Learning in a Credit Economy*

Tiziana Assenza†

Catholic University of Milan and University of Amsterdam, CeNDEF.

Michele Berardi

University of Manchester, School of Social Sciences.

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Abstract

In this paper we analyze a credit economy à la Kiyotaki and Moore (JPE, 1997) enriched with learning dynamics, where both borrowers and lenders need to form expectations about the future price of the collateral. We find that under homogeneous learning, the MSV REE for this economy is E-stable and can be learned by agents, but when heterogeneous learning is allowed and uncertainty in terms of a stochastic productivity is added, expectations of lenders and borrowers can diverge and lead to bankruptcy (default) on the part of the borrowers.

Key words: Credit Economy; Bankruptcy; Learning; Heterogeneity

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†Corresponding author: Tiziana Assenza, ITEMQ, Catholic University of Milan, via Necchi 5, 20123 Milan, Italy. Email: tiziana.assenza@unicatt.it; tel.: +39 (0)2 7234 2483.

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1 Introduction

"Bankruptcy – default – was at the center of the discussion. But in the IMF model – as in the models of most of the macroeconomics textbooks written two decades ago – bankruptcy plays no role. To discuss monetary policy and finance without bankruptcy is like Hamlet without the Prince of Denmark" (J. Stiglitz, "Globalization and its discontents", 2002).

Although the rational representative agent hypothesis is still the cornerstone of most of contemporary macroeconomics, the awareness of its limitations is spreading well beyond the circle of more or less dissenting economists. Even in mainstream macroeconomics, in fact, the representative agent is not as eagerly embraced now as it was in the early years of the debate on micro-foundations in the remote '70s, and it is still adopted mainly for lack of a workable alternative.

In behavioral finance, in contrast, bounded rationality and heterogeneous agents models are becoming a serious alternative to the standard rational representative agent approach, as discussed, e.g., in the extensive surveys of LeBaron (2006) and Hommes (2006). Moreover, in the last decade, bounded rationality and adaptive learning have become increasingly important also in macroeconomics: see, e.g., Evans and Honkapohja (2001) and (2006), Branch and Evans (2006), Branch and McGough (2006), Honkapohja and Mitra (2006) and Berardi (2007).

Learning can represent a form of bounded rationality that still maintains some rigor in the expectation formation process and for this reason has received increasing attention in recent years, in particular in the fields of monetary economics and monetary policy. It has also quickly permeated other branches of macroeconomics, but to the best of our knowledge it has not yet been used to model and analyze information asymmetries in financial markets.

It is clear that people are different in many aspects (e.g., degree of rationality, computational capabilities, information sets, financial conditions, etc.) and heterogeneity is a persistent and non-negligible part of any economic story. Imperfect information and information asymmetries can therefore play an important role in credit/debit relationships, and should to be taken into account when analyzing these issues.

In our paper, we introduce bounded rationality in a credit economy á la Kiyotaki and Moore (1997, KM hereafter), where the expected price of a collateral (land) is used by borrowers (farmers) and lenders (gatherers) to take their decisions. In particular, we drop the assumption of rational expectations and consider agents as adaptive learners that must revise their expectations over time on the basis of the new data becoming available. We first assess the learnability (E-stability) of the fundamental equilibrium for this economy, and then extend the analysis to allow for heterogeneity in the learning processes of lenders and borrowers. We will thus be able to consider an extreme consequence of divergent expectations: bankruptcy.

In the original KM framework, given perfect foresight, if the farmer does not work, land will not yield fruit (due to the idiosyncratic nature of the farmer's technology) and he/she will be unable to reimburse debt. In the event of default, the gatherer can seize the farmer's land and sell it. By assumption, the value of the land will be exactly equal to the service of debt (principal and interest) so that the lender's balance sheet will not be affected by bankruptcy. In this framework, therefore, the borrower can in principle default but the gatherer is not bearing the risk of bankruptcy. Contrary to the KM framework, where, given the structure of the model, bankruptcy does not play any role, we will see that once bounded rationality and uncertainty are introduced into the model, bankruptcy becomes an important element of credit/debit relationships.

The main finding of the paper is that in general, under learning dynamics, the economy is attracted (locally) towards the rational expectations equilibrium, but heterogeneous learning dynamics, when coupled with the possibility of bankruptcy, can have important consequences for the economy.

The paper is organized as follows: in section 2 we briefly recall the benchmark KM model; in section 3 we consider the linearized version of the model and study the dynamic properties of the economy under homogeneous learning; in section 4 we introduce heterogeneity between groups (lenders and borrowers) with respect to the expected price of land, we formalize the role of bankruptcy and we determine the dynamic system that represents the economy; in section 5 we study the consequences of introducing heterogeneous learning rules between farmers and gatherers in two different scenarios: with constant and with stochastic productivity respectively; finally, section 6 concludes. All technical details are in the appendix.

2 The benchmark KM economy

A KM economy consists of two groups of agents: those who are financially constrained (farmers) and the unconstrained ones (gatherers). Agents in both groups produce a perishable good (fruit) by means of a technology that uses land and labor.

A farmer is an agent endowed with inalienable human capital. Therefore, he can get from lenders no more than the value of his collateralizable assets. This is the reason of the financing constraint.[1] A gatherer, on the contrary, does not face financing constraints.

An important consequence of the assumption of idiosyncratic farmer's technology is that the gatherer/lender bears the risk of default. If the farmer withdrew his labour, production would not be carried out, i.e., land would bear no fruit. As a consequence, if the farmer is indebted, he may have an incentive to threaten his creditor to withdraw his labour and repudiate debt. Lenders protect themselves against this threat by collateralizing the farmer's land. This is the reason why the farmer faces a *financing constraint*:

$$b_t \leq \frac{q_{t+1}}{R} K_t^F \tag{1}$$

i.e., the loan he gets (b_t) cannot exceed the value of his collateralizable assets $\left(\frac{q_{t+1}K_t^F}{R}\right)$ – the present value of his current landholding – which plays, in this framework, a role analogous to that of net worth or the equity base in Greenwald and Stiglitz (1993, 2003) and entrepreneurs' savings (internal finance) in Bernanke and Gertler (1989, 1990) and Bernanke, Gertler and

[1] On this issue see Hart and Moore (1994, 1998).

Gilchrist (1999). As a consequence, also in a KM economy production depends upon net worth. In fact, the higher is net worth, the softer is the borrowing constraint and the higher are credit extended, investment and production.

KM assume *preference heterogeneity*: farmers are less patient than gatherers, so that the former are also borrowers and the latter play the role of lenders. Moreover KM assume that there is perfect foresight on the future level of the price of land.

There are two types of goods, output ("fruit") and a collateralizable, durable, non-reproducible asset ("land") whose total supply is fixed ($\bar{K}$). Output can be consumed or lent. If lent, each unit of output yields a constant return $R = 1 + r$ where r is the real interest rate. Output is produced by means of a technology which uses land and labour.

By assumption farmers and gatherers have access to different technologies.

The production function of each farmer is: $y_t^F = (a + \bar{c})K_{t-1}^F$ where y_t^F is output of the farmer in t, a and $\bar{c}$ are positive technological parameters and K_{t-1}^F is land of the farmer in $t-1$. $\bar{c}\,K_{t-1}^F$ is output that deteriorates ("bruised fruit") and is therefore non-tradable.

According to (1), the maximum amount of debt a farmer succeeds in getting "today" b_t is such that the sum of principal and interest Rb_t is equal to the value of the farmer's land when the debt is due, i.e., $q_{t+1}K_t^F$, where q_{t+1} is the (real) price of land at time $t+1$.

The farmer's preferences are represented by a linear utility function

$$U_t^F = \sum_{s=0}^{\infty} \left(\beta^F\right)^s c_{t+s}^F \tag{2}$$

where $\beta^F = \dfrac{1}{1+\rho^F}$ is the farmer's discount factor and c_t^F is his consumption at time t.

The farmer faces also a *flow-of-funds constraint*:

$$y_t^F + b_t \leq q_t(K_t^F - K_{t-1}^F) + Rb_{t-1} + c_t^F, \tag{3}$$

where $(K_t^F - K_{t-1}^F)$ is the farmer's investment in landholding. Preferences are modelled in such a way that farmers consume only non-tradable output, i.e. $c_t^F = \bar{c}\, K_{t-1}^F$. The farmer maximizes (2) subject to the financing constraint and the flow of funds constraint. Solving his optimization problem we get

$$\mu_t K_t^F = a K_{t-1}^F, \tag{4}$$

where $\mu_t = q_t - \dfrac{q_{t+1}}{R}$ is the *downpayment,* i.e., the amount the farmer has to put aside as internal finance to acquire one unit of land. From (4), it follows that the revenues obtained by selling (non-bruised) fruit (aK_{t-1}^F) are employed as downpayment $(\mu_t K_t^F)$. The farmer's demand for land, therefore, is:

$$K_t^F = \frac{a}{\mu_t} K_{t-1}^F. \tag{5}$$

The production function of each gatherer is: $y_t^G = G(K_{t-1}^G)$ where y_t^G is output of the gatherer in t, $G(.)$ is a well behaved production function and

K_{t-1}^G is land of the gatherer in $t-1$. Also the gatherer's preferences are specified by a linear utility function

$$U_t^G = \sum_{s=0}^{\infty} \left(\beta^G\right)^s c_{t+s}^G$$

where $\beta^G = \dfrac{1}{1+\rho^G}$ is the gatherer's discount factor and c_t^G is his consumption at time t. The gatherer faces only a *flow-of-funds constraint*:

$$y_t^G + Rb_{t-1} \leq q_t(K_t^G - K_{t-1}^G) + b_t + c_t^G, \tag{6}$$

where $(K_t^G - K_{t-1}^G)$ is the gatherer's investment in landholding.

Solving the gatherer's optimization problem and assuming, for the sake of simplicity and without loss of generality, that population consists only of one farmer and one gatherer so that $K_t^F = \bar{K} - K_t^G$ we get

$$\mu_t = \frac{G'\left(K_t^G\right)}{R}, \tag{7}$$

where $G'\left(K_t^G\right)$ is the gatherer's marginal productivity.

Substituting this expression into (5) and rearranging we end up with

$$K_t^F = \frac{Ra}{G'\left(K_t^G\right)} K_{t-1}^F, \tag{8}$$

which is a non-linear difference equation in the state variable K_t^F, where $K_t^G = \bar{K} - K_t^F$.

Denoting with a star the steady state value of a variable, plugging the steady state condition $K_t^F = K_{t-1}^F = K^{*F}$ into (5) we obtain $\mu^* = a$. Since

$\mu^* = q^* \left(1 - \frac{1}{R}\right)$, we have $q^* = a\frac{R}{R-1}$ and $b^* = \frac{q^* K^{*F}}{R}$, which imply $b^* = a\frac{K^{*F}}{R-1}$. Substituting these steady state conditions into (7) we obtain $K^{*F} = G'^{-1}(Ra)$. Hence $b^* = \frac{aG'^{-1}(Ra)}{R-1}$ and $Rb^* = a\frac{RK^*}{R-1} = q^* K^*$.

KM log-linearize the economy in the neighborhood of the steady state and show that small shocks to the technological parameter a can produce large and persistent fluctuations in output and asset prices. In their model, in fact, the durable, non reproducible asset (land) plays the dual role of a factor of production for both constrained and unconstrained agents and of collateralizable wealth for financially constrained agents. Therefore the price of the asset affects the borrowers' financing constraint and, at the same time, the size of the borrowers' credit limits feeds back on asset prices.

3 Homogeneous learning

Starting from the economy described above, we now drop the rational expectations (perfect foresight) assumption and endow our agents with an adaptive learning scheme that they use in order to form expectations about the future price of the collateral. We then analyze whether agents would be able to learn over time the correct value of the parameters and thus converge towards rationality. In order to carry out the learning analysis, we first need to linearize the above economy around its steady state.

Using a Cobb-Douglas specification for the production function of the gatherer ($G\left(K_t^G\right) = 2\sqrt{K_t^G}$), and starting from equations (5) and (8), the linearized system representing the dynamics for the economy can be expressed as (see the appendix for details):

$$q_t = \gamma_0 + \gamma_1 K^F_{t-1} + \gamma_2 E_t q_{t+1} \tag{9}$$

$$K^F_t = \gamma_3 + \gamma_4 K^F_{t-1} \tag{10}$$

with:

$$\begin{gathered}
\gamma_0 = \frac{2a}{(aR)^2 \bar{K} + 1} > 0 \\
\gamma_1 = \frac{a^3 R^2}{(aR)^2 \bar{K} + 1} > 0 \\
\gamma_2 = \frac{1}{R} < 1 \\
\gamma_3 = \frac{\left[(aR)^2 \bar{K} - 1\right]^2}{(aR)^2 \left[(aR)^2 \bar{K} + 1\right]} > 0 \\
\gamma_4 = \frac{2}{(aR)^2 \bar{K} + 1} > 0
\end{gathered} \tag{11}$$

which under RE has the minimum state variables (MSV) solution

$$q_t = \Psi_0 + \Psi_1 K^F_{t-1} \tag{12}$$

with

$$\Psi_0 = \frac{\gamma_0}{1-\gamma_2} + \frac{\gamma_1\gamma_2\gamma_3}{(1-\gamma_2)(1-\gamma_2\gamma_4)} \tag{13}$$

$$\Psi 1 = \frac{\gamma_1}{1-\gamma_2\gamma_4}. \tag{14}$$

Note that for this economy to be stationary, we need $\gamma_4 < 1$, i.e., $(aR)^2 \bar{K} > 1$, a restriction that we impose on the structural parameter values.

If we consider agents as adaptive learners, they will have to estimate from data the reduced form equations representing the equilibrium.[2] We assume here that agents (both farmers and gatherers) in their learning process use a model compatible with the law of motion for the economy under RE;[3] therefore they recurrently estimate the relationships (10) and (12) and use them to form their expectations, which inserted into the forward looking equation for q_t determine the actual law of motion (ALM) for the value of land. We assume that agents recurrently estimate the relevant equations using an adaptive learning scheme such as recursive least squares (RLS), and we resort to the E-stability principle which determines a correspondence between convergence of such a learning process in real time and the E-stability of an associated system of differential equations.[4]

The estimated equations (also called perceived laws of motion - PLMs) are

$$q_t = \phi_0 + \phi_1 K_{t-1}^F \tag{15}$$

$$K_t^F = \theta_0 + \theta_1 K_{t-1}^F \tag{16}$$

and we say that learning converges towards rational (perfect foresight) expectations if, over time, the parameter estimates converge to the corresponding values in (11) - (14). Of course estimates for $\theta' s$ converge, as K^F is a

[2] We introduce learning on the linearized model representing the optimality conditions for agents, as it is common in the vast majority of the learning literature in macroeconomics. For a discussion of this modelling strategy and its relation with an alternative approach that has been recently proposed, see Honkapohja, Mitra and Evans (2003).

[3] This is a common assumption in the learning literature. One could consider also perceived laws of motion that are misspecified, but we prefer to keep the departure from rationality at the minimum: if the equilibrium is not learnable with the correct model, it will not be learnable with any misspecified models.

[4] For a detailed analysis of the E-stability principle, see Evans and Honkapohja (2001).

purely backward looking process with no expectations feedback on it, that can easily be estimated using least squares techniques.

As for the equation for price, using (15) and (16) we can compute the expectations to be inserted into (9) and obtain the ALM for q_t:

$$q_t = (\gamma_0 + \gamma_2\phi_0 + \gamma_2\gamma_3\phi_1) + (\gamma_1 + \gamma_2\gamma_4\phi_1)\, K^F_{t-1}. \tag{17}$$

Mapping the PLM into the ALM we obtain the ODEs for ϕ_0 and ϕ_1, whose fixed points represent equilibria for the economy under learning dynamics. The relevant ODEs are

$$\begin{aligned} \dot{\phi}_0 &= \gamma_0 + \gamma_2\phi_0 + \gamma_2\gamma_3\phi_1 - \phi_0 \\ \dot{\phi}_1 &= \gamma_1 + \gamma_2\gamma_4\phi_1 - \phi_1 \end{aligned}$$

whose unique fixed point corresponds to the MSV REE of (12).

Learnability (E-stability) obtains iff these differential equations are stable, i.e., iff $(\gamma_2 - 1)$ and $(\gamma_2\gamma_4 - 1)$ are negative. The first is always realized, since $\gamma_2 = 1/R < 1$. As for the second condition, it can be boiled down to

$$(aR)^2\, \bar{K} > \frac{2-R}{R}$$

which is realized for any parameter values that satisfy the restriction imposed above on γ_4 for stationarity of capital.

4 A KM economy with heterogeneous expectations

In this section we model a simple KM economy under the assumption that agents have heterogeneous expectations about the future level of the asset price. We denote with $q_{t+1}^{e,F}$ and $q_{t+1}^{e,G}$ respectively the expectations in t on the level of the asset price in $t+1$ for the farmer and the gatherer.

As in the benchmark KM economy, we assume preferences heterogeneity between the farmer and the gatherer. The farmer's preferences are represented by a linear utility function

$$U_t^F = \sum_{s=0}^{\infty} \left(\beta^F\right)^s c_{t+s}^F \tag{18}$$

where $\beta^F = \dfrac{1}{1+\rho^F}$ is the farmer's discount factor and c_t^F is his consumption at time t. The farmer's flow of funds constraint is defined as in KM by (3). Moreover, we assume that the farmer will accept any amount of funds that the gatherer is willing to lend, i.e. that the collateral constraint will specify to:

$$b_t \leq \frac{q_{t+1}^{e,G}}{R} K_t^F. \tag{19}$$

This assumption is consistent with the original KM framework where given the farmer's higher discount factor he does not want to postpone production and invests as much as possible.

Now that expectations could differ between farmer and gatherer, we need to take into account the possible consequences on financial incentives.

In order to explicitly introduce the role of heterogeneity in a simple credit economy à la KM we need to specify a voluntary bankruptcy for farmers, one that reflects the incentive for the borrower to pay back his debt to the lender. The intuition is simple: when the borrower needs to decide whether to pay back his debt, he compares the value of the debt with the expected value of the land (which stands as collateral):

$$b_t \lesseqgtr EV^F_{K^F_t},$$

where $EV^F_{K^F_t} = \frac{q^{e,F}_{t+1}}{R} K^F_t$ is the borrower's expected value of land. The farmer decides to pay back the debt only if the value of the debt is equal or smaller than his expected value of the collateral. In the opposite case the farmer will find it convenient to default on his debt and let the lender grab the collateral. Since the credit granted to the borrower depends on the lender's expectations on the value of the collateral, we will see formally below that the voluntary bankruptcy condition reduces to:

$$q^{e,F}_{t+1} < q^{e,G}_{t+1}. \tag{20}$$

If this condition holds, the value of the debt to be repaid at time $t+1$, i.e., $b_t = \frac{q^{e,G}_{t+1}}{R} K^F_t$, is higher than the farmer's expected value of the land $(\frac{q^{e,F}_{t+1}}{R} K^F_t)$, and therefore he would decide to default on its debt (and this decision would be revealed to the lender only at time $t+1$).[5]

In addition to voluntary bankruptcy, there is of course the possibility of

[5] The farmer needs to decide at time t whether or not to default at time $t+1$, since he must decide whether or not to invest in new land.

an involuntary bankruptcy, which can happen when there is a large negative shock to productivity, so that the price of land falls largely and unexpectedly. This is an additional constraint that we chose not to put into our economy. This bankruptcy condition would reduce in fact to a simple threshold for the productivity shock, which would not produce any interesting interaction with the expectation formation processes and would be highly sensitive to the parameterization of the stochastic process for productivity. So while the involuntary bankruptcy can surely represent an additional real-life reason why the economy might not converge towards an equilibrium, we will leave this case out of our analysis.

Summarizing, at time t the farmer pays back the debt of the previous period (Rb_{t-1}) before getting a new loan (b_t) and producing output ($y_t^F = (a+\bar{c}) K_{t-1}^F$). Given the bankruptcy condition he then decides whether to use the loan to invest in land, that he will employ to produce output for the next period ($t+1$), or to "to take the money and run" and consequently repudiate the debt in $t+1$.

In this setting the farmer's maximization problem can be formalized as follows:

$$\max_{c_t^F, b_t} U_t^F = \sum_{s=0}^{\infty} \left(\beta^F\right)^s c_{t+s}^F$$

$$\text{s.t.} \qquad q_t \left(K_t^F - K_{t-1}^F\right) + Rb_{t-1} + c_t^F \leq y_t^F + b_t$$

$$b_t \leq \frac{q_{t+1}^{e,G}}{R} K_t^F$$

$$\frac{q_{t+1}^{e,F}}{R} K_t^F \leq b_t$$

where the last constraint says that the farmer requires a loan at least equal to his expected (present) value of the collateral.

It is immediate to note that if we relax the assumption of heterogeneity in expectations and assume perfect foresight we are back to the benchmark KM model.[6]

Given that $q_t^{e,i} = q_t$, with $i = F, G$, i.e. the price at time t is known to both groups of agents, the Lagrangian specializes as:

$$\begin{aligned} L &= \sum_{s=0}^{\infty} \left(\beta^F\right)^s c_{t+s}^F + \sum_{s=0}^{\infty} \lambda_{t+s}^F \left[(a+\bar{c}) K_{t-1+s}^F + b_{t+s} - q_{t+s}^{e,F}\left(K_{t+s}^F - K_{t-1+s}^F\right) - R b_{t-1+s} - c_{t+s}^F\right] + \\ &\quad + \sum_{s=0}^{\infty} \phi_{t+s}^F \left(\frac{q_{t+1+s}^{e,G}}{R} K_{t+s}^F - b_{t+s}\right) + \sum_{s=0}^{\infty} \gamma_{t+s}^F \left(b_{t+s} - \frac{q_{t+1+s}^{e,F}}{R} K_{t+s}^F\right) \end{aligned}$$

From which we derive the following *FOCs:*

(i) $\dfrac{\partial L}{\partial c_t^F} = 0 \Rightarrow 1 - \lambda_t^F = 0$

(ii) $\dfrac{\partial L}{\partial c_{t+1}^F} = 0 \Rightarrow \beta^F - \lambda_{t+1}^F = 0$

(iii) $\dfrac{\partial L}{\partial b_t} = 0 \Rightarrow \lambda_t^F - \phi_t^F + \gamma_t^F - R\lambda_{t+1}^F = 0$

(iv) $\dfrac{\partial L}{\partial b_{t+1}} = 0 \Rightarrow \lambda_{t+1}^F - \phi_{t+1}^F + \gamma_{t+1}^F - R\lambda_{t+2}^F = 0$

From the *FOCs* above, if all constraints are binding with equality, $\gamma_{t+s}^F > 0$, $\phi_{t+s}^F > 0$, $\lambda_{t+s}^F > 0$, for $s = 0, \ldots.\infty$, and we are back to the homogeneous expectations case where $b_t = \dfrac{q_{t+1}^{e,G}}{R} K_t^F = \dfrac{q_{t+1}^{e,F}}{R} K_t^F$ and therefore $q_{t+1}^{e,G} = q_{t+1}^{e,F}$.

Consider instead the case in which the last condition is not binding, i.e., $\gamma_t^F = 0$, which implies $\phi_t^F = 1 - \beta^F R$; since the financial constraint it is bind-

[6] In order to avoid having to deal also with the possiblity of unvoluntary banckruptcy, which would add further difficulties without adding anything relevant to the problem we want to analyse, we allow for renegotiation of debt between farmer and gatherer in case the expectations of the gatherer turn out to be wrong ex post.

ing iff $\phi_t^F > 0$, we have $R < \dfrac{1}{\beta^F}$.[7] It follows that $b_t = \dfrac{q_{t+1}^{e,G}}{R} K_t^F > \dfrac{q_{t+1}^{e,F}}{R} K_t^F$, i.e. $q_{t+1}^{e,F} < q_{t+1}^{e,G}$. In this case agents have heterogeneous expectations on the price of land, and in particular the value of the debt is greater than the farmer's expected value of the collateral: in this case the farmer will have an incentive to repudiate the debt in the next period, i.e. he will go bankrupt in $t+1$ and the gatherer will grab his land. The last constraint in the farmer's optimization problem can therefore be interpreted as a (voluntary) bankruptcy condition.

By considering the above problem and solving for the farmer's demand for land we get:

$$K_t^F = \frac{a}{\mu_t^{e,G}} K_{t-1}^F \tag{21}$$

where $\mu_t^{e,G} = q_t - \dfrac{q_{t+1}^{e,G}}{R}$ is the expected downpayment the farmer needs to put aside in order to pay back the debt. Note that, in this framework, the amount of downpayment depends on the gatherer's expectations.

As for the gatherer, we build the problem exactly as in the original KM model with the exception that under heterogeneous expectations the perfect foresight value of land will be replaced by the gatherer's expectations. For the sake of clarity, we write also the gatherer's maximization problem, in which he seeks to maximize his utility function subject to the flow of funds constraint:

$$\max_{c_t^G, b_t, K_t^G} U_t^G = \sum_{s=0}^{\infty} \left(\beta^G\right)^s c_{t+s}^G$$

[7] This is exactly the same condition obtained by KM for the financial constraint to be binding.

$$\text{s.t.} \quad q_t(K_t^G - K_{t-1}^G) + b_t + c_t^G \leq y_t^G + Rb_{t-1}$$

$$y_t^G = G\left(K_{t-1}^G\right)$$

The Lagrangian thus specializes as:

$$L = \sum_{s=0}^{\infty} \left(\beta^G\right)^s c_{t+s}^G + \sum_{s=0}^{\infty} \lambda_{t+s}^G \left[G\left(K_{t-1+s}^G\right) + Rb_{t-1+s} - q_{t+s}^{e,G}(K_{t+s}^G - K_{t-1+s}^G) - b_{t+s} - c_{t+s}^G\right]$$

We derive the following *FOCs:*

(i) $\dfrac{\partial L}{\partial c_t^G} = 0 \Rightarrow 1 - \lambda_t^G = 0$

(ii) $\dfrac{\partial L}{\partial c_{t+1}^G} = 0 \Rightarrow \beta^G - \lambda_{t+1}^G = 0$

(iii) $\dfrac{\partial L}{\partial b_t} = 0 \Rightarrow -\lambda_t^G + \lambda_{t+1}^G R = 0$

(iv) $\dfrac{\partial L}{\partial b_{t+1}} = 0 \Rightarrow -\lambda_{t+1}^G + \lambda_{t+2}^G R = 0$

(v) $\dfrac{\partial L}{\partial K_t^G} = 0 \Rightarrow -\lambda_t^G q_t + \lambda_{t+1}^G \left[G'\left(K_t^G\right) + q_{t+1}^{e,G}\right] = 0.$

From the above *FOCs* we get:

$$\mu_t^{e,G} = \frac{G'\left(K_t^G\right)}{R}. \tag{22}$$

Hence, our economy is described by the following dynamic system, which represent respectively the demand for land for farmers and gatherers:

$$\begin{cases} K_t^F = \dfrac{a}{\mu_t^{e,G}} K_{t-1}^F \\ \mu_t^{e,G} = \dfrac{G'\left(K_t^G\right)}{R}. \end{cases}$$

5 Heterogeneous learning

Now that we have a framework that allows for heterogeneity of expectations, we can let farmers and gatherers learn independently from each other. There are a number of different ways in which heterogeneity in learning could be modelled. Agents could have different initial beliefs, they could use different models (PLMs) or different learning algorithms (and of course any combination of the three). We will consider only the last possibility, and in particular we will allow agents to use different gain parameters in their learning schemes.

In order to rewrite the model under heterogeneous expectations, we need to start from the demand for land for farmers and gatherers. The farmers' demand for land is

$$K_t^F = \frac{a}{\mu_t^{e,G}} K_{t-1}^F. \tag{23}$$

Note that the farmer's demand for land depends on gatherer's expectations, because these are those that determine the amount of credit the the farmer will have available for the purchase of land.

The gatherers' demand for land is

$$K_t^G = G'^{-1}\left(R\mu_t^{e,G}\right). \tag{24}$$

To close the model we also need the equilibrium condition for the market of land: $K_t^F + K_t^G = \bar{K}$.

Substituting (23) and (24) in the equilibrium condition for the land market we end up with the following relation:

$$K_{t-1}^F = \frac{Rq_t - q_{t+1}^{e,G}}{aR}\bar{K} - \frac{\left(Rq_t - q_{t+1}^{e,G}\right)^{-1}}{aR}.$$

We then linearize this equation around the steady state for q, i.e. $q_t = q_{t+1} = \dfrac{aR}{R-1}$ and obtain the forward-looking equation for the price of land

$$q_t = \gamma_0 + \gamma_1 K_{t-1}^F + \gamma_2 q_{t+1}^{e,G} \qquad (25)$$

which has the same form and parameter values as under homogeneous learning, except that now the relevant expectations are those of the gatherer alone. The MSV REE for this economy will therefore have once again the form

$$q_t = \phi_0 + \phi_1 K_{t-1}^F$$

which is the functional form we assume all agents will use in their expectations formation process. Farmers and gatherers therefore recursively estimate parameters ϕ_0 and ϕ_1 and use the most recent estimates to form their expectations about q_{t+1}. They will also need an estimate for K_t^F (as this variable is still to be determined at the time agents form their expectations for q_{t+1}), which is obtained by estimating an AR(1) equation for K_t^F (consistent with the equilibrium law of motion for capital) with parameters θ_0 and θ_1: since there is no expectations feedback on this law of motion, this is a simple least squares estimation and the learning process will converge asymptotically to γ_3 and γ_4. Expectations for the price of land for farmers and gatherers can therefore be written as:

$$\begin{aligned} q^{e,F}_{t+1} &= \phi^F_0 + \phi^F_1 \left(\gamma_3 + \gamma_4 K^F_{t-1}\right) \\ q^{e.G}_{t+1} &= \phi^G_0 + \phi^G_1 \left(\gamma_3 + \gamma_4 K^F_{t-1}\right). \end{aligned}$$

Inserting these expectations into the model (25) we can obtain the ensuing ALM for q_t

$$q_t = \left(\gamma_0 + \gamma_2\phi^G_0 + \gamma_2\gamma_3\phi^G_1\right) + \left(\gamma_1 + \gamma_2\gamma_4\phi^G_1\right) K^F_{t-1}$$

and then derive the T-maps from the PLMs to the ALM for the two agents, which give the system of ODEs governing the evolution of the estimated parameters in notional time. For the gatherer we have

$$\begin{aligned} \dot{\phi}^G_0 &= \gamma_0 + \gamma_2\phi^G_0 + \gamma_2\gamma_3\phi^G_1 - \phi^G_0 \\ \dot{\phi}^G_1 &= \gamma_1 + \gamma_2\gamma_4\phi^G_1 - \phi^G_1 \end{aligned}$$

and for the farmer

$$\begin{aligned} \dot{\phi}^F_0 &= \gamma_0 + \gamma_2\phi^G_0 + \gamma_2\gamma_3\phi^G_1 - \phi^F_0 \\ \dot{\phi}^F_1 &= \gamma_1 + \gamma_2\gamma_4\phi^G_1 - \phi^F_1. \end{aligned}$$

Note that there is no feedback from the farmer's expectations (and learning) to the ALM: the learning process for the farmer converges iff the one for the gatherer does, and to the same values. As for the gatherer, the E-stability condition for his learning process is the same as the one we found for homogeneous learning, i.e.,

$$a^2 \bar{K} > \frac{2-R}{R^2}.$$

Since all agents use the same learning algorithms and have the same initial beliefs by assumption, expectations of the two groups remain always the same and the bankruptcy condition never becomes binding in this setting.

5.1 Stochastic productivity and constant gain learning

Up to this point we have been working with a deterministic economy, where no intrinsic uncertainty was present. We now consider the more interesting case in which productivity is stochastic, so that one of the fundamental parameters of our economy keeps changing over time. In particular, we will consider the case in which productivity follows a stationary stochastic process with damping parameter $\rho \in (0,1)$ and innovation $e_t \sim N(1, \sigma_e^2)$:

$$a_t = \bar{a} + \rho a_{t-1} + e_t,$$

where the intercept $\bar{a}$ has been introduced to ensure that the condition for E-stability is satisfied over time.

This change has important implications for the learning analysis. In an economy undergoing changes in its fundamentals, in fact, agents should use a learning scheme that allows for parameter drift, such as a constant gain algorithm, which discounts past observations and gives relatively more importance to new data, thus keeping track of the structural changes in the economy. Therefore, agents need to choose an appropriate value for the gain parameter (or, equivalently, choose the length of the data windows in their regressions), and this is the route through which heterogeneity can enter into the expectations formation processes, since different agents could use different gain parameters. The recursive learning algorithms for the two agents are:

$$\begin{aligned}
\phi_t^G &= \phi_{t-1}^G + g^G \left(R_t^G\right)^{-1} z_t \left(q_t - q_t^{e,G}\right) \\
R_t^G &= R_{t-1}^G + g^G (z_t z_t' - R_{t-1}^G) \\
\phi_t^F &= \phi_{t-1}^F + g^F \left(R^F\right)^{-1} z_t \left(q_t - q_t^{e,F}\right) \\
R_t^F &= R_{t-1}^F + g^F (z_t z_t' - R_{t-1}^F)
\end{aligned}$$

where z_t is the vector of the regressors in the estimated equation

$$z_t = \begin{bmatrix} 1 \\ K_{t-1}^F \end{bmatrix}$$

and g^F and g^G are the gain parameters respectively for the farmer and the gatherer.

Even with a time-invariant economy, parameter estimates coming from a constant gain algorithm can not point-converge to a single value, but they could still converge in distribution around the true value. Once a time-varying productivity in introduced, though, the economic structure is evolving over time, and therefore no convergence at all can be expected. Agents can only hope to "follow" the economy with their (noisy) estimates.

Simulations show that this is in fact what happens if gatherer and farmer use the same gain parameter in their algorithm.

In this heterogeneous setting, though, there is no guarantee that farmer and gatherer will independently choose the same gain parameter in their learning algorithms, and so we consider the more general case in which the gain parameters are different. The farmer could discount past data more or less heavily than the gatherer and in this case, even if the two agents start out with the same initial beliefs, their estimated parameters, and therefore their expectations, will sooner or later diverge.

The constant gain indicates how many data periods agents use in their regressions. A gain of .05, for example, means that agents are using 20 periods, while a gain of .055 corresponds roughly to 18 periods. Even with such a small difference, simulations show that the expectations of the two groups diverge quickly and this can have drastic consequences for the economy by inducing the borrower to default. Even though the learning algorithms are potentially able to keep track of the changes in the economy and the estimated parameters would follow (stochastically) the evolution of the true values, the borrower/lender relationship comes to an abrupt end when the bankruptcy condition becomes binding and the borrower decides to default on the basis of his expectations about the future price of the collateral.

The actual timing of the bankruptcy in the simulations we run depends critically on the difference in the gain parameters and on the variance of the productivity shock that displaces the economy. The bigger is the difference in the gain parameters and the larger the productivity shocks, the sooner bankruptcy arises. With different gains in the learning algorithms, in fact, one of the two agents is able to keep track of the changes in the economy faster than the other: therefore the greater is the difference in the gains and the larger are the shocks that hit the economy, the sooner the estimated parameters for the two agents, and therefore their expectations, will diverge, thus opening the route to bankruptcy.

When a borrower decides to default, the relationship borrower/lender comes to an end; in this economy, where all the borrowers are alike, this would mean that all the borrowers default, the gatherers seize all the land available and farmers disappear from the economy.[8]

In a richer, and more realistic, setting there would be heterogeneity also among borrowers themselves, as well as new entries and exits of borrowers (and lenders) over time, so that the bankruptcy condition would realistically induce a turnover in the borrower/lender relationships. It is sensible to suppose that, under imperfect information, a borrower that has defaulted on a previous debt could still manage to find a lender willing to grant him a new loan, but in a repeated game the reputation of the borrowers would soon become public information available to all lenders and it would be extremely difficult for a "bad" borrower to find new lenders willing to engage in economic relations with him. We do not take these reputations considerations into account in our analysis here, but acknowledge their potential impact on the decision of the borrower to go bankrupt.

[8] To avoid this extreme outcome, it would be enough to introduce an upper limit to the amount of land that can be collateralized, so that even in case of default, farmers would still remain in the market with the residual land they own. As this technicality would not add anything valuable to our analysis, we leave it out for simplicity.

6 Conclusions

In this work we have analyzed a credit economy enriched with learning dynamics. The first finding is that the basic model described in Kiyotaki and Moore (1997) is E-stable, which means that agents, starting from non-rational beliefs but endowed with the correct model for the economy, can learn over time the REE.

We have then extended the basic framework to allow for heterogeneity of expectations and introduced uncertainty in terms of a time-varying productivity: by analyzing heterogeneous learning dynamics in this enriched setting we found that, though in general agents can still learn (in a stochastic sense) the true value of the parameters, farmers may be prevented from doing so by a bankruptcy condition becoming binding over the learning path. This means that the expectations formation processes and the heterogeneity of beliefs between lenders and borrowers can play an important role in a credit economy.

This work shows that learning can introduce important hysteresis into an economy, even when the learning process would actually converge towards an equilibrium in the long run. Short run constraints, in fact, may drive economic agents out of the market while they are learning and before they have got the chance to fully understand the economic structure in which they operate. These phenomena introduce in the economy strong non-linearities and irreversibilities that are often neglected in RE models.

Further work will investigate a number of extensions to the present setting. First, the degree of heterogeneity in the learning schemes could be made endogenous, depending for example on the costs and benefits of using more data in the regressions. Also, a full, blown up analysis that takes into account long-run incentives and reputation effects on the part of the farmer could add useful insights to the findings of this paper.

A Solution under RE of the benchmark KM model

By solving the maximization problems for the farmer and the gatherer in the original KM model under the assumption of perfect foresight, we obtain with the system of equations (5) and (7). In order to solve the model under rational expectation, the system can be rewritten as follows:

$$\begin{aligned} K_t^F &= \frac{a}{\mu_t^e} K_{t-1}^F \\ K_t^G &= G'^{-1}\left(R\mu_t^e\right) \end{aligned}$$

where $\mu_t^e = q_t - \frac{q_{t+1}^e}{R}$ is the expected downpayment with q_{t+1}^e the homogeneous, rational expectation on the future level of the price of land. For the sake of simplicity we specify the gatherer's production function with a Cobb-Douglas such that $y_t^G = G(K_{t-1}^G) = 2\left(K_{t-1}^G\right)^{1/2}$, from which it follows that $G'(K_{t-1}^G) = \left(K_{t-1}^G\right)^{-1/2}$. After substitutions, the system boils down to:

$$K_t^F = \frac{aR}{Rq_t - q_{t+1}^e} K_{t-1}^F \tag{26}$$

$$K_t^G = \left(Rq_t - q_{t+1}^e\right)^{-2} \tag{27}$$

Recalling that the supply of land is constant and equal to $\bar{K}$, in order to determine the equilibrium level of q_t we impose the clearing condition for the land market

$$\bar{K} = K_t^F + K_t^G.$$

After some algebra we end up with the equation

$$K_{t-1}^F = \frac{Rq_t - q_{t+1}^e}{aR}\bar{K} - \frac{\left(Rq_t - q_{t+1}^e\right)^{-1}}{aR}$$

which we linearize around the steady state for q, i. e., $q_t = q_{t+1} = \dfrac{aR}{R-1}$ and obtain

$$K_{t-1}^F \simeq \bar{K} - \frac{1}{(aR)^2} + \frac{1}{a}\left[\bar{K} + \frac{1}{(aR)^2}\right]\left(q_t - \frac{aR}{R-1}\right) - \frac{1}{aR}\left[\bar{K} + \frac{1}{(aR)^2}\right]\left(q_{t+1}^e - \frac{aR}{R-1}\right),$$

which after some more algebra leads to

$$q_t = \gamma_0 + \gamma_1 K_{t-1}^F + \gamma_2 q_{t+1}^e \tag{28}$$

with $\gamma_0 = \dfrac{2a}{(aR)^2\,\bar{K}+1} > 0$; $\gamma_1 = \dfrac{a^3R^2}{(aR)^2\,\bar{K}+1} > 0$ and $\gamma_2 = \dfrac{1}{R} < 1$. Substituting (28) in (26) and rearranging we obtain

$$K_{t-1}^F = \frac{\gamma_0 K_t^F}{a - \gamma_1 K_t^F}$$

which is a non linear relation between K_t^F and K_{t-1}^F. By linearizing it around the steady state for K_t^F, i. e., $K_t^F = K^F = \bar{K} - (aR)^{-2}$ we get

$$K_{t-1}^F \simeq \frac{\gamma_0 \dfrac{(aR)^2\,\bar{K}-1}{(aR)^2}}{a - \gamma_1 \dfrac{(aR)^2\,\bar{K}-1}{(aR)^2}} + \frac{\gamma_0\left[a - \gamma_1\dfrac{(aR)^2\,\bar{K}-1}{(aR)^2}\right] + \gamma_0\gamma_1\dfrac{(aR)^2\,\bar{K}-1}{(aR)^2}}{\left[a - \gamma_1\dfrac{(aR)^2\,\bar{K}-1}{(aR)^2}\right]^2}\left[K_t^F - \frac{(aR)^2\,\bar{K}-1}{(aR)^2}\right]$$

from which we finally obtain

$$K_t^F = \gamma_3 + \gamma_4 K_{t-1}^F \tag{29}$$

with $\gamma_3 = \dfrac{\left[(aR)^2 \bar{K} - 1\right]^2}{(aR)^2 \left[(aR)^2 \bar{K} + 1\right]} > 0$ and $\gamma_4 = \dfrac{2}{(aR)^2 \bar{K} + 1} > 0$.

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