Inflation Expectations, Monetary Policy and Irrational Estimates

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“Today, it seems that we may witnessing the early stages of an “irrational expectations revolution”. What this new revolution needs to succeed, I suggest, is some “irrational expectations econometrics” to make these purely theoretical results seem more relevant and convincing”.

(Peter Ireland, March 2003, Federal Reserve Bank of Atlanta Conference on “Monetary Policy and Learning”)

What elements characterize these “*irrational expectations revolution*” :

- DSGE models (under Rational Expectations)
- Monetary Policy analysis
- Learning

“*Irrational estimates*” to do what?

1. Validation
2. Internal Consistency

Objective

- Describe a theoretical model with the above elements
- Derive some theoretical implications
- Run “irrational estimates” for validation:

  1. look for empirical evidence of learning vs RE

  2. show that Structural change in monetary policy may have determined a learning process in the Euro Area.

…work in process

- Use “irrational estimates” for internal consistency

The Baseline Model:

- IS curve:

$$x_t = \hat{E}_t x_{t+1} - \varphi\left(i_t - \hat{E}_t \pi_{t+1}\right) + g_t$$

$$g_t \text{ i.i.d. } \mathrm{N}\left(0, \sigma_g^2\right)$$

- AS curve:

$$\pi_t = \alpha x_t + \beta \hat{E}_t \pi_{t+1}$$

- Expectations-based reaction functions:

$$i_t = \gamma + \gamma_\pi \hat{E}_t \pi_{t+1} + \gamma_x \hat{E}_t x_{t+1}$$

with $\gamma_x = \varphi^{-1}$

- The economy evolves according to:

$$y_t = Q + F\hat{E}_t y_{t+1} + Sg_t$$

$$y_t = \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \qquad F = \begin{bmatrix} \beta + \alpha\varphi(1-\gamma_\pi) & 0 \\ \varphi(1-\gamma_\pi) & 0 \end{bmatrix}$$

- Rational Expectations Equilibrium:

$$E_t \pi_{t+1} = a_\pi \qquad \pi_t = a_\pi + \alpha g_t$$

- Least Squares Learning:

Perceived Law of Motion (PLM): $\hat{E}_t \pi_{t+1} = a_{\pi,t}$

where: $a_{\pi,t} = a_{\pi,t-1} + t^{-1}\left(\pi_{t-1} - a_{\pi,t-1}\right)$

Actual Law of Motion (ALM): $\pi_t = T\left(a_{\pi,t}\right) + \alpha g_t$

where: $T\left(a_{\pi,t}\right) = -\alpha\varphi\gamma + \left[\beta + \alpha\varphi\left(1-\gamma_\pi\right)\right]a_{\pi,t}$

Ordinary Differential Equation (ODE): $\dfrac{da_\pi}{d\tau} = E\left[T\left(a_\pi\right) + \alpha g - a_\pi\right]$

E-Stability

- If the slope of the *T(.)* mapping is smaller than 1, $\bar{a}_\pi$ is E-stable

Speed of convergence

- If the slope of the *T(.)* mapping is smaller than 1/2

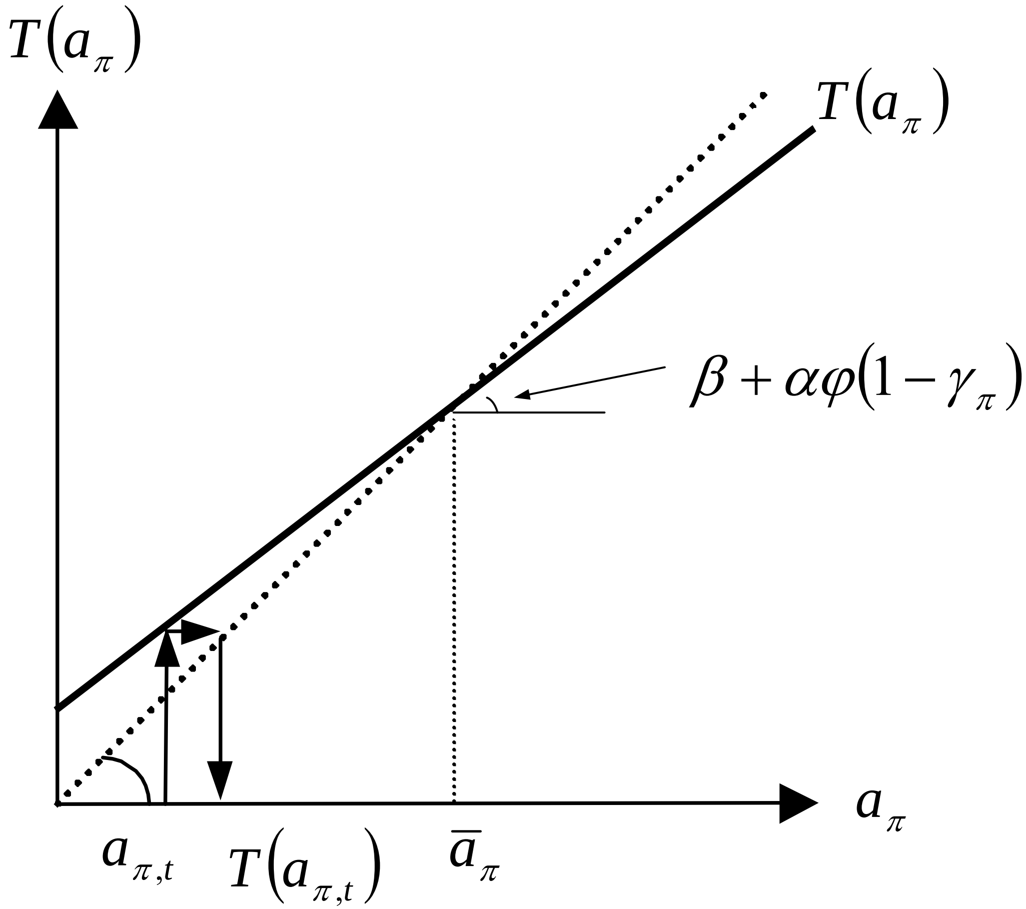
$$\sqrt{t}\left(a_{\pi,t} - \bar{a}_\pi\right) \xrightarrow{D} N\left(0, \sigma_a^2\right)$$

where

$$\sigma_a^2 = \frac{\alpha^2 \sigma_g^2}{1 - \beta - \alpha\varphi\left(1 - \gamma_\pi\right)}$$

The steeper the slope, the larger σ_a^2 , the slower the convergence.

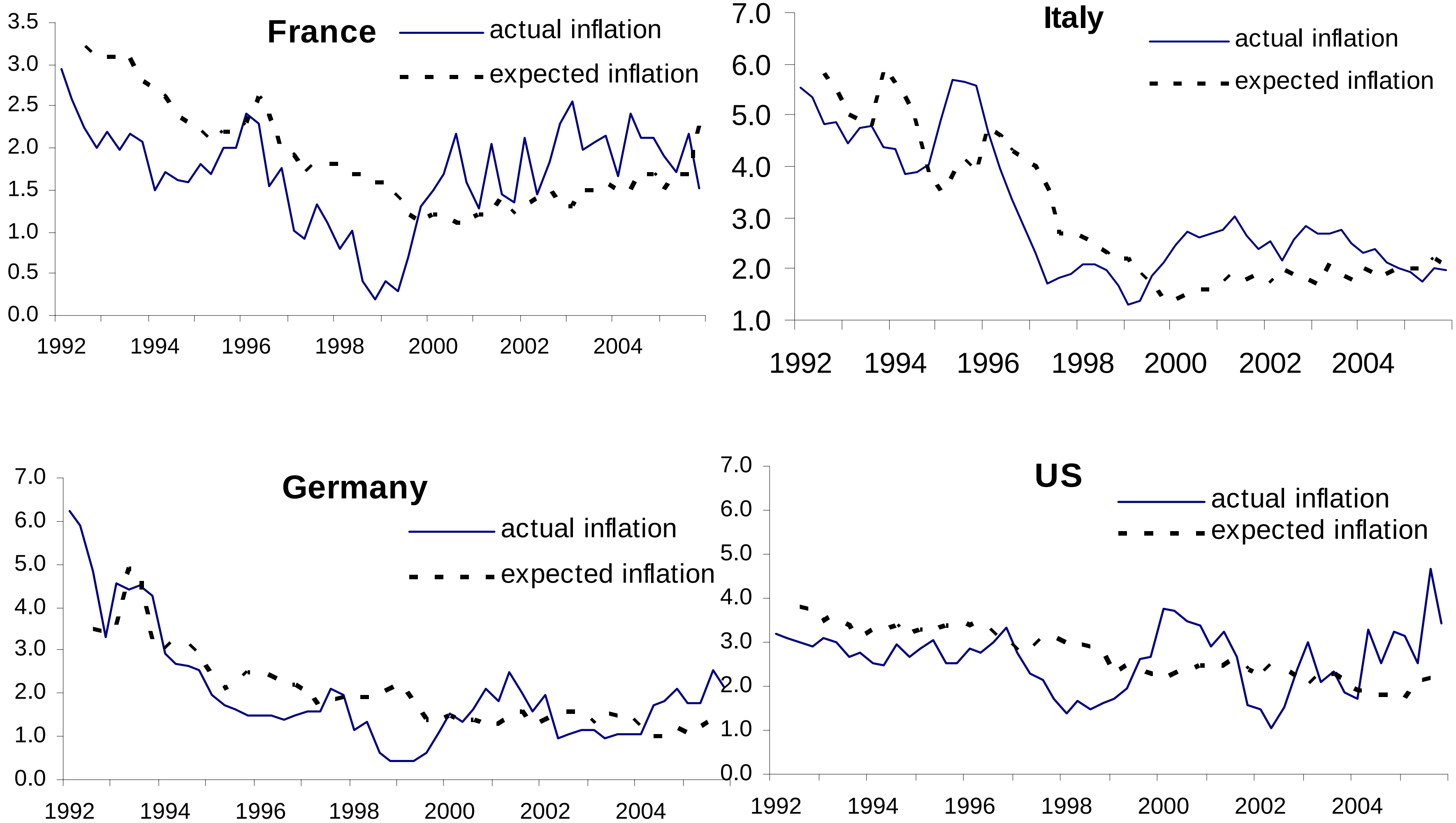
- If the slope of the T(.) mapping is **smaller than 1, but bigger than 1/2**, through simulations we can show that: <u>the steeper the slope, the slower the convergence</u>.



$T(a_\pi)$
$T(a_\pi)$
$\beta + \alpha\varphi(1-\gamma_\pi)$
a_π
$a_{\pi,t}$
$T(a_{\pi,t})$
$\bar{a}_\pi$

Empirical evidence of learning vs RE

Actual Inflation vs Inflation Forecast 1 year before



- The main difficulty in order to distinguish between rational expectations and least squares learning, by looking at data, is that asymptotically the two hypothesis imply the same equilibrium, the REE.

- This implies that we should consider short samples in order to distinguish between the two hypothesis.

- The main approach, in the literature, is to identify departure from rationality as rejections of unbiasedness and efficiency hypothesis.

Unbiasedness test

- Usually the unbiasedness tests are conducted by considering the following simple regression equation:

$$\pi_{t+k} = a_0 + a_1 \hat{E}_t \pi_{t+k} + \varepsilon_{t+k}$$

and test for H_0: $a_0=0$, $a_1=1$.

- However, under learning, we have seen that a_1 gives us information about the speed of convergence:

when a_1 is smaller than 1 but close to 1, we **have slow convergence**:
in every period the **prediction error is very low** (a_0 is close to 0)

- Regressions obtained using quarterly data from *Consensus Forecast* survey on annual expected inflation from 1990 to 2002.

95 percent confidence interval for a_1

Germany	0.992	1.183
France	0.699	0.791
Italy	0.759	0.871
Euro Area	0.981	1.113
United States	0.961	1.144

Two groups of countries:

1) **France and Italy** - we cannot reject learning,
we can reject RE

2) **Germany, US, Euro Area** - we cannot reject "slow" learning,
we cannot reject RE

Test on the ex-post prediction errors

- Another way to test for RE versus learning is to consider directly the **time series of the prediction errors**.

$$u_{t+1} = \pi_{t+1} - \hat{E}_t \pi_{t+1}$$

1) under rational expectations, $E(u_t\, u_{t+1})=0$;

2) under learning, $E(u_t\, u_{t+1})=$ different from 0.

- Problem when considering the k-ahead forecast error:

$$u_{t+k,k} = \pi_{t+k} - \hat{E}_t \pi_{t+k}$$

under rational expectations $E(u_{t,k}\, u_{t+h,k})=0$ only for $h \geq k$ (Hansen and Hodrick, 1980)

- Run regressions: $u_{t+4,4} = b_0 + b_1 u_{t,4} + \varepsilon_t$

95 percent confidence interval for a_1

Germany	0.079	0.163
France	0.416	0.476
Italy	0.086	0.162
Euro Area	0.453	0.462
United States	−0.01	0.030

Three groups of countries:

1) **US** - we cannot reject b_1=0
2) **Germany, Italy** - we cannot reject $b_1 \neq 0$, but b_1 small
3) **France, Euro Area** - we cannot reject $b_1 \neq 0$, but b_1 big

<u>Problem</u>: important information is sacrificed
(all the autocorrelations with lags smaller than 4)

Empirical evidence of a change in rationality

- Look at data before and after the start of stage 3 of the EMU (january 1999), in order to check if an element of discontinuity have been introduced in the area that have affected private agents' forecasting mechanism.

Unbiasedness test: 95 percent confidence interval for a_1

	(1990-1999)		(1999-2004)	
Germany	1.177	1.441	0.855	1.116
France	1.003	1.095	0.866	0.992
Italy	0.916	0.997	0.688	0.813
Euro Area	1.285	1.459	0.785	1.021
United States	1.033	1.209	0.971	1.189

Chow test: we cannot reject the hypothesis of a structural break at a 95 percent significance level for France and Italy.

<u>Test on prediction errors</u>: 95 percent confidence interval for b_1

	(1990-1999)		(1999-2004)	
Germany	0.094	0.216	−0.06	0.071
France	0.165	0.245	0.053	0.134
Italy	−0.20	−0.11	−0.05	0.063
Euro Area	0.64	0.72	−0.02	0.102
United States	−0.15	−0.10	−0.22	−0.10

Chow test: we cannot reject the hypothesis of a structural break at a 95 percent significance level for all countries except Germany.

…back to the theoretical model

Using the **estimates of the parameters** of the IS curve, AS curve and the interest rate rules, we derive measures of the speed of convergence in different countries.

The Baseline Model:

- IS curve: $$x_t = \hat{E}_t x_{t+1} - \varphi\left(i_t - \hat{E}_t \pi_{t+1}\right) + g_t$$

- AS curve: $$\pi_t = \alpha x_t + \beta \hat{E}_t \pi_{t+1} + u_t$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u\,t}, \qquad \varepsilon_{u\,t} \text{ i.i.d. N}\left(0, \sigma^2_{\varepsilon u}\right)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g\,t}, \qquad \varepsilon_{u\,t} \text{ i.i.d. N}\left(0, \sigma^2_{\varepsilon g}\right)$$

- Expectations-based reaction functions: $$i_t = \gamma + \gamma_\pi \hat{E}_t \pi_{t+1} + \gamma_x \hat{E}_t x_{t+1}$$

- The economy evolves according to:

$$y_t = Q + F\hat{E}_t y_{t+1} + Sw_t$$

$$y_t = \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \qquad w_t = \begin{bmatrix} u_t \\ g_t \end{bmatrix} \qquad F = \begin{bmatrix} \beta + \alpha\varphi(1-\gamma_\pi) & \alpha(1-\varphi\gamma_x) \\ \varphi(1-\gamma_\pi) & (1-\varphi\gamma_x) \end{bmatrix}$$

- Rational Expectations Equilibrium:

$$y_t = A + Bw_t, \qquad E_t y_{t+1} = A + B\Psi w_t, \qquad \text{where } \Psi = \begin{bmatrix} \rho_u & 0 \\ 0 & \rho_g \end{bmatrix}$$

- Least Squares Learning:

$$\boxed{\textit{Perceived Law of Motion (PLM):} \quad \hat{E}_t y_{t+1} = A_t + B_t w_t}$$

with:
$$\beta_t = \beta_{t-1} + t^{-1} R_{t-1}^{\ \ -1} z_{t-1}\left(y_{t-1} - z'_{t-1}\,\beta_{t-1}\right)$$
$$R_t = R_{t-1} + t^{-1}\left(z'_{t-1}\,z_{t-1} - R_{t-1}\right)$$

where: $\beta_t = \begin{bmatrix} A_t \\ B_t \end{bmatrix},\ \ z_t = \begin{bmatrix} 1 \\ w_t \end{bmatrix}$

$$\boxed{\textit{Actual Law of Motion (ALM):} \quad y_t = T\left(\beta_{t-1}\right)' z_{t-1}}$$

where: $T\left(A_t, B_t\right) = \left(Q + FA_t, FB_t\Psi + S\right)$

$$\boxed{\textit{Ordinary Differential Equation (ODE):} \quad \frac{d\left(A, B\right)}{d\,\tau} = T\left(A, B\right) - \left(A, B\right)}$$

E-Stability

- If all eigenvalues of F have real part **smaller than 1**, the REE is E-stable

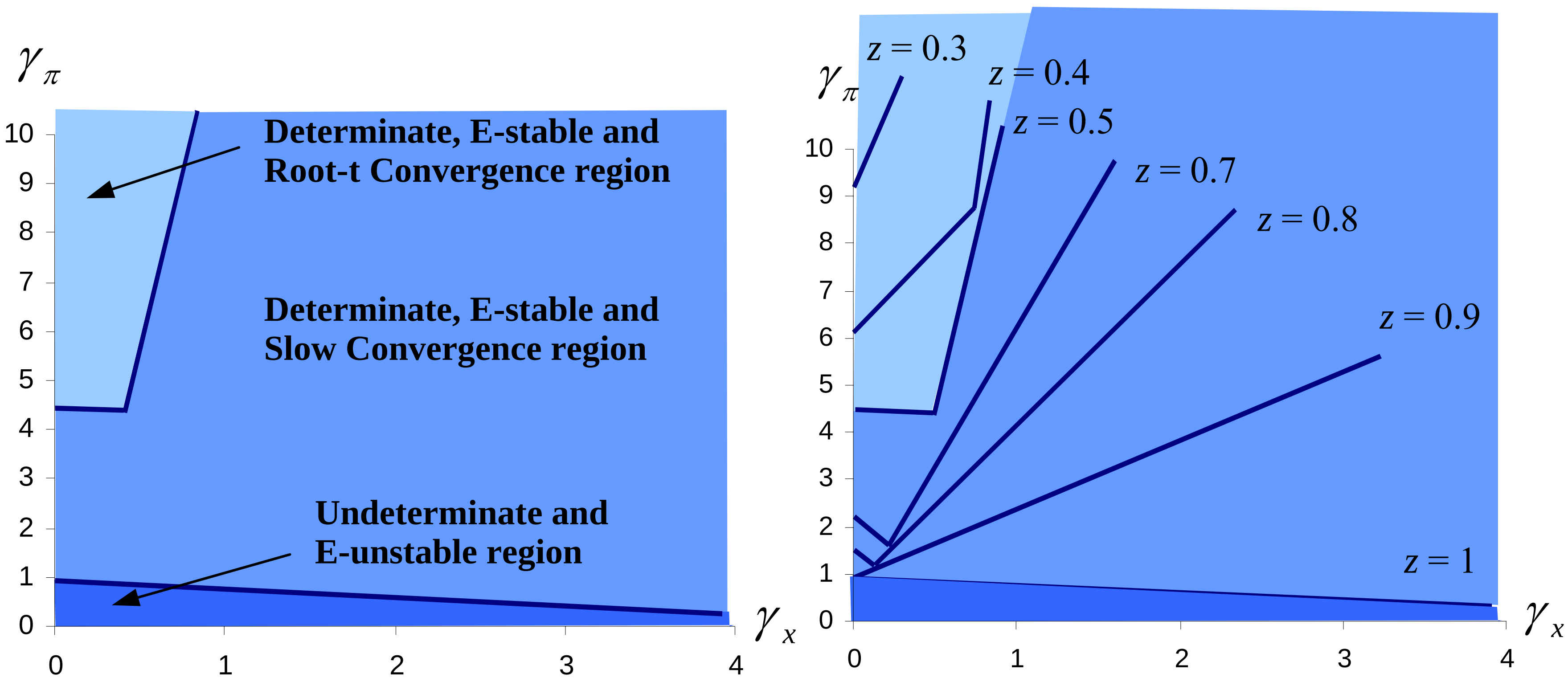
Speed of convergence

- If all eigenvalues of F have real part **smaller than 1/2**

$$\sqrt{t}\left(\beta_t - \overline{\beta}\right) \xrightarrow{D} N\left(0, \Omega_\beta\right)$$

The bigger the real part of the biggest eigenvalue of *F*, the higher Ω_β, the "slower" the convergence.

- If all eigenvalues of F have real part **smaller than 1, but bigger than 1/2**, through simulations we can show that: <u>the bigger the eignevalue, the slower the convergence</u>



γ_π
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
γ_x
Determinate, E-stable and
Root-t Convergence region
Determinate, E-stable and
Slow Convergence region
Undeterminate and
E-unstable region
γ_π
$z = 0.3$
$z = 0.4$
$z = 0.5$
$z = 0.7$
$z = 0.8$
$z = 0.9$
$z = 1$
γ_x

Based on the estimation of Benigno and Lopez-Salido (2002) and Gali, Gertler and Lopez-Salido (2001)

	α	β
Germany	0.104	0.783
France	0.105	0.872
Italy	0.001	1.000
Euro Area	0.099	0.843
United States	0.311	0.872

<u>Problem</u>: these are “rational estimates”

- Compute the speed of convergence, assuming for all countries the same elasticity of intertemporal substitution φ and a Taylor expectations-based reaction function with:

$$\gamma_\pi = 1.5, \quad \gamma_x = 0.5$$

real part of the bigger eigenvalue of *F*

	$\varphi = 0.1$	$\varphi = 1$
Germany	0.914	0.615
France	0.908	0.660
Italy	0.999	0.999
Euro Area	0.894	0.647
United States	0.903	0.608

- Under learning, and under a Taylor (expectations-based) rule the speed of convergence is very low in all countries

- Compute the speed of convergence, under the optimal monetary policy under discretion (Evans and Honkapohja, 2003) :

real part of the bigger eigenvalue of *F*

	$\lambda = 0.1$	$\lambda = 0.5$
Germany	0.707	0.766
France	0.785	0.853
Italy	0.999	0.999
Euro Area	0.767	0.827
United States	0.443	0.730

- Under learning, and under the optimal monetary policy under discretion the speed of convergence is very low in all countries

conclusion:

- For what concerns the evidence in favour of the learning hypothesis versus the RE hypothesis, in the full sample 1990-2002, only the US passes all the tests for rationality, while in the Euro Area, with the exception of Germany results seems in general being in favour of learning.

- The analysis of the effects of the start of stage 3 of EMU, shows contradictory results. The fact that the second subsample, starting after the 1999 break, is very small, could be the main reason of the lack of robustness.

- If, from one side, it seems that after the start of stage 3, a change in forecasting behavior of private agents in the area have occurred, however it is not clear if this change is in the direction of a lower or higher degree of rationality.