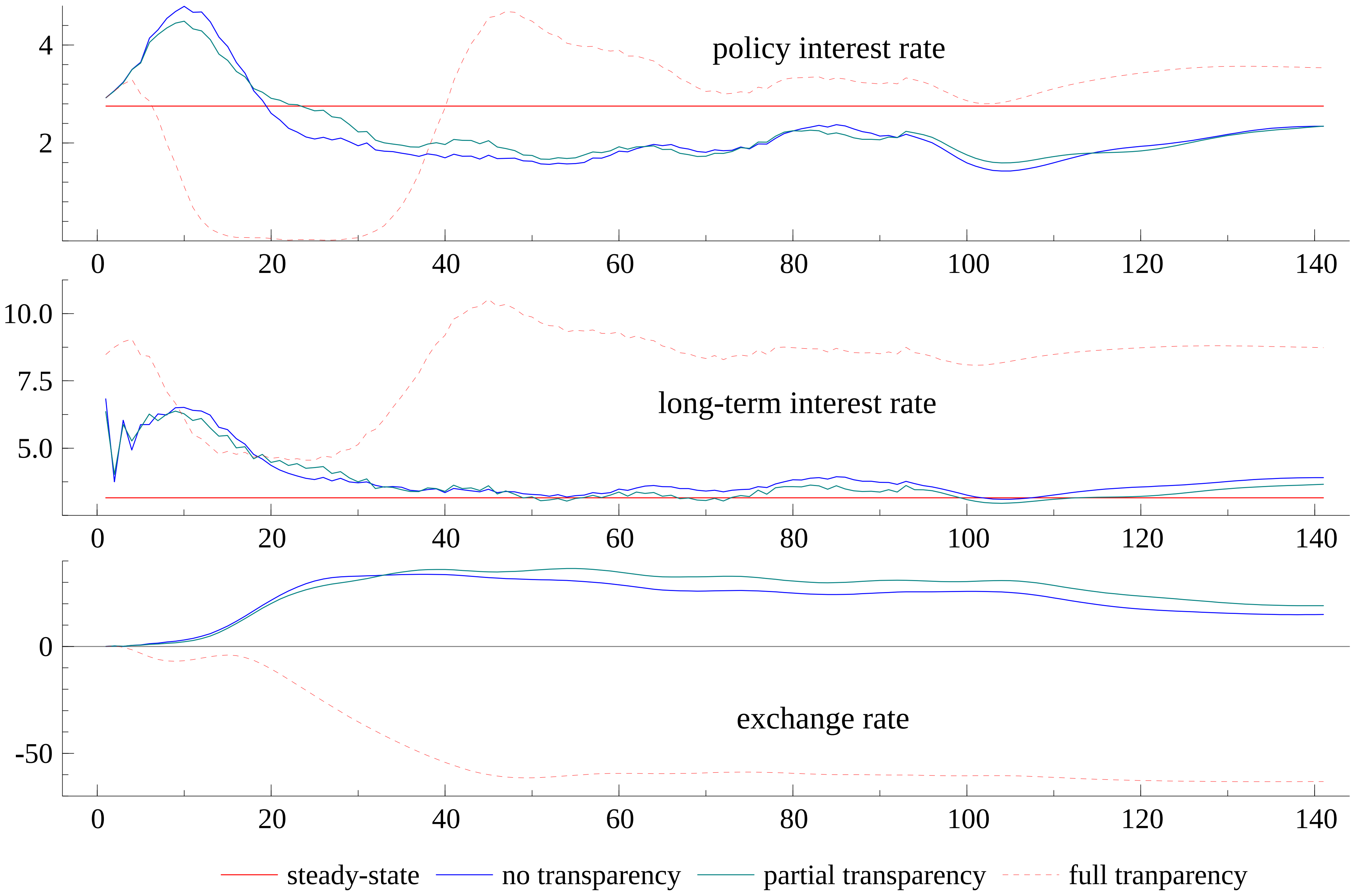

Fig.2b - Interest and exchange rates under no, partial and full transparency (standard deviations)

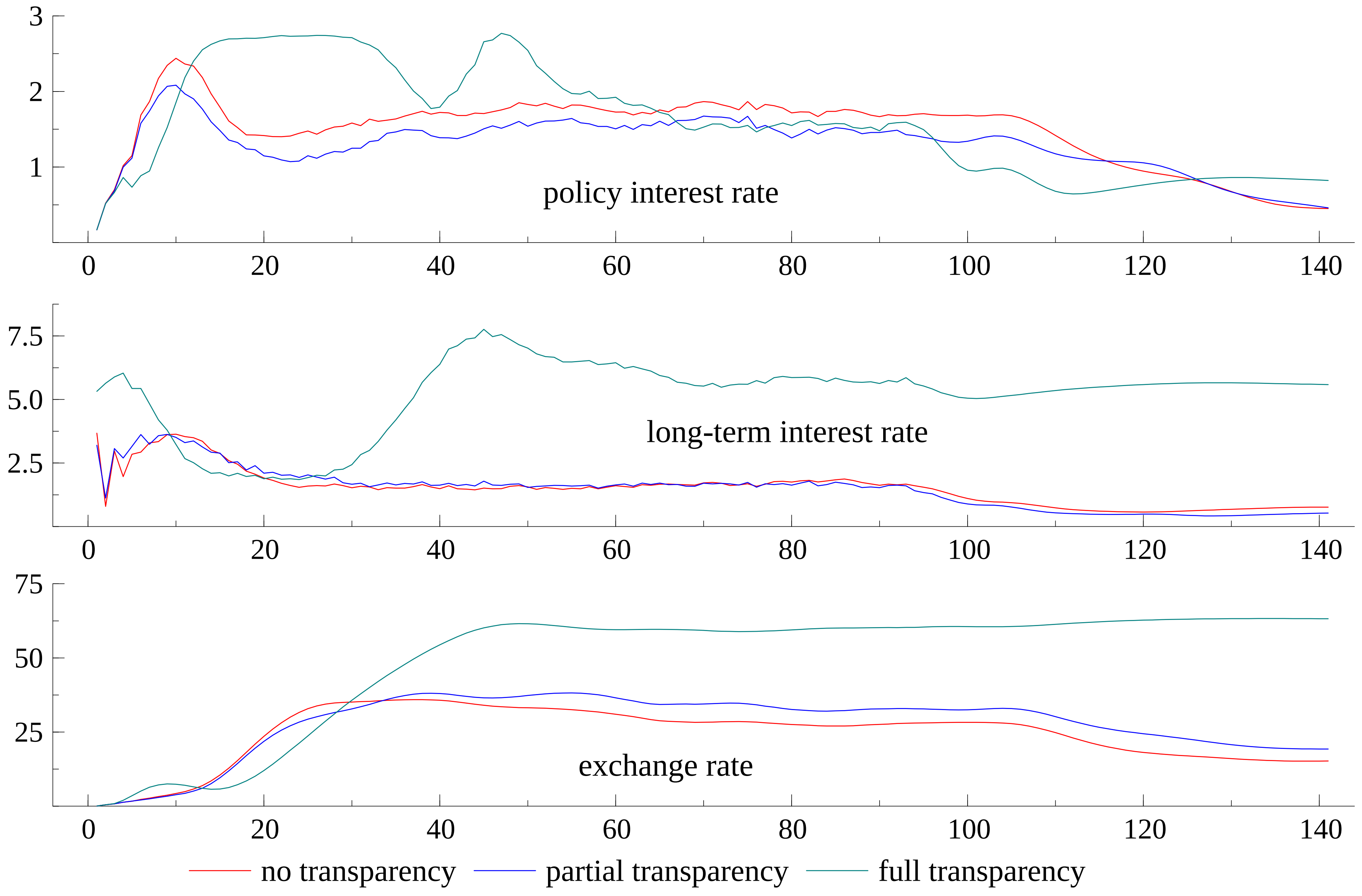

Fig.3a - Central bank's estimates of the natural rates (averages)

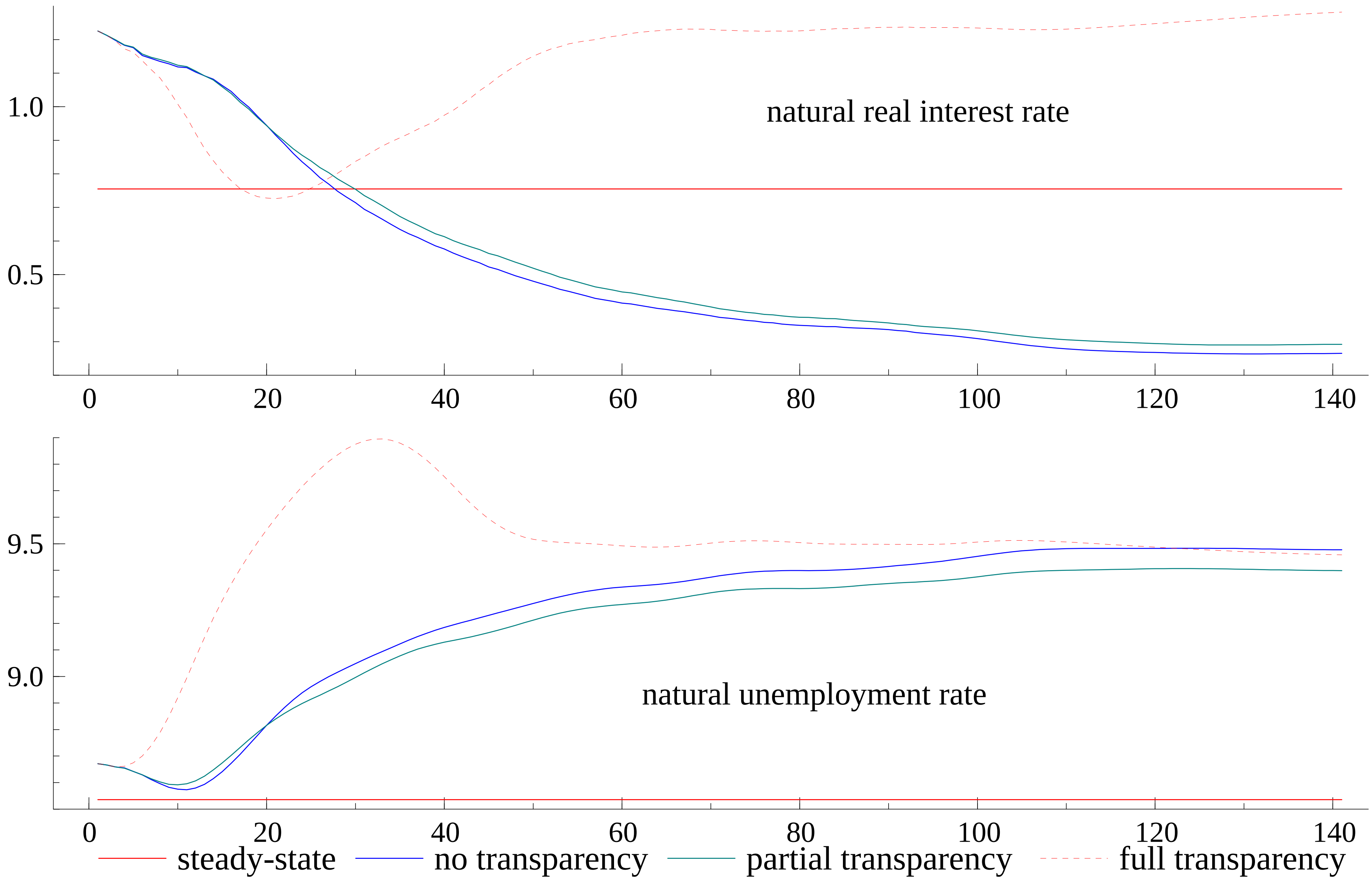

Fig.3b - Central bank's estimates of the natural rates (standard deviations)

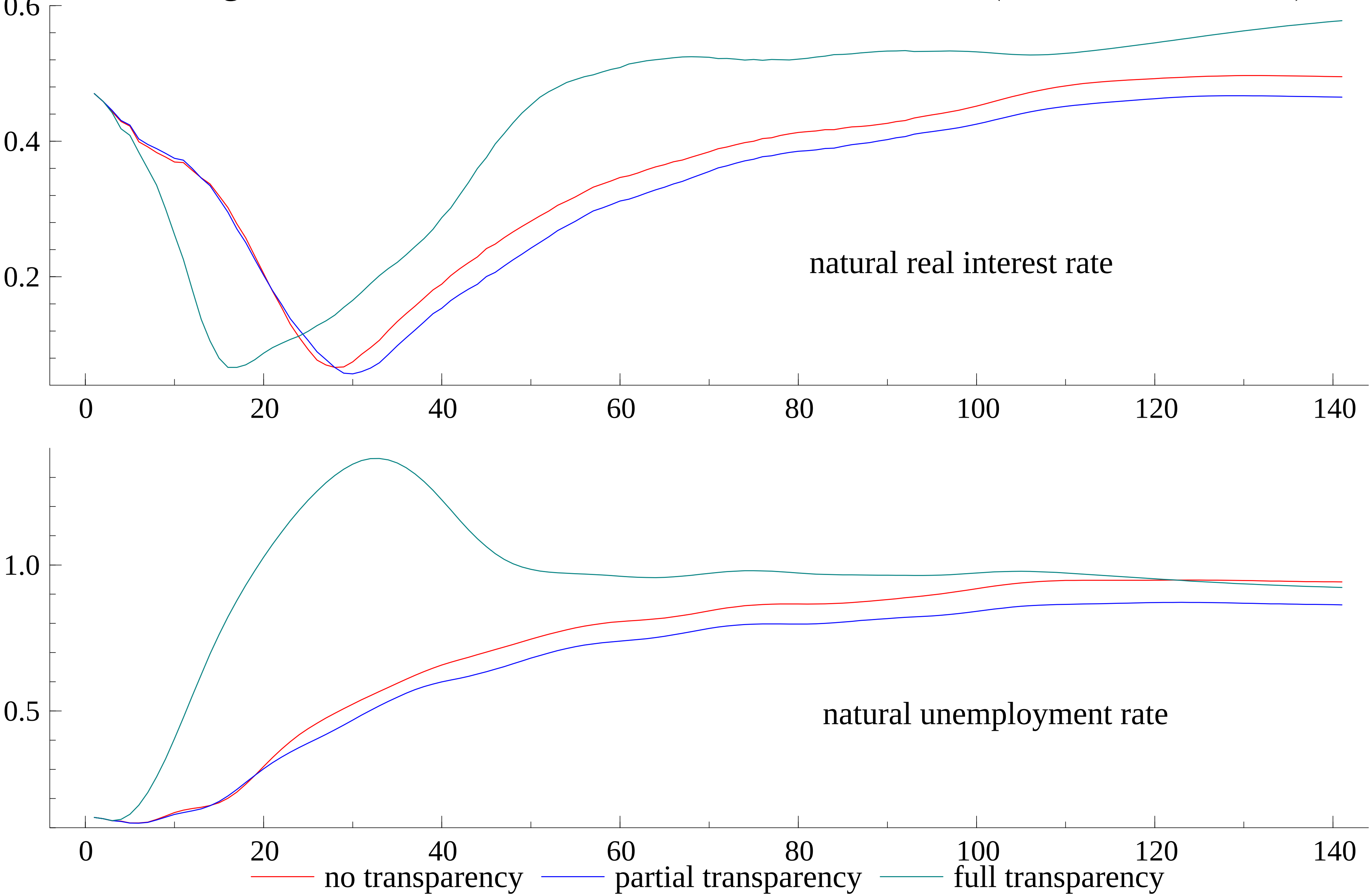

Fig.4 - Coefficients of the PLM of the central bank's policy rule