Investment, Interest Rate Policy, and Equilibrium Stability*

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Abstract

Carlstrom and Fuerst (2005) show that in the presence of investment activity and price stickiness, indeterminacy of equilibrium is induced by forward-looking monetary policy that sets the interest rate in response only to future inflation. In a stochastic version of their model, we find that this indeterminacy problem is due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling, and the problem can be overcome once the forward-looking policy responds also to current output or contains sufficiently strong interest rate smoothing, since this prevents the self-fulfilling expectations. We also show that when E-stability is adopted as the selection criterion from multiple equilibria, even the forward-looking policy generates a locally-unique non-explosive E-stable fundamental rational expectations equilibrium as long as the policy response to expected future inflation is sufficiently strong.

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1 Introduction

Since the time of Keynes and Hicks, macroeconomics has stressed the importance of investment dynamics in business fluctuations. In line with this view, recent analyses show that investment activity induces critical implications for forward-looking monetary policy. In the face of the widespread belief that the Taylor principle (i.e. active policy) is an essential condition for equilibrium determinacy, Dupor (2001) finds that in a continuous time model with investment and sticky prices, local determinacy is ensured by passive policy that sets the interest rate in response only to instantaneous inflation, whose discrete time counterpart is future inflation.[1] In an associated discrete time model, Carlstrom and Fuerst (2005) (henceforth, CF) show that indeterminacy of equilibrium is induced by forward-looking policy that adjusts the interest rate in response only to future inflation, which is in stark contrast with another result of CF that determinacy is guaranteed by active current-looking policy that responds only to current inflation.[2] This indeterminacy problem is also pointed out by Huang and Meng (2007a), although they find that the problem is less severe when the cost share of capital decreases, when the price stickiness or the steady state price markup or inflation rate increases, or when prices are modeled as predetermined variables rather than as non-predetermined ones.[3]

In this paper we address the question of what prescription for the forward-looking monetary policy overcomes the indeterminacy problem, using a stochastic version of CF's model.[4]

[1]A similar result is obtained by Xiao (2007), who uses a discrete time model with an increasing returns to scale production technology.

[2]Sveen and Weinke (2005) find that the active current-looking policy is more likely to induce indeterminacy as prices become stickier. Under the current-looking policy, determinacy depends on the effects of a cost channel of monetary policy relative to an aggregate demand channel, as mentioned later. Benhabib and Eusepi (2005) provide an analysis of global determinacy under the current-looking policy.

[3]While CF use a Calvo (1983) style sticky price model, Huang and Meng (2007a) employ a quadratic price adjustment cost model.

[4]This model assumes the presence of a competitive rental market for capital. Sveen and Weinke (2005) study firm-specific capital and show that a sticky price model with such capital is equivalent in terms of local equilibrium dynamics to an associated rental-capital-market model with a higher degree of price stickiness.

This issue is critical because central banks, inflation-targeting ones in particular, are concerned about expected future inflation rather than actual inflation, as also emphasized by Huang and Meng (2007a). We examine the following two prescriptions. One is whether the problem can be ameliorated if the forward-looking policy adjusts the interest rate in response also to output or contains interest rate smoothing, as empirical studies such as Clarida et al. (2000) and Orphanides (2004) use for a better description of actual monetary policy.[5] Another prescription is: when we adopt E-stability as the criterion for selecting one rational expectations equilibrium (REE) from multiple such equilibria, does the forward-looking policy generate a locally-unique E-stable fundamental REE?[6] As Evans and Honkapohja (2001) show in a broad class of linear stochastic models, if a fundamental REE is E-stable and non-explosive, it is least-squares learnable, i.e. stable under least-squares learning. Therefore, E-stability is an essential condition for any REE to be regarded as plausible, as stressed by McCallum (2003).[7]

As for the first prescription, we show that the indeterminacy problem remains when the forward-looking policy sets the interest rate in response also to expected future output.[8] By contrast, we find that the problem can be overcome if the policy responds to current output or contains sufficiently strong interest rate smoothing. This provides a qualification of CF's conjecture that "[i]ncluding output in the Taylor rule would have only minor effects on the local

[5] Sveen and Weinke (2005) and Benhabib and Eusepi (2005) find that the current-looking policy ensures determinacy with a wider range of model parameters when it responds also to current output.

[6] Throughout the paper, "fundamental" refers to Evans and Honkapohja's (2001) minimal state variable (MSV) solutions to linear RE models so as to distinguish them from McCallum's (1983) original MSV solution. We do not examine E-stability of non-fundamental REE such as sunspot equilibria, which may exist in cases of indeterminacy. For E-stability analysis of these REE, see e.g. Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004), and Evans and McGough (2005), who all use associated models without investment. See also footnote 13. We leave E-stability analysis of non-fundamental REE in our model for future work.

[7] McCallum argues that in cases of indeterminacy there may be a unique non-explosive REE that is E-stable and thus least-squares learnable, whereas a determinate REE that is E-unstable and thus not least-squares learnable is arguably not a plausible candidate for equilibrium that could be observed in the actual economy.

[8] Xiao (2007) shows that in an associated model with a finite labor supply elasticity and a capital adjustment cost, a mild policy response to expected future output can ameliorate the indeterminacy problem.

determinacy conditions" (footnote 7). Before presenting an intuition for our result, we consider what makes the forward-looking policy induce the indeterminacy problem. As its cause, CF focus attention on households' arbitrage activity in bond and capital markets, while Huang and Meng (2007a) stress firms' price setting behavior in monopolistically competitive good markets. Our position is that both of these two are critical to the indeterminacy problem. Any passive forward-looking policy of course induces indeterminacy, and so does even an active policy due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling.[9] To see this, consider a sunspot increase in inflation expectations. The active policy then leads to a rise in the real interest rate, so that the expected future real rental price of capital increases via a no-arbitrage condition between bonds and capital. This raises expected future real marginal cost and hence expected future inflation via an aggregate price adjustment equation. Consequently, the inflationary expectations become self-fulfilling and therefore indeterminacy is induced.[10] With sufficiently strong interest rate smoothing, the active forward-looking policy brings about determinacy. Interest rate smoothing means a policy response to the lagged interest rate and hence makes the forward-looking policy respond also to current and past inflation, so that it guarantees determinacy similarly to the current-looking policy examined by CF.[11] The policy response to current output ameliorates the indeterminacy problem dramatically as long as the policy is active, or more accurately, it satisfies the long-run version of the Taylor principle: in

[9]This cost channel is similar to that in the existing literature such as Christiano and Eichenbaum (1992) and Barth and Ramey (2001) in that a rise in the interest rate increases firms' marginal cost. The difference crucial for equilibrium determinacy is that the *real* interest rate affects *expected future* real marginal cost in our cost channel, while the *nominal* interest rate affects *current* real marginal cost in the literature.

[10]As prices become stickier, the real marginal cost elasticity of inflation decreases, which slightly mitigates the effect of the cost channel and hence the indeterminacy problem. This is in stark contrast with Sveen and Weinke (2005), who obtain the exactly opposite result under the current-looking policy as noted in footnote 2.

[11]Kurozumi and Van Zandweghe (2006) obtain a necessary and sufficient condition for determinacy under monetary policy that sets the interest rate in response to a weighted average of future and current inflation in CF's model, and show that determinacy is more likely with a higher weight on current inflation. They also find that determinacy is likely under interest rate policy that responds only to past inflation.

the long run the nominal interest rate should be raised by more than the increase in inflation.[12] This is because the policy responses to both current consumption and investment subdue any change in the real interest rate stemming from inflation expectations: a rise (decline) in the real interest rate decreases (increases) consumption and investment, both of which reduce the real rate rise (decline) by the policy responses to them. These two types of feedback on policy are absent from the policy response to expected future output, so that the indeterminacy problem remains. We also show that the feedback from current investment rather than consumption is crucial to determinacy, since the latter feedback on policy is limited due to consumption smoothing. This demonstrates that investment dynamics, which have been widely viewed as an important determinant of business fluctuations, are likewise of crucial importance in generating determinacy of REE, suggesting that central banks pay special attention to investment activity.

When we consider our second prescription for the indeterminacy problem, i.e. we adopt E-stability as the criterion for selection from multiple REE, we find that even the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if its inflation coefficient lies in either of the following two intervals, both of which satisfy the Taylor principle. One interval is extremely narrow, in which the inflation coefficient exceeds one and is very close to one. This contains all the inflation coefficients that bring about determinacy of REE. Another interval requires that the inflation coefficient be sufficiently greater than one and its lower bound increase with stickier prices. Any inflation coefficient in these two intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE. Further, if the forward-looking policy adjusts the interest rate in response also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle generates the unique E-stable REE. Therefore, the indeterminacy problem is not critical from the perspective of E-stability or least-squares learnability of fundamental

[12] As Woodford (2003), Bullard and Mitra (2002), and Kurozumi (2006) show with associated models without investment, the long-run version of the Taylor principle is an essential condition for Taylor style interest rate policy to ensure determinacy and E-stability of REE. If the policy responds only to inflation, the long-run version is consistent with the usual Taylor principle.

REE.[13] Our E-stability result is a generalization of Bullard and Mitra (2002), who use an associated model without investment to show that the forward-looking policy yields the unique E-stable REE if and only if it meets the Taylor principle. In the absence of investment activity, monetary policy contains only an aggregate demand channel, whereby any active policy can successfully guide temporarily non-rational expectations toward the rational expectations. In the presence of investment activity, the cost channel emerges and reduces the guiding effect of the demand channel. As a consequence, all non-explosive fundamental REE fail to be E-stable if the inflation coefficient lies in the intermediate interval, if any, between the two intervals of inflation coefficients that generate the unique E-stable REE.

The remainder of the paper proceeds as follows. Section 2 presents a stochastic version of CF's model. Section 3 examines our first prescription for the indeterminacy problem induced by the forward-looking policy. Section 4 investigates the second one. Finally, Section 5 concludes.

2 A stochastic version of Carlstrom and Fuerst's model

We use the same model as CF except in the following two respects. The utility function is assumed to contain uncertain disturbances ξ_t and to be separable between consumption C_t and real money balances M_{t+1}/P_t,[14] where M_{t+1} is nominal balances held at the end of period t and P_t is the price level. The period utility function with leisure $1 - L_t$ then takes the form

$$U(C_t, M_{t+1}/P_t, 1 - L_t; \xi_t) = V(C_t; \xi_t) + W(M_{t+1}/P_t; \xi_t) - L_t.$$

Another difference from CF is the specification of monetary policy. CF study a forward-

[13] The indeterminacy problem may not be critical even if we extend our analysis to non-fundamental REE. Carlstrom and Fuerst (2004) show with an associated model without investment that a sunspot equilibrium is E-stable only if a central bank believes in the sunspot. Clearly, this condition is not practical.

[14] This separability assumption implies that our results can also be obtained with an associated cashless economy model. If the utility functions are non-separable between consumption and real money balances as in CF, a higher degree of the non-separability makes equilibrium indeterminacy more likely under monetary policy that sets the interest rate in response not only to inflation but also to output, as Kurozumi (2006) shows in an associated model without investment.

looking policy that sets the nominal interest rate R_t in response only to expected future inflation $E_t\pi_{t+1}$. We generalize this policy so that it responds also to current output Y_t or expected future output E_tY_{t+1} or contains interest rate smoothing,

$$R_t = (R_{t-1})^{\phi_R}\left[R\left(\frac{E_t\pi_{t+1}}{\pi}\right)^{\phi_\pi}\left(\frac{E_tY_{t+j}}{Y}\right)^{\phi_Y}\right]^{1-\phi_R}, \; j \in \{0,1\}, \; \phi_\pi, \phi_Y \geq 0, \; 0 \leq \phi_R < 1, \tag{1}$$

where E_t is the rational expectation operator conditional on information available in period t and R, π and Y denote steady state values of the interest rate, inflation and output. This generalization is motivated by empirical studies such as Clarida et al. (2000) and Orphanides (2004), who use it for a better description of actual monetary policy.

The equilibrium conditions log-linearized around a steady state are given by[15]

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = -\sigma^{-1}[(\hat{C}_t - g_t) - (E_t\hat{C}_{t+1} - E_tg_{t+1})], \tag{2}$$

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = [1-\beta(1-\delta)](E_t\hat{z}_{t+1} + E_t\hat{Y}_{t+1} - \hat{K}_{t+1}), \tag{3}$$

$$\sigma^{-1}(\hat{C}_t - g_t) = \hat{z}_t + \alpha(1-\alpha)^{-1}(\hat{K}_t - \hat{Y}_t), \tag{4}$$

$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t, \tag{5}$$

$$\hat{Y}_t = s_C\hat{C}_t + s_I\hat{I}_t, \tag{6}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda\hat{z}_t, \tag{7}$$

$$\hat{R}_t = \phi_R\hat{R}_{t-1} + (1-\phi_R)(\phi_\pi E_t\hat{\pi}_{t+1} + \phi_Y E_t\hat{Y}_{t+j}), \quad j \in \{0,1\}. \tag{8}$$

Eq. (2) is the Euler equation for households' optimal consumption decisions with an intertemporal substitution elasticity $\sigma > 0$ and preference shocks g_t, which are assumed to follow a stationary first order autoregressive process with a parameter $|\rho| < 1$ and a white noise ε_t

$$g_t = \rho g_{t-1} + \varepsilon_t. \tag{9}$$

[15] We omit an equilibrium condition for money balances, since the remaining conditions determine local dynamics of REE. We also assume as in CF that fiscal policy is "Ricardian", i.e. it appropriately accommodates consequences of monetary policy for the government budget constraint. We thus leave hidden the government budget constraint and fiscal policy. For analysis of equilibrium determinacy under interest rate policy and non-Ricardian fiscal policy, see e.g. Benhabib et al. (2001), Benhabib and Eusepi (2005), Linnemann (2006), and Kurozumi (2005).

Eq. (2) presents the Fisher relation between the nominal interest rate, expected future inflation, and the real interest rate. Eq. (3) is the no-arbitrage condition between bonds and capital, where z_t is firms' real marginal cost, K_{t+1} is the capital stock at the beginning of period $t+1$, $\beta \in (0,1)$ is a discount factor, and $\delta \in (0,1)$ is the depreciation rate of capital. The right-hand side of (3) can be derived from firms' cost minimization problem, which implies that the real rental price of capital r_t satisfies $r_t = \alpha z_t Y_t / K_t$ in the presence of a competitive rental capital market and a Cobb-Douglas production technology $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ with a cost share of capital $\alpha \in (0,1)$. It also implies that the real wage rate w_t satisfies $w_t = (1-\alpha) z_t (K_t/Y_t)^{\alpha/(1-\alpha)}$, and thus (4) is the labor market condition that matches the wage rate to the marginal rate of substitution between consumption and leisure, where we assume as in CF that the labor supply elasticity is an infinity. Eq. (5) describes capital accumulation and (6) is the resource constraint with steady state output shares of consumption and investment $s_{_C}, s_{_I} \in (0,1)$. Eq. (7) describes Calvo (1983) style staggered price setting of monopolistically competitive firms with indexation to steady state inflation, where the so-called Calvo parameter $\nu \in (0,1)$ (i.e. the probability of not optimally setting prices) gives rise to the real marginal cost elasticity of inflation $\lambda = (1-\nu)(1-\beta\nu)/\nu > 0$.

Here, it is important to stress that in the system of (2)$-$(9) there are two channels of monetary policy, which yield exactly opposite effects on inflation. One is the conventional aggregate demand channel, where Euler equation (2) leads a rise in the real interest rate to dampen consumption and hence output, both of which lower real marginal cost $\hat{z}_t$ via labor market condition (4), thereby reducing current inflation via Phillips curve (7). Another is a cost channel, which is one of the main points of this paper. No-arbitrage condition (3) makes a rise in the real interest rate increase the expected future real rental price of capital $E_t \hat{r}_{t+1}$, which is matched to the expected future marginal product of capital adjusted by expected future real marginal cost, $E_t \hat{Y}_{t+1} - \hat{K}_{t+1} + E_t \hat{z}_{t+1}$, in equilibrium. From (4) we have

$$E_t \hat{z}_{t+1} = \alpha (E_t \hat{Y}_{t+1} - \hat{K}_{t+1} + E_t \hat{z}_{t+1}) + \sigma^{-1}(1-\alpha) E_t [\hat{C}_{t+1} - g_{t+1}].$$

Thus, such an increase in the marginal product of capital raises expected future real marginal cost $E_t\hat{z}_{t+1}$, thereby increasing expected future inflation and hence current inflation via Phillips curve (7). Hence, by this cost channel a rise in the real interest rate increases expected future inflation. This induces a possibility that inflation expectations become self-fulfilling and therefore indeterminacy of REE is induced if monetary policy sets the interest rate in response only to expected future inflation.[16]

The ensuing analysis uses realistic calibrations of model parameters to illustrate conditions for determinacy and E-stability of REE. Table 1 summarizes our baseline calibration. These parameter values are taken from CF so that our results are comparable with theirs. Note that under the baseline calibration the Calvo parameter takes a value of $\nu = 0.57$, so that firms reset optimal prices of their products, on average, once every 2.3 quarters. As noted by Sveen and Weinke (2005) and Benhabib and Eusepi (2005), the actual value of ν is controversial in the empirical literature. Thus we also examine two alternative values, $\nu = 0.67, 0.80$, which imply respectively that $\lambda = 0.18, 0.052$ and firms reset optimal prices, on average, once every three or five quarters.[17]

3 First prescription for the indeterminacy problem

The forward-looking policy, which responds only to expected future inflation, renders REE indeterminate in the model presented above, as shown by CF. In this section we examine our first prescription for this indeterminacy problem: can a policy response to output or interest rate smoothing overcome the problem?

[16] With no rental market, firms accumulate capital and r_t, which denotes the real rental price of capital in the presence of a rental market, represents an average reduction in firms' labor costs due to an additional unit of capital in place in the next period, as Woodford (2003) indicates. Although it differs from the adjusted marginal product of capital, firms' investment decisions still have counteracting effects on current and expected future real marginal cost. Therefore, the cost channel exists with firm-specific capital.

[17] Because of the limited space, we omit to present sensitivity analysis of the other parameters. The qualitative properties of results obtained with the baseline calibration survive in the sensitivity analysis, but of course, the results differ quantitatively.

3.1 Policy response to expected future output

We first analyze the policy response to expected future output, i.e. $j = 1$, $\phi_R = 0$ in (1). With this policy specification, the system of (2)−(9) can be reduced to a system of the form

$$E_t x_{t+1} = A x_t + B g_t, \tag{10}$$

where $x_t = [\hat{\pi}_t\ \hat{C}_t\ \hat{Y}_t\ \hat{K}_t\ \hat{R}_{t-1}]'$ and the coefficient matrix A is given in Appendix A.[18] In this system the first three variables, $\hat{\pi}_t$, $\hat{C}_t$, $\hat{Y}_t$, are non-predetermined while the remaining two, $\hat{K}_t$, $\hat{R}_{t-1}$, are predetermined. Hence, Proposition 1 of Blanchard and Kahn (1980) implies that the forward-looking policy with responses to expected future output generates determinacy of REE if and only if the coefficient matrix A has exactly two eigenvalues inside the unit circle and the other three outside the unit circle.[19] We thus obtain the following result.

Proposition 1 *Suppose that $b_3 = a_2 - (1 - a_1)\phi_Y \neq 0$, where $a_1 = 1 - \beta(1 - \delta)(1 - \alpha)$ and $a_2 = 1 - \beta(1 - \delta)$. Then, if the forward-looking policy adjusts the interest rate in response also to expected future output, i.e. $j = 1$, $\phi_R = 0$ in* (1)*, it generates local determinacy of REE if and only if either of the following two cases is satisfied:* (Case I)

$$\phi_\pi < 1 + \frac{a_2}{\alpha\lambda} - \frac{s_I(1 - a_1)}{\alpha\beta\lambda[s_I + \delta(1 - s_I)]}\phi_Y, \tag{11}$$

$$\phi_\pi + \frac{(1 - \beta)[s_C\sigma + s_I(1 - \alpha)]}{\lambda(1 - s_I)(1 - \alpha)}\phi_Y > 1, \tag{12}$$

$$\phi_Y < \frac{\lambda(a_1 + \alpha)[2s_I + \delta(1 - s_I)]}{(1 + \beta)[s_C\sigma\delta a_2 + s_I(2 - \delta)(2 - \alpha - a_1)]}\left[1 + \frac{2a_2(1 + \beta)}{\lambda(a_1 + \alpha)} - \phi_\pi\right], \tag{13}$$

$$b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|, \tag{14}$$

where b_i, $i = 0, 1, 2$, are given in Appendix B;

(Case II) (14) *and the three strict inequalities opposite to* (11)−(13) *hold.*

Proof See Appendix B. ■

[18]The form of the vector B is omitted since it is not needed in what follows.

[19]To be precise, this condition is sufficient for determinacy but only generically necessary. Throughout the paper, consideration of non-generic boundary cases is omitted.

This proposition confirms CF's result that "the presence of capital makes determinacy essentially impossible" (p. 10) in the case of no policy response to output. While CF present only a necessary condition for determinacy, the following corollary provides a necessary and sufficient condition. Note that Huang and Meng (2007a) also present a necessary and sufficient condition in an associated model with a quadratic price adjustment cost, which yields other features of equilibrium determinacy in sticky price models with investment, e.g. indeterminacy is less likely when the steady state price markup or inflation rate increases or when prices are modeled as predetermined variables rather than as non-predetermined ones.

Corollary 1 *The forward-looking policy, i.e. $\phi_Y = \phi_R = 0$ in* (1)*, brings about local determinacy of REE if and only if its inflation coefficient ϕ_π satisfies*[20]

$$1 < \phi_\pi < 1 + \frac{a_2}{\lambda} \min \left\{ \frac{1-\beta}{\alpha},\ \frac{2(1+\beta)}{a_1 + \alpha} \right\}. \tag{15}$$

This interval is extremely narrow, e.g. $1 < \phi_\pi < 1.0027$ under the baseline calibration.[21] What is the intuition for this indeterminacy problem? As is the case with no investment, any inflation coefficient less than one (i.e. passive policy) induces indeterminacy due to the weakness of the demand channel of monetary policy presented above. Also, indeterminacy is induced by any inflation coefficient greater than an upper bound, which takes such a large value in the absence of investment activity that indeterminacy is unlikely.[22] In our model, this upper bound is given by $1 + 2a_2(1+\beta)/[\lambda(a_1 + \alpha)]$, whose value is 1.52 under the baseline calibration and thus the presence of investment activity lowers the upper bound greatly. In addition, there is

[20]In Proposition 1, no policy response to output, i.e. $\phi_Y = 0$, implies that (Case II) never holds and (12)−(14) can be reduced to (15), since (11) is implied by (14). Corollary 1 holds in a more general case of utility functions that are non-separable between consumption and real money balances as in CF. The proof of this case is provided in Kurozumi and Van Zandweghe (2006).

[21]This interval, though we employ the same calibration, differs slightly from that of CF, $1 < \phi_\pi < 1.0057$, since they use approximate values $a_1 = 0.35$ and $a_2 = 0.03$.

[22]When the aggregate capital stock is fixed over time (i.e. $K_t = K\ \forall t,\ \delta = 0$), this upper bound is given by $1+2(1+\beta)/\{\lambda[1+\alpha\sigma/(1-\alpha)]\}$, which takes a value of 9.0 under the baseline calibration. See also Proposition 4 of Bullard and Mitra (2002) and Proposition 4.5 of Woodford (2003).

another upper bound given by $1 + a_2(1-\beta)/(\lambda\alpha)$, which takes a value extremely close to one, e.g. 1.0027 under the baseline calibration, as noted above. This upper bound arises from the cost channel of monetary policy illustrated above. By this channel an active forward-looking policy makes inflation expectations self-fulfilling and hence induces indeterminacy.

One point of condition (15) is that the indeterminacy problem becomes slightly less severe when the degree of price stickiness, ν, increases, as also indicated by Huang and Meng (2007a).[23] In (15) we can see that an increase in ν reduces only the real marginal cost elasticity of inflation λ and hence raises the upper bound on inflation coefficients that generate determinacy.[24] This is in stark contrast to Sveen and Weinke's (2005) finding that the current-looking policy, which responds only to current inflation, is more likely to induce indeterminacy as prices become stickier. This contrast stems from the way the cost channel induces indeterminacy. Under the forward-looking policy, indeterminacy is only due to this channel. Thus, stickier prices mitigate the effect of the cost channel and hence the indeterminacy problem. Under the current-looking policy, indeterminacy depends on the effect of the cost channel relative to the demand channel. Stickier prices strengthen this relative effect by reducing the effect of the demand channel more than that of the cost channel and as a consequence, indeterminacy is more likely.[25]

We now illustrate the determinacy condition in Proposition 1. Note that (11)$-$(14) are the empirically relevant one, since (Case II) cannot obtain with realistic calibrations of parameters including the baseline one. Condition (12) can be given the following interpretation, which is stressed by Woodford (2003), Bullard and Mitra (2002), and Kurozumi (2006). By (2)$-$(7), each percentage point of permanently higher inflation implies permanently higher output of

[23] Further, an increase in the depreciation rate of capital δ or a decrease in the cost share of capital α mitigates the indeterminacy problem.

[24] With a higher value of $\nu = 0.67$ (0.80), this upper bound takes a slightly larger value of 1.0050 (1.0172).

[25] As shown by Sveen and Weinke (2005), a stronger policy response to current inflation increases the effect of the demand channel more than that of the cost channel and thereby weakens the relative effect, so that determinacy is more likely. Also, determinacy is obtained for an interval of inflation coefficients that exceed one and are extremely close to one, regardless of the degree of price stickiness. This interval corresponds to the one given by condition (15) of Corollary 1, in which the effect of the cost channel is negligible.

$(1-\beta)[s_C\sigma + s_I(1-\alpha)]/[\lambda(1-s_I)(1-\alpha)]$ percentage points. The left-hand side of (12) then shows the long-run rise in the interest rate by policy (8) for each unit permanent increase in inflation. Hence, (12) can be interpreted as the long-run version of the Taylor principle: in the long run the nominal interest rate should be raised by more than the increase in inflation.

Figure 1 shows a region of inflation and output coefficients of the policy that generate determinacy under the baseline calibration. The lower bound on the inflation coefficient ϕ_π is provided by the Taylor principle (12), while the upper bound on the output coefficient ϕ_Y is given by (13). These two bounds arise basically from the demand channel, since we can see the corresponding ones in Figure 3 of Bullard and Mitra (2002) who analyze determinacy in an associated model without investment, in which monetary policy contains only the demand channel. The presence of investment activity gives rise to the upper bound on the inflation coefficient given by (14). The cost channel makes both the upper bounds on inflation and output coefficients severely limit the region of the coefficients that bring about determinacy. As shown in Corollary 1, the forward-looking policy ensures determinacy if and only if its inflation coefficient lies in the extremely narrow interval $1 < \phi_\pi < 1.0027$. Even with the policy response to expected future output, the determinacy region of the coefficients is only slightly widened. The output coefficient of 0.046 generates determinacy for the widest possible interval of the inflation coefficient $0.998 < \phi_\pi < 1.008$. For the output coefficient greater than 0.047, determinacy is impossible to obtain. In short, the policy response to expected future output cannot ameliorate the indeterminacy problem.[26]

3.2 Policy response to current output

We next examine the policy response to current output, i.e. $j = 0$, $\phi_R = 0$ in (1). As shown in Appendix A, this policy specification yields a system of the same form as (10) with a different coefficient matrix A, whose five eigenvalues are a zero and four solutions to a quartic equation

[26] A higher degree of price stickiness enlarges the determinacy region of the policy coefficients slightly, as is the case with no policy response to output. For $\nu = 0.67$ (0.80), the widest possible inflation coefficient interval, which is obtained with the output coefficient of 0.046, is given by $0.994 < \phi_\pi < 1.018$ ($0.983 < \phi_\pi < 1.058$).

$P(\mu) \equiv \mu^4 + h_3\mu^3 + h_2\mu^2 + h_1\mu + h_0 = 0$. Determinacy requires that exactly two eigenvalues lie inside the unit circle and the other three be outside the unit circle. To the best of our knowledge, it is hard to analytically examine conditions for the quartic equation to contain exactly one solution inside the unit circle and the other three outside the unit circle.[27] We thus carry out numerical investigations.

Figure 2 illustrates a region of inflation and output coefficients of the policy that ensures determinacy under the baseline calibration. From the policy specification, the long-run version of the Taylor principle yields the same inequality as the one with the policy response to expected future output, (12), which can also be obtained from $P(1) < 0$. This Taylor principle (12) provides the lower bound on the inflation coefficient ϕ_π and hence it seems to be a necessary condition for determinacy.[28] This lower bound arises from the demand channel, as noted above. The cost channel imposes the upper bounds on the inflation coefficient ϕ_π, given by

$$\begin{aligned} & P(-1) > 0 \\ & \Leftrightarrow\ \phi_\pi < 1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)} + \frac{(1+\beta)\{s_I(1-\alpha)(2-\delta)[1+\beta(1-\delta)] + \delta s_C \sigma a_2\}}{\lambda(a_1+\alpha)[2s_I + \delta(1-s_I)]}\phi_Y, \quad (16) \\ & (1-h_0)(1-h_0^2) - h_2(1-h_0)^2 + (h_3-h_1)(h_1-h_3h_0) < 0. \quad (17) \end{aligned}$$

It seems that these two are necessary conditions for determinacy and the three inequalities (12), (16) and (17) are a sufficient condition (see footnote 27). Figure 2 (i.e. $0 \leq \phi_\pi \leq 3$) shows that if the output coefficient ϕ_Y exceeds 0.2, the policy guarantees determinacy as long as it meets the Taylor principle (12).[29] Therefore, the active forward-looking policy with responses

[27] We conjecture: if the forward-looking policy adjusts the interest rate in response also to current output, i.e. $j = 0$, $\phi_R = 0$ in (1), it generates local determinacy of REE if it satisfies (12), (16) and (17). Note that these three inequalities can be reduced to condition (15) in the case of no policy response to output.

[28] Proposition 4.5 of Woodford (2003) shows that in an associated model without investment, the corresponding condition is a necessary condition under which the policy response to current output ensures determinacy.

[29] A higher degree of price stickiness enlarges the determinacy region of the policy coefficients. For instance, when the output coefficient is 0.5, the inflation coefficient must exceed 0.971 under the baseline calibration, i.e. $\nu = 0.57$, while with $\nu = 0.67$ (0.80) this lower bound on the inflation coefficient is reduced to 0.947 (0.821). Meanwhile, the upper bound on the inflation coefficient increases as prices become stickier.

to current output can overcome the indeterminacy problem.

How does the policy response to current output overcome the indeterminacy problem? The key point is that such a policy response prevents self-fulfilling inflation expectations and thereby the active forward-looking policy generates determinacy. To see this, consider a sunspot increase in inflation expectations. Under an active forward-looking policy, the real interest rate rises and then increases expected future inflation by the cost channel. This induces a possibility that the inflationary expectations become self-fulfilling and indeterminacy is generated. But, once the policy adjusts the interest rate in response also to current output, there is feedback on the real interest rate from movements in current consumption and investment. This subdues the real interest rate rise stemming from the inflationary expectations in the following two ways. A rise in the real interest rate discourages current consumption, so that such a rate rise can be reduced by the policy response to current consumption. Also, a real interest rate rise decreases current investment, which reduces this rate rise as long as the policy responds to current investment. In these two ways, the policy response to current output overcomes the indeterminacy problem.

Here, we address the question of which policy response of these two is crucial to such dramatic amelioration of the problem. We first examine the policy response to current consumption by replacing policy (8) with

$$\hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_C \hat{C}_t. \tag{18}$$

Analyzing the system's coefficient matrix A given in Appendix A yields the next proposition.

Proposition 2 *If the forward-looking policy sets the interest rate in response also to current consumption as in* (18)*, it generates local determinacy of REE if and only if it satisfies*

$$1 - \frac{1-\beta}{\lambda(1-\alpha)}(\sigma\phi_C) \; < \; \phi_\pi \; < \; 1 + \frac{a_2}{\lambda} \min\left\{ \frac{1-\beta+\sigma\phi_C}{\alpha}, \; \frac{(1+\beta)(2+\sigma\phi_C)}{a_1+\alpha} \right\}, \tag{19}$$

where $a_1 = 1 - \beta(1-\delta)(1-\alpha)$ *and* $a_2 = 1 - \beta(1-\delta)$.

Proof See Appendix C. ■

Like (12), the first inequality in (19) can be interpreted as the long-run version of the Taylor principle. Figure 3 displays a region of inflation and consumption coefficients of policy (18) that ensure determinacy under the baseline calibration.[30] As is the case with policy responses to output, the lower bound on the inflation coefficient ϕ_π arises from the demand channel, while its upper bound is induced by the cost channel. Figure 3 shows that a more vigorous policy response to current consumption widens the interval of inflation coefficients that bring about determinacy. If $\phi_C = 0.5$, this interval is $0.98 < \phi_\pi < 1.14$, which is wider than $1 < \phi_\pi < 1.0027$ in the case of no policy response to consumption. One point of condition (19) is that both the lower and upper bounds contain the term $\sigma\phi_C$. This implies that the intertemporal substitution elasticity of consumption, σ, is a crucial factor in generating determinacy. As σ increases, consumption becomes more responsive to changes in the real interest rate and as a consequence, the policy response to current consumption becomes more important for determinacy with the policy response to current output. Under realistic calibrations of σ, however, this is not the case, as shown in Figure 3.[31]

We next consider the policy response to current investment by replacing (8) with

$$\hat{R}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_I \hat{I}_t. \tag{20}$$

As is the case with the policy response to current output, it seems hard to analytically examine conditions for determinacy, since the policy specification (20) yields a system of the same form as (10) with a different coefficient matrix A whose five eigenvalues are a zero and four solutions to a quartic equation $Q(\mu) \equiv \mu^4 + j_3\mu^3 + j_2\mu^2 + j_1\mu + j_0 = 0$, as shown in Appendix A.[32] Thus,

[30] Figure 3 provides the range of the consumption coefficient given by $0 \leq \phi_C \leq 2.1$, which corresponds to that of the output coefficient in Figure 2 (i.e. $0 \leq \phi_Y \leq 3$) because $0 \leq s_C\phi_Y \leq 0.7 \times 3 = 2.1$.

[31] A higher degree of price stickiness widens the determinacy region of the policy coefficients. With the consumption coefficient of 0.5, determinacy with $\nu = 0.67$ (0.80) requires the inflation coefficient to lie in the interval $0.96 < \phi_\pi < 1.25$ ($0.86 < \phi_\pi < 1.84$).

[32] We conjecture: if the forward-looking policy adjusts the interest rate in response also to current investment as in (20), it ensures local determinacy of REE if it satisfies (21)−(23). Note that these three inequalities can be reduced to condition (15) in the case of no policy response to investment.

we study determinacy numerically. Figure 4 illustrates a region of inflation and investment coefficients of policy (20) that ensure determinacy under the baseline calibration.[33] The policy specification (20) implies that the long-run version of the Taylor principle yields

$$\phi_\pi + \frac{(1-\beta)(s_C\sigma + 1 - \alpha)}{\lambda(1-s_I)(1-\alpha)}\phi_I \;>\; 1, \tag{21}$$

which can also be obtained from $Q(1) < 0$. This Taylor principle (21) provides the lower bound on the inflation coefficient ϕ_π and thus it seems to be a necessary condition for determinacy. This lower bound arises from the demand channel, while the cost channel imposes the upper bounds on the inflation coefficient ϕ_π, given by

$$\begin{aligned}
&Q(-1) > 0 \\
&\Leftrightarrow\; \phi_\pi < 1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)} + \frac{(1+\beta)(2-\delta)\{(1-\alpha)[1+\beta(1-\delta)] - s_C\sigma a_2\}}{\lambda(a_1+\alpha)[2s_I + \delta(1-s_I)]}\phi_I, \qquad (22)\\
&(1-j_0)\left(1-j_0^2\right) - j_2\left(1-j_0\right)^2 + \left(j_3 - j_1\right)\left(j_1 - j_3 j_0\right) < 0. \qquad (23)
\end{aligned}$$

It seems that these two are necessary conditions for determinacy and the three inequalities (21)−(23) are a sufficient condition (see footnote 32). As is the case with the policy response to current output, we can see that determinacy is likely. In Figure 4 (i.e. $0 \le \phi_\pi \le 3$), if the investment coefficient ϕ_I exceeds 0.1, policy (20) generates determinacy as long as it satisfies the Taylor principle (21).[34] Therefore, the policy response to current investment ameliorates the indeterminacy problem dramatically.

The findings above suggest that the policy response to current investment rather than consumption is crucial for determinacy with the policy response to current output under realistic calibrations. This is because current consumption has a dampened response to changes in the

[33] Figure 4 provides the range of the investment coefficient given by $0 \le \phi_I \le 0.9$, which corresponds to that of the output coefficient in Figure 2 (i.e. $0 \le \phi_Y \le 3$) because $0 \le s_I\phi_Y \le 0.3 \times 3 = 0.9$.

[34] With stickier prices, the determinacy region of the policy coefficients enlarges. For instance, when the investment coefficient is 0.5, the inflation coefficient must exceed 0.956 under the baseline calibration, i.e. $\nu = 0.57$, while with $\nu = 0.67$ (0.80) this lower bound on the inflation coefficient is reduced to 0.920 (0.729). Moreover, the upper bound on the inflation coefficient increases as prices become stickier.

real interest rate due to consumption smoothing, so that determinacy requires a large policy response to consumption. Current investment responds more sharply to real interest rate changes, and hence there is stronger feedback from current investment on interest rate policy, which overcomes the indeterminacy problem. This desirable property is inherited by the policy response to current output. Investment dynamics have been widely viewed as an important determinant of business fluctuations, despite a relatively small share of investment spending in aggregate demand. Our finding shows that investment dynamics are likewise of crucial importance in generating determinacy of REE. This suggests that central banks pay special attention to movements in investment activity.

3.3 Interest rate smoothing

We proceed to examine whether interest rate smoothing, i.e. $\phi_Y = 0$ in (1), ameliorates the indeterminacy problem. The system of (2)$-$(9) can be reduced to a system of the same form as (10) with a different coefficient matrix A given in Appendix A. By investigating this coefficient matrix, we obtain the following proposition.

Proposition 3 *The forward-looking policy with interest rate smoothing, i.e. $\phi_Y = 0$ in (1), generates local determinacy of REE if and only if it satisfies*

$$1 < \phi_\pi < \frac{1+\phi_R}{1-\phi_R}\left[1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)}\right], \tag{24}$$

$$|d_2| > 3 \quad or \quad d_0(d_0 - d_2) + d_1 - 1 > 0, \tag{25}$$

where $a_1 = 1-\beta(1-\delta)(1-\alpha)$, $a_2 = 1-\beta(1-\delta)$, and d_i, $i = 0, 1, 2$, are given in Appendix D.[35]

Proof See Appendix D. ■

This determinacy condition is illustrated with the baseline calibration in Figure 5, which displays a region of coefficients of inflation and interest rate smoothing that satisfy the condition. The first inequality in (24) is the Taylor principle and is due to the demand channel

[35]Like Corollary 1, this proposition holds in a more general case of utility functions that are non-separable between consumption and real money balances as in CF. The proof of this case is available upon request.

of monetary policy. It gives rise to the lower bound on the inflation coefficient ϕ_π. The cost channel of monetary policy induces the upper bounds on the inflation coefficient, given by the second inequalities in (24) and (25). For weak interest rate smoothing whose degree is less than a certain threshold given by $\alpha\beta a_2/[a_1(\alpha\lambda+a_2)] = 0.20$, the upper bound on the inflation coefficient is provided by the second inequality in (25), which severely limits the determinacy region of the coefficients, as is the case with no interest rate smoothing. With $\phi_R = 0.1$, determinacy is obtained for an inflation coefficient in the interval of $1 < \phi_\pi < 1.010$. Once interest rate smoothing is sufficiently strong, i.e. $\phi_R > 0.20$, the second inequality in (24) determines the upper bound. In the case of $\phi_R = 0.5$, determinacy is guaranteed by any inflation coefficient in the interval of $1 < \phi_\pi < 4.55$. Hence, determinacy is likely with a range of the inflation coefficient and is more likely with more inertial interest rate policy.

As prices become stickier, the determinacy region of the policy coefficients widens in some direction, as is the case with no interest rate smoothing, while it also narrows in that the threshold of interest rate smoothing for determinacy increases. With a higher degree of price stickiness of $\nu = 0.67$, weak interest rate smoothing, in which ϕ_R is less than an increased threshold of 0.33, yields a slightly wider determinacy region (e.g. $1 < \phi_\pi < 1.012$ for $\phi_R = 0.1$), and high interest rate smoothing with $\phi_R > 0.33$ generates a wider determinacy region (e.g. $1 < \phi_\pi < 6.13$ for $\phi_R = 0.5$). But, with a much higher degree of price stickiness of $\nu = 0.80$, the threshold of interest rate smoothing increases, i.e. 0.58, so that in the case of $\phi_R = 0.5$ the determinacy region becomes much narrower, $1 < \phi_\pi < 1.10$.

What is the intuition for this determinacy with sufficiently strong interest rate smoothing? Interest rate smoothing means a policy response to the lagged interest rate and hence makes the forward-looking policy respond also to current and past inflation. As shown by CF and Kurozumi and Van Zandweghe (2006), equilibrium determinacy is possible with the current-looking or backward-looking policy, which sets the interest rate in response only to current or past inflation. Also, Sveen and Weinke (2005) show that the current-looking policy is more likely to induce indeterminacy as prices become stickier. Therefore, the forward-looking policy with interest rate smoothing inherits these properties from the current-looking and backward-looking policies. In sum, interest rate smoothing helps the forward-looking policy generate determinacy of REE, although its amelioration of the indeterminacy problem is not so effective as the policy response to current output.

4 Second prescription for the indeterminacy problem

We turn next to our second prescription for the indeterminacy problem: when E-stability is adopted as the criterion for selecting one from multiple REE, does the forward-looking policy generate a locally-unique E-stable fundamental REE?[36] Following the literature, our E-stability analysis is based on the so-called "Euler equation" approach suggested by Honkapohja et al. (2003). Specifically, the rational expectation operator E_t is replaced with a possibly non-rational one $\hat{E}_t$ in the system of (2)−(9) with $\phi_Y = \phi_R = 0$. Also, this system can be reduced to a system of the form

$$Fy_t = G\hat{E}_t y_{t+1} + H\hat{K}_t + Jg_t, \tag{26}$$

where $y_t = [\hat{\pi}_t \ \hat{C}_t \ \hat{Y}_t \ \hat{K}_{t+1}]'$ and the coefficient matrices F, G, H are given in Appendix E.[37] Then, fundamental RE solutions to system (26) are given by

$$y_t = \bar{c} + \bar{\Phi}\hat{K}_t + \bar{\Gamma}g_t, \tag{27}$$

where the coefficient matrices are determined by

$$\bar{c} \ = \ 0_{4\times 1}, \quad H \ = \ (F - G\bar{\Phi}[0_{1\times 3} \ 1])\bar{\Phi}, \quad \bar{\Gamma} \ = \ \{F - G\bar{\Phi}[0_{1\times 3} \ 1] - \rho G\}^{-1} J.$$

Note that $\bar{\Gamma}$ is uniquely determined given a $\bar{\Phi}$, but $\bar{\Phi}$ is not generally uniquely determined, which induces multiplicity of fundamental REE.

[36] Recall that in this paper we refer to Evans and Honkapohja's (2001) MSV solutions to linear RE models as fundamental and do not undertake E-stability analysis of non-fundamental REE (see footnote 6).

[37] The form of the vector J is omitted, since it is not needed in what follows.

Following Section 10.5 of Evans and Honkapohja (2001), we analyze E-stability of fundamental REE.[38] Corresponding to fundamental RE solutions (27), all agents are assumed to be endowed with a perceived law of motion (PLM) of y_t

$$y_t = c + \Phi \hat{K}_t + \Gamma g_t. \tag{28}$$

Using a forecast from the PLM and the relation $\hat{K}_{t+1} = [0_{1\times 3}\ 1] y_t$ to substitute $\hat{E}_t y_{t+1}$ out of (26) leads to an actual law of motion (ALM) of y_t

$$\begin{aligned} y_t \quad = \quad & F^{-1} G (I + \Phi [0_{1\times 3}\ 1]) c + F^{-1} (G \Phi [0_{1\times 3}\ 1] \Phi + H) \hat{K}_t \\ & + F^{-1} \{ G (\Phi [0_{1\times 3}\ 1] \Gamma + \rho \Gamma) + J \} g_t \end{aligned} \tag{29}$$

provided that F is invertible. Here, I denotes a conformable identity matrix. Then, a mapping T from the PLM (28) to the ALM (29) can be defined by

$$\begin{aligned} T(c, \Phi, \Gamma) \quad = \quad & \big(F^{-1} G (I + \Phi [0_{1\times 3}\ 1]) c,\ F^{-1} (G \Phi [0_{1\times 3}\ 1] \Phi + H), \\ & F^{-1} \{ G (\Phi [0_{1\times 3}\ 1] \Gamma + \rho \Gamma) + J \} \big) . \end{aligned}$$

For a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ to be E-stable, the matrix differential equation

$$\frac{d}{d\tau}(c, \Phi, \Gamma) = T(c, \Phi, \Gamma) - (c, \Phi, \Gamma)$$

[38] System (26) contains a predetermined variable $\hat{K}_t$, so that we can consider two learning environments, which are studied respectively in Section 10.3 and 10.5 of Evans and Honkapohja (2001). One environment allows agents to use current endogenous variables in expectation formation, whereas another does not. In this paper we present only E-stability analysis with the latter environment, as in Bullard and Mitra (2002). This is because any inflation coefficient that generates a locally-unique non-explosive E-stable fundamental REE in the latter environment does so in the former one, as Kurozumi (2006) shows in an associated model without investment. An intuition for this is that in forming future expectations, agents have more information by the current endogenous variables and hence E-stability is more likely in the former environment than in the latter one. Another reason for our focus on the latter environment is that the former induces a problem with simultaneous determination of the expectations and current endogenous variables, which is critical to equilibrium under non-rational expectations as indicated by Evans and Honkapohja (2001) and Bullard and Mitra (2002).

must have local asymptotic stability at the solution, where τ denotes a notional time. Then, we have

$$\begin{aligned} DT_c(c, \Phi) &= F^{-1}G(I + \Phi[0_{1\times 3}\ 1]), \\ DT_\Phi(\Phi) &= F^{-1}G([0_{1\times 3}\ 1]\Phi I + \Phi[0_{1\times 3}\ 1]), \\ DT_\Gamma(\Phi, \Gamma) &= F^{-1}G(\rho I + \Phi[0_{1\times 3}\ 1]). \end{aligned}$$

Therefore, it follows that a fundamental RE solution $(\bar{c}, \bar{\Phi}, \bar{\Gamma})$ is E-stable if and only if all eigenvalues of three matrices, $DT_c(\bar{c}, \bar{\Phi})$, $DT_\Phi(\bar{\Phi})$, $DT_\Gamma(\bar{\Phi}, \bar{\Gamma})$, have real parts less than one. We summarize this result in the following lemma.

Lemma 1 *Suppose that the coefficient matrix F is invertible. A fundamental RE solution to the system of* (2)$-$(9) *with the forward-looking policy (i.e. $\phi_Y = \phi_R = 0$) is E-stable if and only if all eigenvalues of three matrices, $F^{-1}G(\gamma I + \bar{\Phi}[0_{1\times 3}\ 1])$, $\gamma = 1$, ρ, $\bar{\Phi}_4$, have real parts less than one, where $\bar{\Phi}_4$ is the fourth element of the RE solution vector $\bar{\Phi}$.*

With this lemma, we investigate E-stability of fundamental REE numerically, since it seems impossible to analytically solve the matrix equation for $\bar{\Phi}$ in fundamental RE solutions (27) and thus to obtain explicit conditions for the E-stability. To compute (27), we use the method of Klein (2000) and McCallum (1998), which is a generalization of Blanchard and Kahn (1980). As pointed out by McCallum, different non-explosive fundamental REE are obtained for different groupings of stable generalized eigenvalues of the matrix pencil for system (26).[39]

The E-stability analysis shows that in the presence of investment activity, the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if its inflation coefficient lies in either of the following two intervals, both of which satisfy the Taylor principle, i.e. $\phi_\pi > 1$. One interval is extremely narrow, where the inflation coefficient exceeds one and is very close to one. This contains the interval of inflation coefficients that bring about determinacy of REE, given by condition (15) of Corollary 1. Under the baseline calibration,

[39] In cases of indeterminacy, the baseline calibration shows order one or two indeterminacy and hence two or three distinct fundamental REE.

the interval for the unique E-stable REE is $1 < \phi_\pi < 1.0032$, which includes the one for determinacy, $1 < \phi_\pi < 1.0027$. Another interval requires that the inflation coefficient be sufficiently greater than one and its lower bound increase with stickier prices. This interval is $\phi_\pi > 1.26$ under the baseline calibration.[40] Any policy response to expected future inflation in these two intervals succeeds in guiding temporary equilibria under non-rational expectations toward the unique E-stable REE by the demand channel of monetary policy. As noted in footnote 25, the effect of the cost channel of monetary policy is negligible for inflation coefficients extremely close to one, and hence such policy responses yield the unique E-stable REE. Also, when the inflation coefficient is sufficiently greater than one, the effect of the cost channel relative to the demand channel is weak enough to generate the unique E-stable REE. Further, if the forward-looking policy responds also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle (12) generates a unique non-explosive E-stable fundamental REE including the determinate REE, which is also E-stable. These results suggest that the indeterminacy problem induced by the forward-looking policy, which is emphasized by CF, is not critical from the perspective of E-stability or least-squares learnability of fundamental REE.

Our E-stability results are a generalization of Bullard and Mitra (2002), who use an associated model without investment to show that the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE if and only if it meets the Taylor principle. In the presence of investment activity, the cost channel emerges and reduces the guiding effect of the demand channel. As a consequence, all non-explosive fundamental REE fail to be E-stable if the policy response to expected future inflation lies in the intermediate interval, if any, between the two intervals of the inflation coefficients that generate the unique E-stable REE.

[40] As prices become stickier, both the upper bound of the narrower interval and the lower bound of the wider interval increase, e.g. $1 < \phi_\pi < 1.0059$ (1.0202), $\phi_\pi > 1.54$ (2.71) for $\nu = 0.67$ (0.80).

5 Concluding remarks

In the presence of investment activity and price stickiness, indeterminacy of REE is induced by forward-looking monetary policy that sets the interest rate in response only to expected future inflation, as first shown by CF. This indeterminacy problem is due to a cost channel of monetary policy, whereby inflation expectations become self-fulfilling. We have examined two prescriptions for the problem. The first prescription has shown that the indeterminacy problem can be ameliorated once the forward-looking policy adjusts the interest rate in response also to current output or contains sufficiently strong interest rate smoothing, as empirical studies use it for a better description of actual monetary policy. In particular, the policy response to current output dramatically overcomes the indeterminacy problem in two ways: via policy responses to current consumption and investment. Both of these policy responses subdue changes in the real interest rate stemming from inflation expectations, thereby preventing self-fulfilling inflation expectations and hence indeterminacy. We have also demonstrated that the policy response to current investment rather than consumption is crucial to the determinacy with the policy response to current output in our model, since feedback from current consumption on interest rate policy is limited due to consumption smoothing. The second prescription has shown that when we adopt E-stability as the REE selection criterion, even the forward-looking policy generates a locally-unique non-explosive E-stable fundamental REE as long as its inflation coefficient is sufficiently strong. Further, if the policy adjusts the interest rate in response also to current output, almost every pair of the inflation and output coefficients that meets the long-run version of the Taylor principle generates the unique E-stable REE.

We use a stochastic version of CF's model. In the actual economy, aggregate variables such as consumption and investment display more considerable persistence than in our model. In order to fit models to actual data, recent business cycle literature such as Christiano et al. (2005) and Smets and Wouters (2003) allows for habit formation in preferences for consumption, a finite labor supply elasticity, staggered nominal wage setting in monopolistically competitive labor markets, adjustment costs in investment or capital, variable capital utilization, and so forth, which all are absent in our model. A few recent studies incorporate some of these features into our fundamental model to investigate equilibrium determinacy numerically. Xiao (2007) uses such a model incorporating a finite labor supply elasticity and a capital adjustment cost and then shows that a mild policy response to expected future output helps the forward-looking policy ensure determinacy. Huang and Meng (2007b) employ a similar model to Xiao to find that under an empirically reasonable labor supply elasticity, the policy response to current output fails to make the forward-looking policy generate determinacy, but once staggered nominal wage setting is incorporated into their model, the role of such a policy response in guaranteeing determinacy is greatly enhanced. These studies suggest that one topic of our future research is to examine what empirically relevant extension of our fundamental model may or may not help the forward-looking policy bring about equilibrium determinacy.

Another topic is E-stability analysis of non-fundamental REE. In this paper we have investigated only fundamental REE. Some readers may consider this focus unappealing. Carlstrom and Fuerst (2004), however, show that a sunspot equilibrium is E-stable only if a central bank believes in the sunspot, using an associated model without investment. Because this condition is not practical, our focus on fundamental REE might be plausible. To make sure of the validity of our focus, we will examine E-stability of non-fundamental REE in our model, following recent analyses with associated models without investment, such as Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004), and Evans and McGough (2005).

Appendix

A Coefficient matrices in systems of form (10)

Let $a_1 = 1 - \beta(1-\delta)(1-\alpha)$ and $a_2 = 1 - \beta(1-\delta)$. When the forward-looking policy responds also to expected output, i.e. $j = 1$, $\phi_R = 0$ in (1), the coefficient matrix A of system (10) is given by

$$A = [A_{mn}] = \begin{bmatrix} \frac{1}{\beta} & -\frac{\lambda}{\beta\sigma} & -\frac{\alpha\lambda}{\beta(1-\alpha)} & \frac{\alpha\lambda}{\beta(1-\alpha)} & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} & 0 \\ A_{31} & A_{32} & A_{33} & A_{34} & 0 \\ 0 & -\frac{\delta s_C}{s_I} & \frac{\delta}{s_I} & 1-\delta & 0 \\ A_{51} & A_{52} & A_{53} & A_{54} & 0 \end{bmatrix}, \tag{30}$$

where

$$\begin{aligned}
&A_{21} = \widetilde{A}_{21}, \quad A_{22} = \widetilde{A}_{22} + 1, \quad A_{23} = \widetilde{A}_{23}, \quad A_{24} = \widetilde{A}_{24}, \quad \widetilde{A}_{2n} = \sigma[A_{1n}(\phi_\pi - 1) + A_{3n}\phi_Y], \\
&A_{31} = \widetilde{A}_{31}, \quad A_{32} = \widetilde{A}_{32} + \frac{a_2[A_{42} - (1-\alpha)/\sigma]}{a_2 + (a_1 - 1)\phi_Y}, \quad A_{33} = \widetilde{A}_{33} + \frac{a_2 A_{43}}{a_2 + (a_1 - 1)\phi_Y}, \\
&A_{34} = \widetilde{A}_{34} + \frac{a_2 A_{44}}{a_2 + (a_1 - 1)\phi_Y}, \quad \widetilde{A}_{3n} = \frac{A_{1n}(1 - a_1)(\phi_\pi - 1)}{a_2 + (a_1 - 1)\phi_Y}, \\
&A_{51} = \widetilde{A}_{51}, \quad A_{52} = \widetilde{A}_{52}, \quad A_{53} = \widetilde{A}_{53}, \quad A_{54} = \widetilde{A}_{54}, \quad \widetilde{A}_{5n} = A_{1n}\phi_\pi + A_{3n}\phi_Y.
\end{aligned}$$

When the forward-looking policy responds also to current output, i.e. $j = 0$, $\phi_R = 0$ in (1), the system takes the same form as (10) with the same coefficient matrix A as (30), except

$$\begin{aligned}
&A_{21} = \bar{A}_{21}, \quad A_{22} = \bar{A}_{22} + 1, \quad A_{23} = \bar{A}_{23} + \sigma\phi_Y, \quad A_{24} = \bar{A}_{24}, \quad \bar{A}_{2n} = \sigma A_{1n}(\phi_\pi - 1), \\
&A_{31} = \bar{A}_{31}, \quad A_{32} = \bar{A}_{32} + A_{42}, \quad A_{33} = \bar{A}_{33} + A_{43} + \frac{(1-\alpha)\phi_Y}{a_2}, \quad A_{34} = \bar{A}_{34} + A_{44}, \\
&\bar{A}_{3n} = (1-\alpha)\left[\frac{A_{1n}(\phi_\pi - 1)}{a_2} - \frac{A_{2n}}{\sigma}\right], \\
&A_{51} = \bar{A}_{51}, \quad A_{52} = \bar{A}_{52}, \quad A_{53} = \bar{A}_{53} + \phi_Y, \quad A_{54} = \bar{A}_{54}, \quad \bar{A}_{5n} = A_{1n}\phi_\pi.
\end{aligned}$$

The characteristic equation of the coefficient matrix A is given by

$$\mu\, P(\mu) = \mu\left(\mu^4 + h_3\mu^3 + h_2\mu^2 + h_1\mu + h_0\right) = 0,$$

where

$$
\begin{aligned}
h_3 &= -2-\frac{1}{\beta}-\frac{\delta(1-s_I)}{s_I}+\frac{\lambda a_1}{\beta a_2}(\phi_\pi-1)-\frac{1-a_1}{a_2}\phi_Y,\\
h_2 &= \frac{1}{\beta}+\left(1+\frac{1}{\beta}\right)\left[1+\frac{\delta(1-s_I)}{s_I}\right]-\frac{\lambda}{\beta a_2}\left\{\alpha+a_1\left[1+\frac{\delta(1-s_I)}{s_I}\right]\right\}(\phi_\pi-1)\\
&\quad+\left\{\frac{(1-a_1)[2-\delta+\beta(1-\delta)^2]}{\beta a_2(1-\delta)}+\frac{\delta s_C\sigma}{s_I}\right\}\phi_Y,\\
h_1 &= -\left[1+\frac{\delta(1-s_I)}{s_I}\right]\left[\frac{1}{\beta}-\frac{\lambda\alpha}{\beta a_2}(\phi_\pi-1)\right]-\left\{\frac{(1-a_1)[1+\beta(1-\delta)(2-\delta)]}{\beta^2 a_2(1-\delta)}+\frac{\delta s_C\sigma}{\beta s_I}\right\}\phi_Y,\\
h_0 &= \frac{1-a_1}{\beta^2 a_2}\phi_Y.
\end{aligned}
$$

When the forward-looking policy responds also to current consumption as in (18), the system takes the same form as (10) with the same coefficient matrix A as the one with the policy response to current output, except $A_{22}=\bar{A}_{22}+\sigma\phi_C+1$, $A_{23}=\bar{A}_{23}$, $A_{32}=\bar{A}_{32}+A_{42}+(1-\alpha)\phi_C/a_2$, $A_{33}=\bar{A}_{33}+A_{43}$, $A_{52}=\bar{A}_{52}+\phi_C$, $A_{53}=\bar{A}_{53}$.

When the forward-looking policy responds also to current investment as in (20), the system takes the same form as (10) with the same coefficient matrix A as the one with the policy response to current output, except $A_{22}=\bar{A}_{22}-\sigma s_C\phi_I/s_I+1$, $A_{23}=\bar{A}_{23}+\sigma\phi_I/s_I$, $A_{32}=\bar{A}_{32}+A_{42}-(1-\alpha)s_C\phi_I/(s_I a_2)$, $A_{33}=\bar{A}_{33}+A_{43}+(1-\alpha)\phi_I/(s_I a_2)$, $A_{52}=\bar{A}_{52}-s_C\phi_I/s_I$, $A_{53}=\bar{A}_{53}+\phi_I/s_I$. The characteristic equation of the coefficient matrix A is given by

$$\mu\, Q(\mu)=\mu\left(\mu^4+j_3\mu^3+j_2\mu^2+j_1\mu+j_0\right)=0,$$

where

$$
\begin{aligned}
j_3 &= -2-\frac{1}{\beta}-\frac{\delta(1-s_I)}{s_I}+\frac{\lambda a_1}{\beta a_2}(\phi_\pi-1)-\frac{1-a_1-s_C\sigma a_2}{s_I a_2}\phi_I,\\
j_2 &= \frac{1}{\beta}+\left(1+\frac{1}{\beta}\right)\left[1+\frac{\delta(1-s_I)}{s_I}\right]-\frac{\lambda}{\beta a_2}\left\{\alpha+a_1\left[1+\frac{\delta(1-s_I)}{s_I}\right]\right\}(\phi_\pi-1)\\
&\quad+\left\{\frac{(1-a_1)[2-\delta+\beta(1-\delta)^2]}{\beta s_I a_2(1-\delta)}-\frac{s_C\sigma[1+\beta(1-\delta)]}{\beta s_I}\right\}\phi_I,\\
j_1 &= -\left[1+\frac{\delta(1-s_I)}{s_I}\right]\left[\frac{1}{\beta}-\frac{\lambda\alpha}{\beta a_2}(\phi_\pi-1)\right]-\left\{\frac{(1-a_1)[1+\beta(1-\delta)(2-\delta)]}{\beta^2 s_I a_2(1-\delta)}-\frac{s_C\sigma(1-\delta)}{\beta s_I}\right\}\phi_I,\\
j_0 &= \frac{1-a_1}{\beta^2 s_I a_2}\phi_I.
\end{aligned}
$$

In the case of the forward-looking policy with interest rate smoothing, i.e. $\phi_Y=0$ in (1), the system takes the same form as (10) with the same coefficient matrix A as the one with the policy response to current output, except $A_{23} = \bar{A}_{23}$, $A_{25} = \sigma\phi_R$, $\bar{A}_{2n} = \sigma A_{1n}[(1-\phi_R)\phi_\pi - 1]$, $A_{33} = \bar{A}_{33} + A_{43}$, $A_{35} = (1-a_1)\phi_R/a_2$, $\bar{A}_{3n} = (1-\alpha)\{A_{1n}[(1-\phi_R)\phi_\pi - 1]/a_2 - A_{2n}/\sigma\}$, $A_{53} = \bar{A}_{53}$, $A_{55} = \phi_R$, $\bar{A}_{5n} = A_{1n}(1-\phi_R)\phi_\pi$.

B Proof of Proposition 1

For the system's coefficient matrix A given in Appendix A, we can show that its five eigenvalues are two zeros and three solutions to the cubic equation

$$b_3\mu^3 + b_2\mu^2 + b_1\mu + b_0 = 0,$$

where

$$\begin{aligned}
b_3 &= a_2 - (1-a_1)\phi_Y, \quad a_1 = 1-\beta(1-\delta)(1-\alpha), \quad a_2 = 1-\beta(1-\delta), \\
b_2 &= -a_2\left[2 + \frac{1}{\beta} + \frac{\delta(1-s_I)}{s_I}\right] + \frac{\lambda a_1}{\beta}(\phi_\pi - 1) + \left\{(1-\alpha)[2-\delta+\beta(1-\delta)^2] + \frac{\delta s_C \sigma a_2}{s_I}\right\}\phi_Y, \\
b_1 &= \frac{a_2}{\beta}\left[2 + \beta + \frac{\delta(1+\beta)(1-s_I)}{s_I}\right] - \frac{\lambda}{\beta}\left\{\alpha + a_1\left[1 + \frac{\delta(1-s_I)}{s_I}\right]\right\}(\phi_\pi - 1) \\
&\quad - \left\{(1-\alpha)\left[\frac{1}{\beta} + (1-\delta)(2-\delta)\right] + \frac{\delta s_C \sigma a_2}{\beta s_I}\right\}\phi_Y, \\
b_0 &= -\frac{a_2}{\beta}\left[1 + \frac{\delta(1-s_I)}{s_I}\right] + \frac{\lambda\alpha}{\beta}\left[1 + \frac{\delta(1-s_I)}{s_I}\right](\phi_\pi - 1) + \frac{1-a_1}{\beta^2}\phi_Y.
\end{aligned}$$

Because the policy generates local determinacy of REE if and only if the matrix A has exactly two eigenvalues inside the unit circle and the other three outside the unit circle, it follows that the necessary and sufficient condition for determinacy is that all three solutions to the cubic equation are outside the unit circle. Hence, determinacy requires that the cubic equation have non-zero solutions. This implies that $b_0 \neq 0$ and the cubic equation can be rewritten as, letting $\widetilde{\mu} = 1/\mu$,

$$b_0\widetilde{\mu}^3 + b_1\widetilde{\mu}^2 + b_2\widetilde{\mu} + b_3 = 0.$$

The necessary and sufficient condition is now that all three solutions to this new cubic equation are inside the unit circle. Then, from the Schur-Cohn criterion,[41] it follows that determinacy

[41] See e.g. Proposition 5.3 of LaSalle (1986).

is obtained if and only if either of the following two cases is satisfied.

$$\text{(Case I)} \quad b_0 < 0,\ b_0 + b_1 + b_2 + b_3 < 0,\ b_0 - b_1 + b_2 - b_3 < 0,\ b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|;$$

$$\text{(Case II)} \quad b_0 > 0,\ b_0 + b_1 + b_2 + b_3 > 0,\ b_0 - b_1 + b_2 - b_3 > 0,\ b_0^2 - b_3^2 > |b_0 b_2 - b_1 b_3|.$$

Then, the first three inequalities in (Case I) can be reduced to (11)−(13), respectively.

C Proof of Proposition 2

For the system's coefficient matrix A given in Appendix A, we can show that its five eigenvalues are two zeros, $1 + \delta(1/s_I - 1) > 1$, and two solutions to the quadratic equation

$$\mu^2 + c_1 \mu + c_0 = 0,$$

where $c_1 = -1 - 1/\beta + \lambda a_1(\phi_\pi - 1)/(\beta a_2) - \sigma\phi_C$, $c_0 = 1/\beta - \alpha\lambda(\phi_\pi - 1)/(\beta a_2) + \sigma\phi_C/\beta$, $a_1 = 1 - \beta(1-\delta)(1-\alpha)$, and $a_2 = 1 - \beta(1-\delta)$.

From an analogous argument to that in the proof of Proposition 1, it follows that the necessary and sufficient condition for local determinacy of REE is that both solutions to the quadratic equation are outside the unit circle. By Proposition C.1 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

$$\text{(Case I)} \quad c_0 > 1, \quad c_0 + c_1 > -1, \quad c_0 - c_1 > -1;$$

$$\text{(Case II)} \quad c_0 + c_1 < -1, \quad c_0 - c_1 < -1.$$

The three inequalities in (Case I) can be reduced to (19). To complete the proof, it suffices to show that (Case II) never obtains. To see this, the two inequalities in (Case II) can be reduced to

$$\phi_\pi - 1 \ < \ -\frac{\sigma(1-\beta)}{\lambda(1-\alpha)}\phi_C, \quad \phi_\pi - 1 \ > \ \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)} + \frac{\sigma a_2(1+\beta)}{\lambda(a_1+\alpha)}\phi_C.$$

Combining these two yields a contradiction

$$0 \ < \ \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)} + \frac{\sigma a_2(1+\beta)}{\lambda(a_1+\alpha)}\phi_C \ < \ \phi_\pi - 1 \ < \ -\frac{\sigma(1-\beta)}{\lambda(1-\alpha)}\phi_C \ < \ 0.$$

D Proof of Proposition 3

For the system's coefficient matrix A given in Appendix A, we can show that its five eigenvalues are 0, $1+\delta(1/s_I - 1) > 1$, and three solutions to the cubic equation

$$\mu^3 + d_2\mu^2 + d_1\mu + d_0 = 0,$$

where $d_2 = -1 - 1/\beta + \lambda a_1[(1-\phi_R)\phi_\pi - 1]/(\beta a_2) - \phi_R$, $d_1 = 1/\beta - \alpha\lambda[(1-\phi_R)\phi_\pi - 1]/(\beta a_2) + [\lambda a_1/(\beta a_2) + 1 + 1/\beta]\phi_R$, $d_0 = -[\alpha\lambda/(\beta a_2)+1]\phi_R/\beta$, $a_1 = 1-\beta(1-\delta)(1-\alpha)$, and $a_2 = 1-\beta(1-\delta)$.

From an argument similar to that in the proof of Proposition 1, it follows that the necessary and sufficient condition for local determinacy of REE is that one solution to the cubic equation is inside the unit circle and the other two are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

(Case I) $1 + d_2 + d_1 + d_0 < 0,\ \ -1 + d_2 - d_1 + d_0 > 0;$

(Case II) $1 + d_2 + d_1 + d_0 > 0,\ \ -1 + d_2 - d_1 + d_0 < 0,\ \ |d_2| > 3$ or $d_0(d_0 - d_2) + d_1 - 1 > 0.$

The first two inequalities in (Case II) can be reduced to (24). To complete the proof, it suffices to show that (Case I) never obtains. To see this, the two inequalities in (Case I) can be reduced to

$$\phi_\pi \ > \ \frac{1+\phi_R}{1-\phi_R}\left[1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)}\right], \quad \phi_\pi \ < \ 1.$$

Combining these two yields a contradiction

$$1 \ < \ \frac{1+\phi_R}{1-\phi_R}\left[1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)}\right] \ < \ \phi_\pi \ < \ 1.$$

E Coefficient matrices in system (26)

The coefficient matrices F, G, H of system (26) are given by, letting $a_2 = 1-\beta(1-\delta)$,

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{a_2}{1-\alpha} \\ 0 & -s_C & 1 & -\frac{s_I}{\delta} \\ 1 & -\frac{\lambda}{\sigma} & -\frac{\lambda\alpha}{1-\alpha} & 0 \end{bmatrix}, \quad G = \begin{bmatrix} -\sigma(\phi_\pi - 1) & 1 & 0 & 0 \\ -(\phi_\pi - 1) & \frac{a_2}{\sigma} & \frac{a_2}{1-\alpha} & 0 \\ 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ -\frac{s_I(1-\delta)}{\delta} \\ -\frac{\lambda\alpha}{1-\alpha} \end{bmatrix}.$$

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Table 1: Baseline calibration

β	discount factor	0.99
σ	intertemporal substitution elasticity of consumption	1
λ	real marginal cost elasticity of inflation	1/3
α	cost share of capital	1/3
δ	depreciation rate of capital	0.02
s_C	steady state output share of consumption	0.7
s_I	steady state output share of investment	0.3
ρ	autoregression parameter for preference shocks	0.35

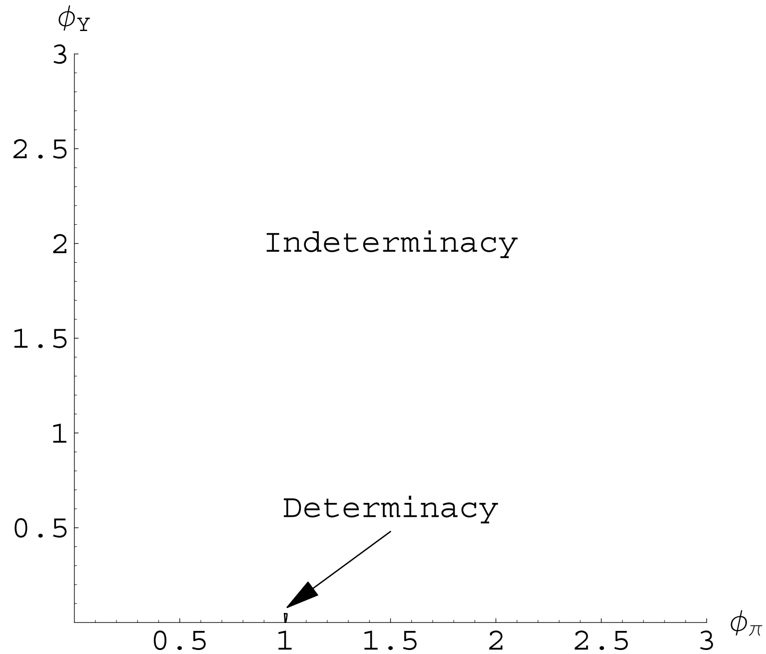

Figure 1: Region of policy coefficients generating local determinacy of REE: case of expected future output

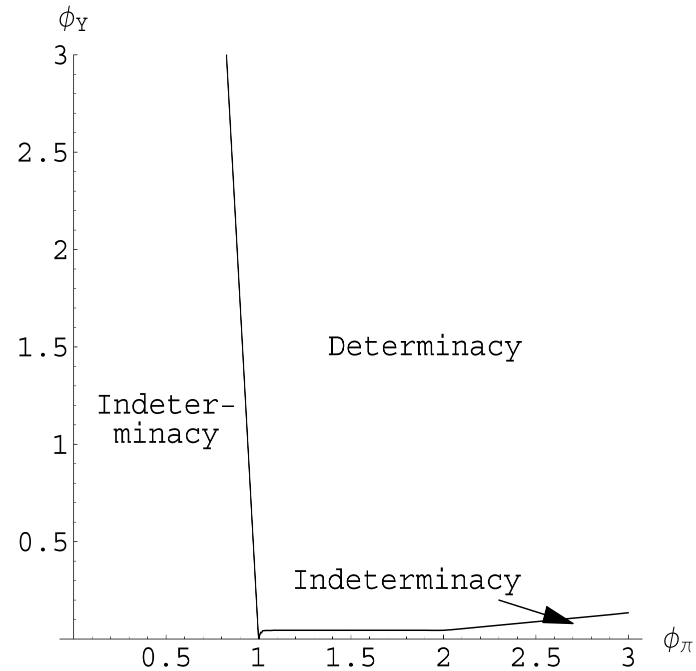

Figure 2: Region of policy coefficients generating local determinacy of REE: case of current output

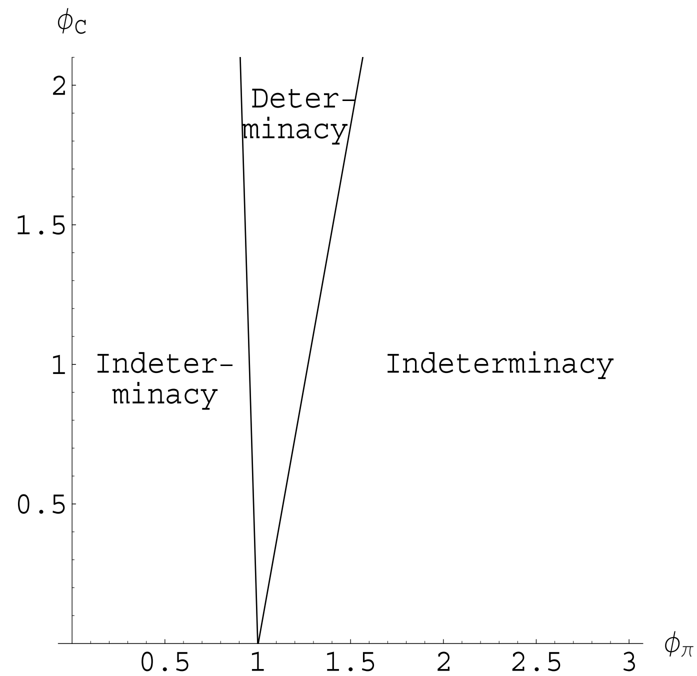

Figure 3: Region of policy coefficients generating local determinacy of REE: case of current consumption

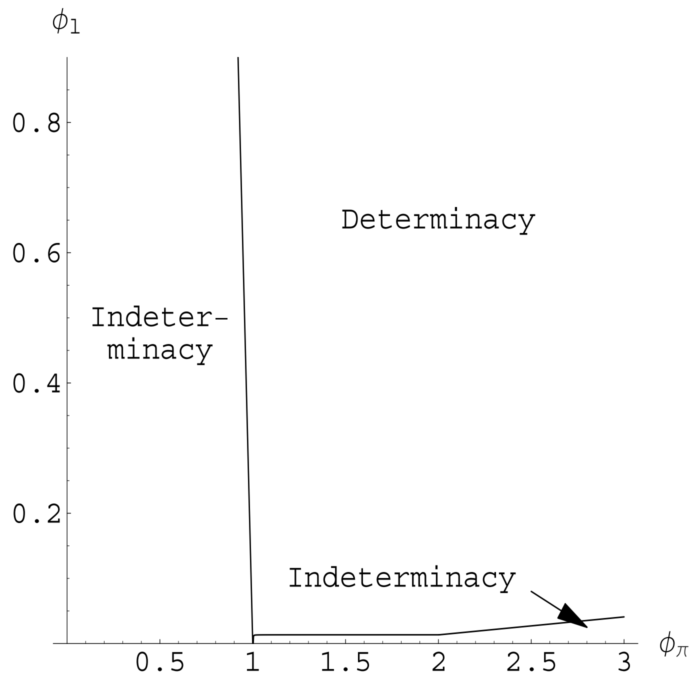

Figure 4: Regions of policy coefficients ensuring local determinacy of REE: case of current investment

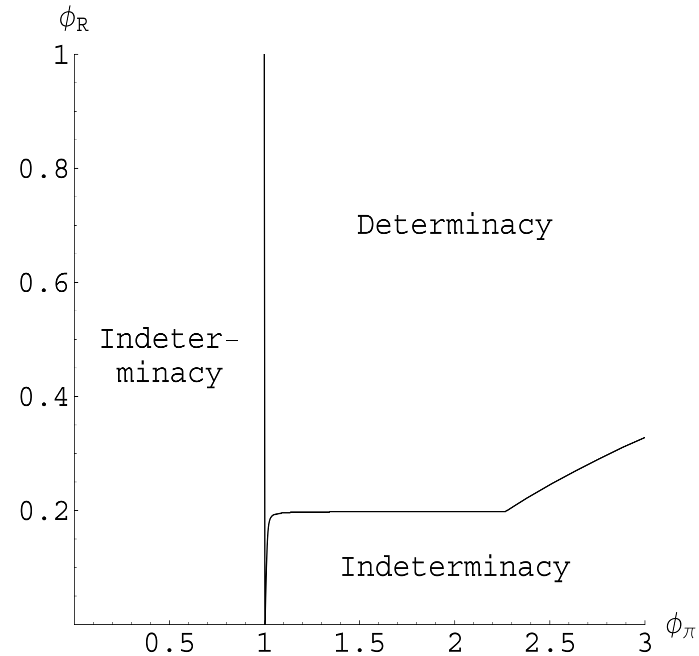

Figure 5: Region of policy coefficients generating local determinacy of REE: case of interest rate smoothing