How credible is the Federal Reserve? A structural estimation of policy re-optimizations

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Abstract

Using a Markov-switching Bayesian likelihood approach, the paper proposes a new measure of the degree of credibility of the Federal Reserve. We estimate a medium-scale macroeconomic model, where the central bank has access to a commitment technology, but where a regime-switching process governs occasional re-optimizations of announced plans. The framework nests the commonly used discretion and commitment cases, while allowing for a continuum of intermediate cases. Our estimates reject both full-commitment and discretion. We instead identify occasional re-optimization episodes that are consistent with changes in Federal Reserve policymakers and operating procedures. Finally, through counterfactual analysis we assess the role of credibility over the past four decades.

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“Whether we have the credibility to persuade markets that we’ll follow through is an empirical question.”

Ben Bernanke, Federal Reserve Chairman, September 13th 2012.

1 Introduction

Both academics and policymakers agree on the importance of central bank credibility in conducting monetary policy. Over the past few decades significant effort has been devoted to enhance credibility in monetary policy, through the creation of independent central banks, the adoption of clear policy objectives, improved transparency and communication strategies, among other measures. Whether central banks are indeed credible, however, remains a largely open question. This paper proposes a novel measure of central bank credibility, and provides new evidence about the credibility of the Federal Reserve over the past few decades.

The term credibility is used in practice to refer to a multiplicity of different concepts. As surveyed by Blinder (2000), academics and policymakers identify the term “credibility” with various different measures, such as “transparency”, “independence”, “aversion to inflation”, etc. Our definition of credibility coincides with the notion of “commitment”, as in the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983). The presence of a policy trade-off (e.g. stabilizing inflation vs. output), combined with the forward looking nature of economic agents, makes it desirable for the central bank to commit to a policy plan. By committing to a plan, the central bank can shape agents’ expectations in a way that improves the short-run policy tradeoffs. However, once those short-run benefits have been reaped, there is an ex-post temptation to deviate from the original plan, and to re-optimize. Credibility is then defined as the ability to resist the temptation to re-optimize. This definition is widely accepted in the monetary policy literature, and is also consistent with the central bank having a “a history of doing what it says it will do”, which both academics and policymakers selected as the most important factor in building central bank credibility in the survey by Blinder (2000).

The monetary policy literature has typically considered two alternative (and extreme) scenarios about the central bank ability to commit. It has either assumed that the central bank always follows its announced plans (commitment case), or that it always deviates (discretion case). This paper adopts a more flexible approach based on the works by Roberds (1987), Schaumburg and Tambaletti (2007) and Debortoli and Nunes (2010) that nests...
commitment and discretion as special cases, while allowing for a continuum of intermediate cases—i.e. the so-called loose commitment setting.\textsuperscript{1} The central bank has the ability to commit to its future plans, but it may occasionally give in to the temptation to revise its plans. Both the central bank and the private sector are aware of the possibility of policy re-optimizations, and take it into account when forming expectations.

In particular, we consider a model where the behavior of the central bank is allowed to change over time, according to a two-state regime-switching process. In each period, with probability $\gamma$ the central bank follows its previous plan, while with probability $1 - \gamma$ it makes a new plan. The probability $\gamma \in [0, 1]$ can then be interpreted as a measure of credibility. In the limiting case where $\gamma = 1$ there would be no re-optimization (the case of commitment), while if $\gamma = 0$ there would be a re-optimization in every period (the case of discretion). Intermediate values of $\gamma$ describe levels of credibility in between the two extremes.\textsuperscript{2} This setting is meant to capture the fact that central bankers understand the benefits of credibility, but at the same time there could be situations when a central bank disregards its commitments. Possible candidates for such events are changes in the dominating views within a central bank due to time-varying composition of its decision-making committee or varying intensity of outside pressures by politicians and the financial industry.\textsuperscript{3} This paper aims at identifying and characterizing policy re-optimizations, but abstracts from analyzing their specific sources.

The empirical analysis is conducted within the medium-scale model for the US economy of Smets and Wouters (2007) (henceforth SW). That model can be viewed as the backbone of the estimated models developed at central banks in recent years, and used for monetary policy analysis and forecasting. We depart from that model in two important ways. First, monetary policies are chosen optimally by a central bank operating under loose commitment, rather than being described by a simple (Taylor-type) rule. Second, we deal with a version of

\textsuperscript{1}Roberds (1987) used the term “stochastic replanning” while Schaumburg and Tambalotti (2007) used the term “quasi-commitment”.

\textsuperscript{2}Equivalently, that probability can be thought of as a continuous variable measuring the durability of the Federal Reserve’s promises, where longer durability corresponds to higher levels of credibility.

\textsuperscript{3}In the case of the United States, the reserve bank presidents serve one-year terms as voting members of the FOMC on a rotating basis, except for the president of the New York Fed. Furthermore, substantial turnover among the reserve bank presidents and the members of the Board of Governors arises due to retirement and outside options. With the (up to) seven members of the Board of Governors being nominated by the U.S. President and confirmed by the U.S. Senate, the composition of views in the FOMC may be affected by the views of the political party in power at the time of the appointment. Chappell et al. (1993) and Berger and Woitek (2005) find evidence of such effects in the U.S. and Germany, respectively. Also, the book by Havrilesky (1995) provides evidence on when politicians tried to influence monetary policy, and when the Federal Reserve did and did not respond.
the SW model with regime-switching. On top of the regime-switching process driving policy re-optimizations described earlier, we also allow the variance of the shock processes to shift over time to control for additional potential sources of time variation. Estimation is carried out using a Bayesian Markov Chain Monte Carlo (MCMC) algorithm.

Two main results emerge from the estimates. First, the model supports the idea that the Federal Reserve is to some extent credible, but that credibility is not perfect. This result differs from the existing literature, as it signals that both the commonly used assumptions of commitment and discretion are rejected by the data. Within a variety of different exercises, the posterior mode of the unconditional probability of commitment is estimated to be about 0.80, with fairly narrow confidence intervals. Such a value could be viewed as closer to either commitment or discretion, depending on the metric used. In order to provide a clearer interpretation of our result, we perform counterfactual simulations in which the central bank is assumed to operate either under commitment or under discretion throughout the entire sample. Our analysis shows that during the 1970s that actual data is close to the counterfactual path under discretion. But the fall in inflation in the early 1980s and the subsequent low levels are closer to the counterfactual path under commitment. Overall, these issues highlight the importance of using our general framework, where sometimes the dynamics of the economy are better described by the case of commitment, while at other times the case of discretion is better.

The second contribution of the article is the identification of historical episodes when the Federal Reserve likely abandoned its commitments, as measured by the (smoothed) probability of re-optimization. We find that policy re-optimizations likely occurred with the appointments of Arthur Burns, G. William Miller and Paul Volcker but not with the appointments of Alan Greenspan and Ben Bernanke. Re-optimization episodes were also likely around changes in operating procedures of the Federal Reserve, specifically during the reserves targeting experiment conducted under Volcker in the early 1980s and the FOMC policy to start announcing the target for the Federal Funds rate around 1994. Additionally, we find a re-optimization episode in 2008, around the start of the quantitative easing policy under Ben Bernanke. An alternative interpretation of our re-optimization episodes is to view them as a source of monetary policy shocks. According to this perspective, we find that typically the deviations from commitment during the 1970s have implied policies that are relatively more expansionary, while deviations in the 1990s and 2000s imply policies that are relatively more contractionary.

Alternative approaches to measure central banks’ credibility have been proposed in the
literature. For instance, through an index-based aggregation of information contained in bylaws and questionnaires, Cukierman (1992) develops some indicators of independence, transparency and aversion to inflation. Our measure is instead based on the notion of commitment that is only partially related to those indicators. Also, as initially proposed in Svensson (1994), several studies have inferred a measure of “inflation-target” credibility by looking at the deviations of inflation expectations from the central bank’s inflation target.\(^4\) Those studies focus on credibility as a device to mitigate the long-run inflation bias. Here we take a complementary approach, and study instead the role of commitment as a device to improve the short-run policy trade-offs – the so called “stabilization - bias”. In this respect, our structural estimation approach allows us to disentangle commitment problems from other factors that may influence agents’ expectations, like generic sources of economic fluctuations.

Our work is also related to the empirical literature on optimal monetary policy. For the most part, that literature has abstracted from assessing the empirical plausibility of alternative commitment settings, by focusing either on commitment or discretion.\(^5\) Few exceptions are the recent works of Givens (2012) and Coroneo et al. (2013), who compare estimates of models with commitment and discretion, and conclude that the data favor the specification under discretion. Kara (2007) obtains a structural estimate of the degree of commitment through a least-square estimation of a monetary policy rule obtained within the framework of Schaumburg and Tambalotti (2007), and provides evidence against the cases of commitment and discretion. To the best of our knowledge, this paper is the first empirical study that considers a framework where the central bank’s behavior regarding its previous commitment may change over time, with occasional switches between re-optimizations and continuations of previous plans. In contemporary and independent work Chen et al. (2013) perform a similar Bayesian regime-switching estimation of a simple monetary policy model, and study switches between active and passive regimes. We instead focus on the commitment problem, and perform our analysis within a state-of-the-art DSGE model.

From a methodological viewpoint, our setting is closely related to recent empirical studies in the DSGE regime-switching literature [see Liu et al. (2011) and Bianchi (2012)] that analyze regime-switching in the inflation target or coefficients of the monetary policy rule. Similar to our paper, they allow the variances of the shocks to switch over time. But

\(^4\)Following this idea, Gurkaynak et al. (2005) argue that uncertainty about the inflation target may drive the high volatility of long-term interest rates in response to monetary policy announcements.

\(^5\)Some examples are the works of Dennis (2004), Söderström et al. (2005), Salemi (2006), Ilbas (2010) and Adolfson et al. (2011).
these studies have a reduced-form Taylor rule specification for monetary policy while in our setting central bank formulates an optimal plan, that is occasionally abandoned. The restrictions implied by optimal policy under loose commitment allow us to distinguish policy re-optimization episodes from other types of regime-switches.

The rest of the paper is organized as follows. Section 2 describes the baseline model, while Section 3 discusses the formulation of optimal policy in the loose commitment framework. Section 4 describes the estimation procedure, and Section 5 outlines the main results. Section 6 provides some concluding remarks. Additional details regarding the estimation procedure and robustness exercises are contained in an appendix.

2 The model

As discussed in the introduction, the distinctive feature of our model concerns the way monetary policy is designed. The underlying economy is instead described by a standard system of linearized equations

$$A_{-1}x_{t-1} + A_0x_t + A_1E_t x_{t+1} + Bv_t = 0$$

where $x_t$ denotes a vector of endogenous variables, $v_t$ is a vector of zero-mean, serially uncorrelated, normally distributed exogenous disturbances and $A_{-1}$, $A_0$, $A_1$ and $B$ are matrices whose entries depend (non-linearly) on the model’s structural parameters. The term $E_t$ denotes rational expectations with respect to those innovations, conditional on the information up to time $t$.

The analysis is conducted within the model of Smets and Wouters (2007). The model includes monopolistic competition in the goods and labor market, nominal frictions in the form of sticky price and wage settings, allowing for dynamic inflation indexation. It also features several real rigidities – habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The model describes the behavior of 14 endogenous variables: output ($y_t$), consumption ($c_t$), investment ($i_t$), labor ($l_t$), the capital stock ($k_t$), with variable utilization rate ($z_t$) and associated capital services ($k^*_t$), the wage rate ($w_t$), the rental rate of capital ($r_t^k$), the nominal interest rate ($r_t$), the value of capital ($q_t$), price inflation $\pi_t$, and measures of price-markups ($\mu^p_t$) and wage-markups ($\mu^w_t$). The model dynamics are driven by six structural shocks: two shocks – a price-markup

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6Monopolistic competition is modeled following Kimball (1995), while the formulations of price and wage stickiness follow Yun (1996) and Erecg et al. (2000).
(\varepsilon^p_t) and wage-markup (\varepsilon^w_t) shock – follow an ARMA(1,1) process, while the remaining four shocks – total factor productivity (\varepsilon^a_t), risk-premium (\varepsilon^b_t), investment-specific technology shock (\varepsilon^i_t) and government spending shock (\varepsilon^g_t) – follow an AR(1) process. All the shocks are uncorrelated, with the exception of a positive correlation between government spending and productivity shocks, i.e. \text{Corr}(\varepsilon^g_t, \varepsilon^a_t) = \rho_{ag} > 0.7 The model can be cast into eq. (1) defining \(x_t\) as a 22x1 vector containing all the variables described above (i.e. endogenous variables, structural shocks and corresponding MA components), and \(v_t\) as vector containing the i.i.d. innovations to the structural shocks.

We depart from the original SW formulation in two fundamental ways. First, we account for changes in the volatility of the exogenous shocks. Recent studies (see Primiceri (2005), Sims and Zha (2006) and Cogley and Sargent (2006) among others) find that exogenous shocks have displayed a high degree of heteroskedasticity. For our purposes, ignoring this heteroskedasticity would potentially lead to inaccurate inference: the time variation in the volatility of the shocks could be mistakenly attributed to policy re-optimization episodes, thus biasing our measure of credibility. To deal with this, we model heteroskedasticity using a Markov-switching process. Specifically,

\[
v_t \sim N(0, Q_{s_t}^o),
\]

where the variance-covariance matrix \(Q_{s_t}^o\) depends on an unobservable state \(s_t^o \in \{h, l\}\) that describes the evolution of the volatility regime. While in principle one could consider a process with more states, a specification with two states has been found to fit the data best in estimated regime-switching DSGE models [see Liu et al. (2011) and Bianchi (2012)]. The Markov-switching process for volatilities \(s_t^o\) evolves independently from the regime-switching process that governs re-optimizations \(s_t\), described in detail in the next section. The transition matrix for \(s_t^o\) is given by

\[
P^o = \begin{bmatrix}
    p_h & (1 - p_h) \\
    (1 - p_l) & p_l
\end{bmatrix}
\]

The second and more important departure from the original SW model concerns the behavior of the central bank. Rather than including a (Taylor-type) interest rate rule, and the associated monetary policy shock, we explicitly solve the central bank’s decision problem.

\footnote{All the variables are expressed in deviations from their steady state. For a complete description of the model, the reader is referred to the original Smets and Wouters (2007) paper.}
As discussed in the next section, this allows us to describe the central bank’s commitment problem, and to characterize the nature of policy re-optimizations. Throughout our analysis, it is assumed that the central bank’s objectives are described by a (period) quadratic loss function

\[ x_t' W x_t \equiv \pi_t^2 + w_y \tilde{y}_t^2 + w_r (r_t - r_{t-1})^2 \]  

(2)

where \( \tilde{y}_t \) is the output-gap – the deviations of output from the level that would prevail in the absence of nominal rigidities and mark-up shocks. Without loss of generality, the weight on inflation is normalized to one so that \( w_y \) and \( w_r \) represent the weights on output gap and interest rate relative to inflation, and will be estimated from the data. According to eq. (2), the central bank’s inflation target coincides with the steady state level of inflation \( \bar{\pi} \), while the output target coincides with its “natural” counterpart. This formulation is consistent with the natural rate hypothesis, i.e. that monetary policy cannot systematically affect average output. It is also consistent with the original SW specification, where because of price and wage indexation, the steady state inflation does not produce any real effect. As a result, the central bank’s credibility problems do not lead to an average inflation-bias, but only to a stabilization-bias in response to economic shocks, as illustrated in Clarida et al. (1999).8 \( r_t \) is the nominal interest rate and the last term in the loss function provides an incentive for the central bank to smooth the interest rate.

The choice of a simple loss function like 2) is not only consistent with the Federal Reserve’s mandate but is also attractive from an empirical perspective. A common approach in the literature is to describe the central bank behavior through simple rules, that are known for their good empirical properties. Here we adopt a similar approach, and adopt a simple loss function that has been shown to realistically describe the behavior of the Federal Reserve (see e.g. Rudebusch and Svensson (1999), or more recently Ilbas (2010) and Adolfson et al. (2011)). We then investigate to what extent the central bank was credible in implementing such empirically plausible objectives. One alternative would be to consider a theoretical loss function, consistent with the representative agent’s preferences. However, there are several reasons why the central bank’s objectives may not reflect the preferences of the underlying society [see e.g. Svensson (1999)]. For instance, and following Rogoff (1985), appointing a central banker who is more averse towards inflation than the overall public may be desirable in the limited commitment settings considered here.9

\[ ^8 \text{Notice however that since the markup-up shocks are allowed to follow an ARMA(1,1), the stabilization-bias could potentially be very persistent, and closely resemble an average inflation bias.} \]

\[ ^9 \text{We verified that a version of the model with a utility-based welfare criterion [e.g. Benigno and Woodford (2012)] provides a much poorer empirical fit. Also, we verified that our simple loss function has been shown} \]
The system of equations (1) implies that current variables ($x_t$) depend on expectations about future variables ($E_t x_{t+1}$). This gives rise to the time-inconsistency problem at the core of our analysis. The central bank’s plans about the future course of policy could indeed have an immediate effect on the economy, as long as those plans are embedded into the private sector expectations. Having reaped the gains from affecting expectations, the central bank has an ex-post incentive to disregard previous plans, and freely set its policy instruments.

The literature has typically considered one of two dichotomous cases to deal with the time-inconsistency problem: full-commitment or discretion. In this paper we use a more general setting that nests both these frameworks. Following Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010), it is assumed that the central bank has access to a loose commitment technology. In particular, the central bank is able to commit, but it occasionally succumbs to the temptation to revise its plans. Both the central bank and private agents are aware of the possibility of policy re-optimizations and take it into account when forming their expectations.

More formally, at any point in time monetary policy can switch between two alternative scenarios, governed by the unobservable state $s_t \in \{0; 1\}$. If $s_t = 1$, previous commitments are honored. Instead, if $s_t = 0$, the central bank makes a new (state-contingent) plan over the infinite future, disregarding all the commitments made in the past. The variable $s_t$ evolves according to a two-state stochastic process, with transition matrix

\[
P = \begin{bmatrix}
Pr(s_t = 1|s_{t-1} = 1) & Pr(s_t = 0|s_{t-1} = 1) \\
Pr(s_t = 1|s_{t-1} = 0) & Pr(s_t = 0|s_{t-1} = 0)
\end{bmatrix} = \begin{bmatrix}
\gamma & 1 - \gamma \\
1 - \gamma & \gamma
\end{bmatrix}
\]

and where $\gamma \in [0, 1]$. The limiting case where previous promises are always honored (i.e. $\gamma = 1$) coincides with the canonical commitment setting. Instead, if $\gamma = 0$ the central bank operates under discretion.

Notice that $s_t$ constitutes an independent switching process, where $Pr(s_t|s_{t-1} = 1) = Pr(s_t|s_{t-1} = 0)$. In other words, honoring commitments in a given period does not make a policy re-optimization more or less likely in the future.\textsuperscript{10} As a result, there is a direct and to approximate well the preferences of the representative agents, leading to minimal losses in comparison to the ideal Ramsey plan. Results are available upon request to the authors.

\textsuperscript{10}In standard monetary regime-switching models, a process like $s_t$ displays instead some degree of persistence, capturing the fact that once a monetary regime (e.g. Dovish or Hawkish) takes office, it is likely to remain in power for a prolonged period of time.
intuitive mapping between a single parameter of the model – the probability of commitment $\gamma$ – and the degree of central bank’s credibility: the higher is $\gamma$, the more credible is the central bank.\textsuperscript{11}

As is common in the DSGE regime-switching literature, we maintain the assumption that $s_t$ is an exogenous process. Accordingly, we are interpreting policy re-optimizations as exogenous shocks influencing the behavior of the central bank, in a similar fashion to common monetary policy shocks. The validity of this assumption could be questioned on the grounds that central banks deliberately choose to abandon their commitments in specific situations, e.g. when unusually large shocks hit the economy.\textsuperscript{12} That criticism would be especially valid if the central bank had to commit to specific targets for its variables of interest: it would be very costly, if not impossible, to achieve those targets in turbulent times. In our setting, however, the central bank has more flexibility. This is because the responses to the shocks $v_t$ are always part of the central bank state-contingent plan.\textsuperscript{13} Thus, if a particular adverse shock materializes, there would be no need to deviate from the original plan. Thus, in our setting it is not obvious that a specific history of structural shocks would make re-optimization more or less likely. In section 5.3 we formally check this intuition. Granger causality tests show the model state variables do not have a statistically significant predictive power for the estimated (smoothed) probability of re-optimization, thus supporting the validity of our assumption.

### 3.1 The central bank’s problem and policy re-optimizations

The problem of central bank when making a new plan can then be written as

$$x_{t-1}'Vx_{t-1} + d = \min_{\{x_t\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} (\beta \gamma)^t [x_t' W x_t + \beta (1 - \gamma) (x_t' V x_t + d)]$$

(3)

subject to

$$A_{-1} x_{t-1} + A_0 x_t + \gamma A_1 E_t x_{t+1} + (1 - \gamma) A_1 E_t x_{t+1}^{reop} + B v_t = 0 \ \forall t$$

(4)

\textsuperscript{11}Such a mapping would be less straightforward if we were to adopt a more general Markov-Switching process. In that case, it would indeed be necessary to distinguish between conditional and unconditional measures of credibility, that would depend on two regime-switching probabilities. Also, following that approach would significantly complicate the solution to the central bank problem.

\textsuperscript{12}Admittedly, it would be ideal to let policy re-optimizations to depend on the model’s state variable. However, this approach would add a substantial computational problem. Debortoli and Nunes (2010) consider a simple model where the probability of commitment depends on endogenous state-variables, and show that the differences with respect to the exogenous probability case are minimal. That specification, however, require adopting a non-linear solution method, that would render unfeasible our estimation exercise.

\textsuperscript{13}From a practical viewpoint, a central bank could commit to a state-contingent plan adopting a simple policy rule, or specifying policy targets with “escape - clauses” [see e.g. Mishkin (2009)]
The terms $x_{t-1}' V x_{t-1} + d$ summarize the value function at time $t$. Since the problem is linear quadratic, the value function is given by a quadratic term in the state variables $x_{t-1}$, and a constant term $d$ reflecting the stochastic nature of the problem. The objective function is given by an infinite sum discounted at the rate $\beta \gamma$ summarizing the history in which re-optimizations never occur. Each term in the summation is composed of two parts. The first part is the period loss function. The second part indicates the value the policymaker obtains if a re-optimization occurs in the next period. The sequence of constraints (4) corresponds to the structural equations (1), with the only exception that expectations of future variables are expressed as the weighted average between two terms: the allocations prevailing when previous plans are honored ($x_{t+1}$), and those prevailing when a re-optimization occurs ($x_{t+1}^{\text{reop}}$). This reflects the fact that private agents are aware of the possibility of policy re-optimizations, and take this possibility into account when forming their expectations.14

We solve for the Markov-Perfect equilibrium of the above economy, where the equilibrium choices $x_{t+1}^{\text{reop}}$ only depend on natural state-variables. We can thus express the expectations related to the re-optimizations state as $E_t x_{t+1}^{\text{reop}} = \tilde{F} x_t$, where the matrix $\tilde{F}$ is a matrix of coefficients to be determined, and is taken as given by the central bank.15

The presence of the (unknown) matrix $\tilde{F}$ complicates the solution of the central bank problem. For any given $\tilde{F}$, the solution to the central bank’s problem can be derived using the recursive techniques described in Kydland and Prescott (1980) and Marcet and Marimon (2011). The associated system of first-order conditions could then be solved using a standard solution algorithm for rational expectations models [e.g. Sims (2002)]. However, a Markov-Perfect equilibrium additionally requires the matrix $\tilde{F}$ to be consistent with the policies actually implemented by the central bank. This requires us to solve a fixed point problem.16

The solution to the central bank’s problem takes the form

$$
\begin{bmatrix}
x_t \\
\lambda_t
\end{bmatrix} = F_{st} \begin{bmatrix}
x_{t-1} \\
\lambda_{t-1}
\end{bmatrix} + G v_t
$$

14To simplify the notation, we have dropped regime dependence and replaced $x_{t+1}^{\text{reop}}|s_t = 0$ with the more compact term $x_{t+1}^{\text{reop}}$.

15We are therefore ruling out the possibility of reputation and coordination mechanism as those described for instance in Walsh (1995)

16Methods to solve for Markov-Perfect equilibria are described in Backus and Driffill (1985), Söderlind (1999), and Dennis (2007). Debortoli et al. (2012) extended those methodologies to analyze loose commitment problems in large-scale models. The algorithm makes use of the fact that in equilibrium it must be that $x_{t}^{\text{reop}} = F_{xx} x_{t-1} + G_{x} v_{t}$. Rational expectations then implies that $E_t x_{t+1}^{\text{reop}} = F_{xx} x_t$. Thus, one must solve the fixed point problem such that $F_{xx} = \tilde{F}$. 

11
where \( \lambda_t \) is a vector of Lagrange multipliers attached to the constraints (4), with initial condition \( \lambda_{t-1} = 0 \). In particular, the Lagrange multipliers \( \lambda_{t-1} \) contain a linear combination of past shocks \( \{v_{t-1}, v_{t-2}, \ldots, v_{t-1}\} \), summarizing the commitments made by the central bank before period \( t \).\(^{17}\) Therefore, the effects of policy re-optimizations can be described by the state dependent matrices

\[
F_{(s_t=1)} = \begin{bmatrix} F^{xx} & F^{x\lambda} \\
F^{\lambda x} & F^{\lambda\lambda} \end{bmatrix}, \quad F_{(s_t=0)} = \begin{bmatrix} F^{xx} & 0 \\
F^{\lambda x} & 0 \end{bmatrix}. \quad (6)
\]

In particular, notice that the unobservable state \( s_t \) only affects the columns of the matrices \( F_{s_t} \) describing the responses to \( \lambda_{t-1} \). This is because a policy re-optimization implies that previous commitments are disregarded. Therefore, when policies are re-optimized it is as if the current variables are not affected by \( \lambda_{t-1} \). On the contrary, the policy responses to the state variables \( x_{t-1} \) and to the shocks \( v_t \) remain the same, regardless of whether the central bank re-optimizes or not.

The above formulation allows us to provide some intuition about the nature of policy re-optimizations. From a reduced-form perspective, a policy re-optimization implies that macroeconomic variables cease to display dependence on a subset of the historical data – summarized in our model by the vector \( \lambda_{t-1} \). From a more structural perspective, policy re-optimizations could instead be viewed as a particular type of monetary policy shock, say \( e_t^{\text{reop}} \), described by

\[
e_t^{\text{reop}} = x_t^{\text{reop}} - x_t = -F^{x\lambda}\lambda_{t-1}. \quad (7)
\]

Notice, however, that while the timing of these “re-optimization shocks” is exogenous – as for standard monetary policy shocks – the sign and magnitude of their impact are instead endogenous, and depend on the history of past shocks summarized by \( \lambda_{t-1} \). For example, if a re-optimization shock occurs when \( \lambda_{t-1} \) is large (small) the shock will have a large (small) impact on the economy. As we will see later, this has implications for how accurately the specific re-optimization episodes are identified in the data.

Our setting bears some similarities to the models recently developed in monetary regime-switching literature (see e.g. Davig and Leeper (2007), Farmer et al. (2009), Liu et al. (2011) and Bianchi (2012)).\(^{18}\) In those models, an exogenous shock switches the economy

\(^{17}\)For this reason the Lagrange multipliers \( \lambda_t \) are often referred to as co-state variables.

\(^{18}\)Note that since the same type of central bank is optimally choosing policy, our framework does not display an indeterminacy problem. There would need to be an additional layer of uncertainty or mismeasurement to give rise to the possibility of indeterminacy.
from one regime to another where the conduct of policy is different. In our model, the
re-optimization shocks are better thought of as starting a new commitment regime, where
another re-optimization shock in the future will end it and start yet another. Nevertheless, as
it happens in standard rational expectations regime-switching models, what happens under a
certain regime depends on what agents expect is going to happen under alternative regimes,
and on the probability of switching to a new regime. For instance, it is worth noticing that
the probability of commitment, $\gamma$, not only enters into the transition matrix $P$, but it also
affects the state-space matrices $F_{st}$ and $G$.

An important difference is that our two policy regimes are described by the same struc-
tural parameters. In other words, allowing for occasional re-optimizations does not require
introducing any additional parameters, besides the switching probability $\gamma$. As indicated by
equation (6) policy re-optimizations only impose specific zero-restrictions on the law of mo-
tion of the model states. Such restrictions distinguish policy re-optimizations episodes from
more generic sources of non-linearities, like switches in structural parameters, or switches in
policymakers’ preferences.

An alternative approach would be to consider that policy re-optimizations are related to
switches in the central bank’s preferences, like from a “Hawkish” to a ”Dovish” regime. We
consider this as a promising area for future research, since such a framework would allow
the identification of the deep sources of regime switches in Taylor rule parameters commonly
found in the literature.\textsuperscript{19} Our setting does not distinguish between re-optimizations within a
policy regime, from those happening because of a regime change. Nevertheless, our inferred
re-optimizations episodes can be compared with changes in members of the FOMC and
operating procedures of the Federal Reserve, as discussed in Section 5.2.

4 Estimation

The law of motion (5) can be combined with a system of observation equations:

$$ x_{t}^{obs} = A + Hx_{t} $$

where $x_{t}^{obs}$ denotes the observable variables, the matrix $H$ maps the state variables into
the observables, and $A$ is a vector of constants. For comparability with SW, the model is

\textsuperscript{19}See for instance, Lakdawala (2013) for a continuous time-varying preference approach and the discussion
in Debortoli and Nunes (2013), comparing the implications of switches in central banks’ preferences with
changes in Taylor-rule parameters within a baseline New Keynesian model.
estimated using seven quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment, the real wage, log hours worked, the log difference of the GDP deflator and the federal funds rate. Also, the monetary policy shock in SW is replace by an i.i.d. measurement error, so that the number of shocks is the same as the number of observable variables. This is required to ensure that we have enough shocks to avoid the stochastic singularity problem in evaluating the likelihood.

The estimation is carried out using Bayesian likelihood approach. The likelihood function for a standard DSGE model can be evaluated using the standard Kalman Filter. Given the regime-switching nature of our model, the standard Kalman filter needs instead to be augmented with the Hamilton (1989) filter, following the procedure described in Kim and Nelson (1999). The likelihood function is then combined with the prior to obtain the posterior distribution. The detailed steps in evaluating the likelihood function, together with the outline of the Bayesian estimation algorithm are provided in the appendix.

We estimate a total 42 parameters, while fixing 6 parameters.\textsuperscript{20} Table 1-3 summarize the priors used for the estimated parameters. For the common structural parameters as well as for the shock processes we use the same priors used in SW,\textsuperscript{21} Regarding the three new parameters describing the central bank behavior, we proceed as follows. For the probability of commitment $\gamma$ we use a uniform prior on the interval $[0,1]$, as we do not want to impose any restrictive prior beliefs about whether the optimal policy is conducted in a setting that is closer to commitment or discretion. Thus the posterior of $\gamma$ will be entirely determined by the data. For the loss function parameters $w_y$ and $w_r$ we instead choose fairly loose Gamma priors.

The data sample in the baseline estimation runs from 1966:Q1-2012:Q2. There may be concern about using the data from 2007 onwards that includes the financial crisis and periods where the zero lower bound was binding. It is important to note that our specification remains agnostic about the specific monetary policy instrument used by the central bank. In principle, one could abstract from considering the zero-bound on the nominal interest rate, and use data that goes through the financial crisis, which would be a challenge if monetary policy were described by a rule for the nominal interest rate. As a robustness

\textsuperscript{20}As in SW, the depreciation rate $\delta$ is fixed at .025, spending-GDP ratio $g_y$ at 18%, steady-state markup in the labor market at 1.5 and curvature parameters in the goods and labor markets at 10. We additionally fix the real wage elasticity of labor supply $\sigma_l = 1$, as it greatly improves the convergence of our MCMC algorithm, and is loosely estimated in the original SW paper. In the robustness section in the appendix, we show that changing this value from 1 does not change the results

\textsuperscript{21}In our model the regime switching variance specification introduces two values for the standard deviation of each shock, as well as two parameters of the transition matrix ($P^{vo}$).
check, we estimate the posterior mode of the model where the data sample does not include
the financial crisis and find very similar results. Additionally we estimate the model using
long-term interest rates (instead of the fed funds rate) which did not face the zero lower
bound constraint. All these results are discussed below.

5 Results

5.1 Parameter Estimates

Table 1 reports the priors and the posterior mode, mean and 95% error bands for the
structural parameters. Despite the different modeling choices and sample data, our estimates
are very similar to those obtained in SW. A notable exception is that we find a smaller degree
of dynamic indexation, thus pointing to a smaller role of this component in driving the
inflation dynamics. Similar considerations hold for the parameters of the shock processes, as
summarized in Table 2. The standard deviations are not directly comparable to SW, since
we allow them to switch over time. But the weighted average of our estimated standard
deviations across the two regimes is very similar to the SW estimates. The parameters
related to the price-markup shock process are somewhat different, as both the autoregressive
parameter $\rho_p$ and the MA parameter $\mu_p$ are estimated to be larger than in SW. In this
respect our findings are closer to the results of Justiniano and Primiceri (2008) , who also
find different persistence of price-markup shocks, and end up using an i.i.d. specification.\(^\text{22}\)
In general, and as shown in Figure 1, the contributions of the different shocks to historical
fluctuations is consistent with the original SW findings. Markup shocks play a major role
in explaining the historical behavior of inflation, while demand-type shocks are the most
important drivers of output fluctuations. Overall, these considerations reassure us that the
empirical properties of the model are not altered by our different formulation of the central
bank behavior.

Let’s now turn our attention to the parameters describing the central bank’s behavior,
summarized in Table 3. Our estimates for the two weight parameters in the central bank’s
loss function fall in the estimated range in the literature. Available estimates of those weight
tends to be very sensitive with respect to the particular model and data sample. For instance,
estimates range from values of $[w_r = 0.005, w_y = 0.002]$ in Favero and Rovelli (2003) to the

\(^{22}\)As discussed in Appendix A-1, when we estimated the posterior mode of the model using an AR(1)
specification we found a very low coefficient on the auto-regressive term similar to the i.i.d. setup of Justiniano
and Primiceri (2008), while all the other estimates remains very similar to the benchmark case.
Table 1: Prior and Posterior Distribution of Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distr.</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{l}$</td>
<td>Normal</td>
<td>0.000</td>
<td>0.234</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Gamma</td>
<td>0.620</td>
<td>0.241</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Normal</td>
<td>0.400</td>
<td>0.241</td>
</tr>
<tr>
<td>$\bar{\beta}$</td>
<td>Gamma</td>
<td>0.250</td>
<td>0.241</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta</td>
<td>0.300</td>
<td>0.192</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Normal</td>
<td>0.500</td>
<td>0.697</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Normal</td>
<td>4.000</td>
<td>6.316</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Normal</td>
<td>1.500</td>
<td>1.771</td>
</tr>
<tr>
<td>$h$</td>
<td>Beta</td>
<td>0.700</td>
<td>0.765</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Normal</td>
<td>1.250</td>
<td>1.614</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.809</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.783</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.626</td>
</tr>
<tr>
<td>$w_y$</td>
<td>Gamma</td>
<td>1.000</td>
<td>0.015</td>
</tr>
<tr>
<td>$w_r$</td>
<td>Gamma</td>
<td>1.000</td>
<td>1.824</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Uniform</td>
<td>0.500</td>
<td>0.811</td>
</tr>
</tbody>
</table>

Note: The table reports the prior and the estimated posterior mean, mode and 95% confidence bands for the model structural parameters.
Table 2: Prior and Posterior Distribution of shock processes

<table>
<thead>
<tr>
<th></th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Prior Std. Dev</th>
<th>Prior Mode</th>
<th>Posterior Mean</th>
<th>Posterior 5%</th>
<th>Posterior 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviations in high and low regimes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^l_a$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.343</td>
<td>0.352</td>
<td>0.307</td>
<td>0.403</td>
</tr>
<tr>
<td>$\sigma^l_b$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.158</td>
<td>0.158</td>
<td>0.123</td>
<td>0.194</td>
</tr>
<tr>
<td>$\sigma^l_g$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.356</td>
<td>0.359</td>
<td>0.303</td>
<td>0.418</td>
</tr>
<tr>
<td>$\sigma^l_I$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.412</td>
<td>0.415</td>
<td>0.350</td>
<td>0.488</td>
</tr>
<tr>
<td>$\sigma^l_p$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.146</td>
<td>0.149</td>
<td>0.129</td>
<td>0.173</td>
</tr>
<tr>
<td>$\sigma^l_w$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.274</td>
<td>0.279</td>
<td>0.243</td>
<td>0.318</td>
</tr>
<tr>
<td>$\sigma^l_m$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.064</td>
<td>0.066</td>
<td>0.055</td>
<td>0.079</td>
</tr>
<tr>
<td>$\sigma^h_a$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.643</td>
<td>0.652</td>
<td>0.562</td>
<td>0.759</td>
</tr>
<tr>
<td>$\sigma^h_b$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.292</td>
<td>0.296</td>
<td>0.228</td>
<td>0.372</td>
</tr>
<tr>
<td>$\sigma^h_g$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.650</td>
<td>0.660</td>
<td>0.568</td>
<td>0.766</td>
</tr>
<tr>
<td>$\sigma^h_I$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.569</td>
<td>0.573</td>
<td>0.472</td>
<td>0.689</td>
</tr>
<tr>
<td>$\sigma^h_p$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.221</td>
<td>0.227</td>
<td>0.194</td>
<td>0.264</td>
</tr>
<tr>
<td>$\sigma^h_w$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.346</td>
<td>0.355</td>
<td>0.293</td>
<td>0.428</td>
</tr>
<tr>
<td>$\sigma^h_m$</td>
<td>Inv Gamma</td>
<td>0.1</td>
<td>2</td>
<td>0.315</td>
<td>0.322</td>
<td>0.279</td>
<td>0.378</td>
</tr>
<tr>
<td>diag($P^{vol,l}$)</td>
<td>Beta</td>
<td>0.8</td>
<td>0.16</td>
<td>0.934</td>
<td>0.916</td>
<td>0.858</td>
<td>0.961</td>
</tr>
<tr>
<td>diag($P^{vol,h}$)</td>
<td>Beta</td>
<td>0.8</td>
<td>0.16</td>
<td>0.883</td>
<td>0.857</td>
<td>0.759</td>
<td>0.934</td>
</tr>
</tbody>
</table>

**MA parameters ($\mu$) and AR parameters ($\rho$)**

<table>
<thead>
<tr>
<th></th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Prior Std. Dev</th>
<th>Prior Mode</th>
<th>Posterior Mean</th>
<th>Posterior 5%</th>
<th>Posterior 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_w$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.894</td>
<td>0.874</td>
<td>0.789</td>
<td>0.936</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.986</td>
<td>0.977</td>
<td>0.947</td>
<td>0.995</td>
</tr>
<tr>
<td>$\rho_{wa}$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.425</td>
<td>0.430</td>
<td>0.295</td>
<td>0.567</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.999</td>
<td>0.999</td>
<td>0.997</td>
<td>1.000</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.447</td>
<td>0.456</td>
<td>0.308</td>
<td>0.617</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.940</td>
<td>0.940</td>
<td>0.908</td>
<td>0.968</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.769</td>
<td>0.776</td>
<td>0.710</td>
<td>0.841</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.947</td>
<td>0.927</td>
<td>0.880</td>
<td>0.958</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Beta</td>
<td>0.1</td>
<td>0.2</td>
<td>0.996</td>
<td>0.991</td>
<td>0.978</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Note: The table reports the prior, and the estimated posterior mean, mode and 95% confidence bands for the parameters describing the shock processes and the diagonal elements of transition matrix for the volatility regimes. The superscripts $l$ and $h$ refer to the low volatility and high volatility regimes.
Figure 1: Historical Decomposition

Note: The figure reports the contribution of different shocks to the historical fluctuations of output-growth and inflation. Demand shocks denote the combination of risk-premium, investment-specific and government expenditure shocks.
Table 3: Prior and Posterior Distribution of Monetary Policy Parameters

<table>
<thead>
<tr>
<th></th>
<th>Prior Distr.</th>
<th>Prior Mean</th>
<th>Prior St. Dev</th>
<th>Prior Mode</th>
<th>Posterior Mean</th>
<th>Posterior 5%</th>
<th>Posterior 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_y ) (Output-Gap weight)</td>
<td>Gamma</td>
<td>1.000</td>
<td>1.000</td>
<td>0.015</td>
<td>0.017</td>
<td>0.010</td>
<td>0.026</td>
</tr>
<tr>
<td>( w_r ) (Interest rate weight)</td>
<td>Gamma</td>
<td>1.000</td>
<td>1.000</td>
<td>1.824</td>
<td>2.248</td>
<td>1.403</td>
<td>3.288</td>
</tr>
<tr>
<td>( \gamma ) (Prob. of Commitment)</td>
<td>Uniform</td>
<td>0.500</td>
<td>0.290</td>
<td>0.811</td>
<td>0.815</td>
<td>0.777</td>
<td>0.851</td>
</tr>
</tbody>
</table>

Note: The table reports the prior, and the estimated posterior mean, mode and 95% confidence bands for the parameters describing the central bank behavior.

Values of \([w_r = 4.517, w_y = 2.941]\) in Dennis (2006). The posterior mode of our estimates \([w_r = 1.824, w_y = 0.015]\) falls in the middle of this range.\(^{23}\)

Most interestingly, the last row of Table 3 shows that the posterior mode for the probability of commitment \(\gamma\) equals 0.81, while Figure 2 shows the entire marginal posterior distribution. The narrow confidence bands indicate that the parameter \(\gamma\) is estimated quite precisely. The main implication of our result is that the data clearly rejects both the commonly used setups of commitment (\(\gamma = 1\)) and discretion (\(\gamma = 0\)). This result is robust to a variety of different specifications. For instance, and as discussed more extensively in Appendix A-1, we considered different subsamples (e.g. the pre-financial crisis sample) and different measures of the interest rates (e.g. the Long-Term interest rate) to address potential concerns related to a binding zero-lower bound on the Federal Funds rate. We also used different priors for \(\gamma\) (Gamma and Beta priors rather than Uniform) to have an uninformative prior even if credibility is measured according to a different metric. In all the cases considered, the estimated values \(\gamma\) remains close to 0.8, thus rejecting both commitment and discretion. The same conclusion holds when estimating separately the alternative commitment settings, and comparing the corresponding marginal likelihoods – see Appendix A-2.

5.2 Counterfactual analysis

The specific value of \(\gamma\), per se is not indicative of whether the central bank has a high or low level of credibility. On the one hand, a probability of commitment of 80% could be viewed as sufficiently close to the ideal commitment case. On the other hand, the use of quarterly data implies that the Federal Reserve is expected to re-optimize on average once

---

\(^{23}\)Allowing for an additional term in the loss function that involves interest rate variability tends to reduce the estimate of \(w_r\), see e.g. Ilbas (2010).
Figure 2: Posterior distribution of $\gamma$

Note: The figure shows the marginal posterior distribution of $\gamma$, the probability of commitment.
every 5-quarters, arguably a relatively short commitment horizon.

However, counterfactual exercises can shed light on the nature of our results. The main question we ask is what would happen under alternative commitment scenarios. To that end we perform counterfactual simulations of the model assuming that the central bank operates either under commitment \( (\gamma = 1) \) or under discretion \( (\gamma = 0) \). The remaining parameters of the model are left unchanged.

Table 4 shows how commitment affects the unconditional second moments for some relevant variables. In general, the relative standard deviations and the cross-correlations with output in a model with \( \gamma = .81 \) are closer to the discretion than to the commitment case. The last line of the table also reports the implied welfare losses with respect to the commitment case, measured in terms of equivalent permanent increase in the inflation rate.\(^{24}\) According to that measure, the total gains of passing from discretion to commitment are equivalent to a permanent decrease in the inflation rate of 1.2% per year. Most of those gains – corresponding to a 1% permanent reduction in inflation – could still be achieved if increasing credibility from \( .081 \) to \( 1 \). We can thus conclude that the economy would behave quite differently if the central bank had perfect commitment, and thus there is still some scope to improve credibility.

Next we look at counterfactual paths of inflation and output growth within our sample period under the assumption of discretion and commitment. The structural shocks are fixed at the values estimated under the loose commitment setting (i.e., the baseline model). Figure 3 displays these counterfactual paths. For output growth, both the counterfactuals under commitment and discretion do not display big differences compared to the data. For inflation, in the period from the mid-1970s to early-1980s the counterfactual under discretion is closer to the data. Inflation under commitment is lower during this period, but not low enough to conclude that the Federal Reserve acting under commitment could have avoided the ”Great Inflation” of the 1970s. On the contrary, since the early-1980s we see that the commitment counterfactual path tracks the inflation data almost perfectly. Under discretion, inflation would have been more volatile, especially so in the past decade. Our exercise thus suggests that starting from the early 1980s the Federal Reserve behaved very similarly to a perfectly committed central bank, and that commitment largely contributed to reduce inflation fluctuations.

More generally, these counterfactual exercises further confirm the idea that the dynamics

\(^{24}\)Such a measure is often used to gauge losses for the objective functions employed by central banks and is described, for instance, in Jensen (2002).
Figure 3: Counterfactual analysis

Note: The figure reports counterfactual simulations under commitment and discretion, in response to the estimated structural shocks.
Table 4: Second Moments and Welfare

<table>
<thead>
<tr>
<th></th>
<th>Model Commitment (\gamma = .81)</th>
<th>Discretion</th>
<th>US Data 1966-2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. of Output</td>
<td>10.80</td>
<td>10.92</td>
<td>10.80</td>
</tr>
<tr>
<td>Relative Standard deviation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fed Fund Rate</td>
<td>0.051</td>
<td>0.085</td>
<td>0.095</td>
</tr>
<tr>
<td>Price Inflation</td>
<td>0.035</td>
<td>0.070</td>
<td>0.081</td>
</tr>
<tr>
<td>Wage Inflation</td>
<td>0.068</td>
<td>0.095</td>
<td>0.101</td>
</tr>
<tr>
<td>Hours</td>
<td>0.534</td>
<td>0.541</td>
<td>0.540</td>
</tr>
<tr>
<td>Cross-correlations (w.r.t. output)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fed Fund Rate</td>
<td>0.052</td>
<td>-0.478</td>
<td>-0.471</td>
</tr>
<tr>
<td>Price Inflation</td>
<td>-0.046</td>
<td>-0.551</td>
<td>-0.637</td>
</tr>
<tr>
<td>Wage Inflation</td>
<td>0.071</td>
<td>-0.319</td>
<td>-0.423</td>
</tr>
<tr>
<td>Hours</td>
<td>0.397</td>
<td>0.417</td>
<td>0.404</td>
</tr>
<tr>
<td>Welfare Loss</td>
<td>0.000</td>
<td>1.090</td>
<td>1.209</td>
</tr>
</tbody>
</table>

Note: The first three columns report the model unconditional moments of selected variables under alternative commitment settings. Parameters are set at the estimated posterior mode. The last column reports the corresponding values for the US data 1966-2012.

of the macro variables are not easily captured by either discretion or commitment. The data is sometimes closer to commitment, sometimes closer to discretion, and sometimes does not even lie in between the commitment and discretion extremes.

5.3 Policy re-optimizations episodes

In addition to the measure of credibility discussed above, our estimates also give us an indication of specific historical episodes when the Federal Reserve likely abandoned its commitments. This can be done by looking at the evolution of the (smoothed) probabilities of re-optimization, as reported in the top panel of Figure 4. Such smoothed probabilities refer to inference about which regime was prevalent based on using all available information in the sample.

The probability of re-optimization mostly hovers around 0.2 with only a handful of spikes. At a first glance, this may appear inconsistent with our estimated value of \(\gamma = 0.81\), as
one would expect that the probability of re-optimization should be close to one 20% of the time, and close to zero the remaining 80%. However, this does not happen in our model for two reasons. First, in our model $\gamma$ not only governs the switching re-optimization variable, but it also affects the expectations regarding the endogenous state variables in equation 4. The fact the there are fewer re-optimizations episodes than suggested by the unconditional probability could then reflect the separate roles played by “beliefs” about credibility (affecting expectations), and actual re-optimizations. Second, and as illustrated in Section 4, our model identifies re-optimization episodes when there are large differences in the path of variables with or without re-optimizations $x_t - x_t^{reop}$. If such difference is small, as it would be cases during prolonged periods with moderate fluctuations, it would be almost impossible to distinguish re-optimizations from continuations of past plans. In such cases, the smoothed probability remains at the unconditional average, i.e. at a level of about 0.2.

We can isolate five dates when re-optimizations were more likely than continuations of previous plans – i.e. the probability of re-optimization exceeds 50%. Those dates are (i) 1969:Q4, (ii) 1980:Q3, (iii) 1984:Q4, (iv) 1989:3 and (v) 2008:Q3. If we lower the cutoff threshold to 40% then we get two additional dates (vi) 1979:Q1 and (vii) 1993:Q1. A natural test for the validity of our results is to contrast these dates with existing narrative accounts of the US monetary policy history. The first two episodes coincide with the appointment of new Federal Reserve Chairmen: Arthur Burns in early 1970 and Paul Volcker in mid 1979. In late 1980 there is another re-optimization, corresponding to a view that there has been a policy-reversal during 1980, and that the Volcker policy was credible and effective only from late 1980 or early 1981 (see e.g. Goodfriend and King (2005)). We see another re-optimization in 1984 which could potentially correspond to the end of the experiment of targeting non-borrowed reserves that was undertaken in the first few years under Volcker. Only two episodes are identified over the 20-year long Greenspan tenure. A first re-optimization occurred in 1989, close to the “Romer and Romer” date of December 1988 (see Romer and Romer (1989)). A second re-optimization is identified in 1993. Arguably, this could be related with the major policy change of February 1994 when the Federal Reserve began explicitly releasing its target for the federal funds rate, along with statements of the committee’s opinion on the direction of the economy. Those announcements were intended to convey information about future policies, as an additional tool to influence current economic outcomes. The

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25For instance, Figure 8 shows those differences for output growth, inflation and the Fed Funds rate, while the estimation procedure entails accounting for this difference for all the seven observables.
Figure 4: Smoothed Probabilities: Re-optimization and High Volatility Regime

Note: The figure shows the smoothed probability of being in a re-optimization state (upper panel), and of being in the high-volatility regime (lower panel) for the posterior mode estimates. Shaded areas correspond to the NBER recessions, and the vertical solid lines indicate the appointment of a new Federal Reserve Chairman.
last re-optimization is identified in 2008, when the Federal Reserve started adopting unusual policy decisions like the purchases of mortgage-backed securities and other long-term financial assets. Thus overall it appears that some of our dates align with changes in Federal Reserve chairmen while others correspond to changes in operating procedures of the Federal Reserve.

Moreover, there does not seem to be any systematic correspondence between re-optimizations and recessions, or switches in the volatility regime. This can be seen in the bottom panel of figure 4, showing the smoothed probabilities of being in a high volatility regime. The identified periods of high-volatility are consistent with canonical analyses of US business-cycles, but are not aligned with our re-optimization episodes. The 70s and the early 80s are characterized by long and recurrent episodes of high-volatility. The probability of high-volatility surges in correspondence with well-known oil shock episodes: the OPEC oil embargo of 1973-1974, the Iranian revolution of 1978-1979 and Iran-Iraq war initiated in 1980. From 1984 onwards, the economy entered a long period with low-volatility – the Great-Moderation – interrupted by the bursting of the dot-com bubble in 2000, and by the events in the aftermath of September 11, 2001. Finally, periods with high-volatility are clearly identified in correspondence with the Great-Recession and financial crisis of 2008-2009.

There may also be a concern that the switching is actually driven by the state of the economy. We address this concern showing that the observable variables of the model do not help forecast changes in the probability of re-optimization. More specifically, we run Granger Causality tests regressing the smoothed probability on four of its lags, and four lags of the seven macro variables. Results are reported in Table 5. The first row labeled ”All” considers the restriction that jointly all the macro variables have no forecasting power for the probability of commitment. The p value shows that we cannot reject this at the 10% level. The next rows show the tests of whether individually each of the variables can forecast the probability of re-optimization. At the 10% level we fail to reject for all the variables except for investment where the p value is extremely close to 0.1. These tests suggest that our assumption about exogenous switching appears to be a reasonable one.

5.4 What are the effects of policy re-optimizations?

As discussed earlier, the effects of a re-optimization are state-dependent, in the sense that they crucially depend on the history of economic shocks preceding the re-optimization period. Figures 5 - 7 illustrate this phenomenon showing the impulse responses to a technology, government spending and wage markup shock. The solid line shows the path under the

\[26\] See for example the recent historical survey of Hamilton (2011).
Table 5: Granger Causality Test

<table>
<thead>
<tr>
<th></th>
<th>F statistic</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>1.3993</td>
<td>0.1038</td>
</tr>
<tr>
<td>GDP</td>
<td>1.1258</td>
<td>0.3466</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.2326</td>
<td>0.9197</td>
</tr>
<tr>
<td>Investment</td>
<td>2.0064</td>
<td>0.0965</td>
</tr>
<tr>
<td>Wage</td>
<td>1.4927</td>
<td>0.2072</td>
</tr>
<tr>
<td>Hours</td>
<td>0.9282</td>
<td>0.4493</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.6068</td>
<td>0.6583</td>
</tr>
<tr>
<td>Fed Funds</td>
<td>1.1415</td>
<td>0.3393</td>
</tr>
</tbody>
</table>

Note: The table reports the F-values and p-values of Granger causality tests. The unrestricted regression involves regressing the smoothed probability on four of its lags and four lags of all the seven macro variables, and the restricted regression imposes zeros on the coefficients of those macro variables.

assumption that a re-optimization never occurs (even though agents expect it to occur with probability $1 - \gamma$). The line with ‘dots’ refers instead to the scenario where a re-optimization occurs after 5 quarters, but not after that. The difference between the two lines thus measures the effects of a policy re-optimization that occurs in period $t = 5$. A few interesting observations stand out. A re-optimization that occurs after a technology shock involves a reduction in inflation relative to continuing past plans. Whereas a re-optimization occurring after a markup shock involves an increase in inflation relative to continuing past plans. Similarly, the response of the interest rate and output gap to re-optimization shocks is also dependent on which shock has occurred previously.

For comparability purposes, Figures 5 - 7 also show commitment (dashed green line) and discretion (dashed - dotted line) cases. It can be noticed that is not always the case that the responses under loose commitment (blue lines) are in between the discretion and commitment cases. This is because there is uncertainty about the timing of future re-optimizations, a feature that is unique to our framework.

Our re-optimizations could also be viewed as a particular class of monetary policy shocks. Within our model, a deviation from previous commitment, like a generic monetary policy shock, constitutes an exogenous and unanticipated change in the course of policy. But there is an important difference between policy re-optimizations and generic monetary shocks. For example, suppose the economy was hit by a sequence of increases in oil prices, and that the Federal Reserve had committed to keep the interest rate high over a certain horizon. In that
Figure 5: Impulse responses to a Technology Shock

Note: Impulse responses to a 1 standard deviation shock under alternative commitment settings. The line with "dots" indicates the responses under "loose commitment", assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.
Figure 6: Impulse responses to a Demand Shock

Note: Impulse responses to a 1 standard deviation government expenditure shock under alternative commitment settings. The line with “dots” indicates the responses under “loose commitment”, assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.
Figure 7: Impulse responses to a Wage-Markup Shock

Note: Impulse responses to a 1 standard deviation wage markup shock under alternative commitment settings. The line with “dots” indicates the responses under “loose commitment”, assuming that a policy re-optimization occurs after 5 quarters, and there is no policy re-optimization thereafter.
case, a policy re-optimization would bring about more expansionary policy than expected. On the contrary, in an economy affected by negative demand shocks, the central bank would commit to keep the interest rate low, and a re-optimization would then be associated with an unanticipated contractionary policy. Thus, whether a re-optimization has a positive or a negative impact on the variables of interest depends on the entire sequence of shocks previously experienced by the economy.

It then seems useful to analyze the effects of re-optimizations over our sample period. To that end, figure 8 illustrates the effects of deviating from a promised plan on a given date. Specifically it shows the difference, $x_t|s_t = 0, x_{t-1} - [x_t|s_t = 1, x_{t-1}]$ for output growth, inflation and the nominal interest rate. The thought experiment is the following: if a re-optimization were to occur at each period in our sample, how would the values of output, inflation and interest rates be different relative to the case where the previous commitment is honored? Policy re-optimizations would have made output and inflation higher until the early 1980s, but would have had a negligible effect (or lowered them) during the Great Moderation. This is because in periods with high volatility the central bank needs to make significant commitments to its future actions to stabilize the economy. Those commitments constitute a relevant burden in subsequent periods, and abandoning past commitment would lead to a radically different outcomes. Instead, in a low volatility economy the central bank carries over less relevant commitments, and there is less need to stabilize the economy. As a consequence, the effects of abandoning past commitments are relatively small.

Regarding the episodes discussed above, Figure 8 shows that the re-optimizations of the ’70s and ’80s are all associated with an increase in the level of inflation and output growth. In other words, those re-optimizations implied a “looser” policy than under the commitment plan. The two re-optimizations of 1993 and 2009 are instead associated with a more contractionary policy. This suggests that Quantitative Easing does constitute a deviation from a commitment plan, but in the sense that monetary policy should have been more expansionary than it actually was. This conforms to the common view that as the economy hit the zero-lower bound, quantitative easing was a necessary but insufficient tool.

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27Keep in mind that this exercise is conducted conditioning on the estimated parameters being consistent with an unconditional probability of commitment being equal to our estimated value of 0.81.
Note: The figure shows the effects of re-optimizations over time, measured as the difference between the value conditional on re-optimization and the value conditional on continuation of previous commitment, i.e. $[x_t|s_t = 0, x_{t-1}] - [x_t|s_t = 1, x_{t-1}]$. Vertical lines indicate the episodes where $\text{Prob}(s_t = 0) > 50\%$. 
6 Conclusion

This paper proposes a structural econometric approach to measure the degree of the Federal Reserve’s credibility, within a standard medium-scale macroeconomic model. Monetary policy choices are modeled according to a loose commitment setting, where deviations from commitment plans are governed by a regime-switching process.

The conventional approach to think about central banks’ credibility is to distinguish between two polar cases: commitment and discretion. Our results depict a very different picture regarding the actual behavior of the Federal Reserve. Over the past four decades, the Fed displayed a certain ability to commit, but its credibility was not perfect. There have been periods where the Fed honored its commitments, but also episodes when it did not. Such episodes sometimes line up closely with changes of Fed chairmen, and other times with changes in the operational procedures of the Federal Reserve.

Moreover, counterfactual exercises show that the behavior of the Federal has changed over time. During the ’70s, the actual inflation dynamics would indicate a discretionary behavior. Instead, starting from the mid 1980s inflation dynamics are more consistent with a commitment behavior. But it would be misleading to conclude that there has been a one-time change from discretion to commitment. The Federal Reserve has occasionally deviated from its commitment plans throughout the entire sample. If anything, the main difference is that while the deviations in the 70’s implied more expansionary policies, the deviations in the 90’s and 2000’s has been more contractionary. In this respect, our result are consistent with earlier studies in the monetary policy literature arguing that the Fed moved from a passive to an active regime, and the associated “good policies” contributed to the Great Moderation.

According to our model, there is still some scope to increase the credibility of the Federal Reserve. Credibility gains would reduce the fluctuations in measures of inflation and economic activity, and thus enhance welfare. Our study, however, remains silent about the specific sources of credibility problems, and thus on the possible ways to correct them. For instance, under the helm of chairman Ben Bernanke, the Federal Reserve has taken several measures to better communicate with the public about current and future policy actions. In 2012 the Federal Reserve announced an official inflation target of 2%. Additionally they started releasing individual forecasts of the FOMC members’ relating to economic activity. Looking forward, our approach could be used to assess the effectiveness of these type of policies.
References


Appendix

A-1 Robustness Checks

In this section we discuss a variety of robustness checks. Results are summarized in Table 6, where the first column describes how each specification differs from the benchmark case considered in the main text. For practical purposes, the table only reports the posterior mode estimates of the policy parameters $w_y$, $w_r$ and $\gamma$. The complete results are available upon request to the authors.

To facilitate the comparison, row (a) reports again the benchmark results. Row (b) contains the estimates excluding data since the beginning of the financial crisis, so that the data sample is restricted to end at Q2:2007. This is because, we want to make sure that our main results are not affected by features of the financial crisis that are not explicitly included in our model, like unconventional policies and the zero-lower bound constraint. The estimates of $\gamma$ and $w_y$ remain very close to the benchmark case, whereas the estimate of $w_r$ is lower at 0.87. This lower estimate is most likely due to the fact that the Federal Reserve has kept the Fed funds rate constant (around zero) from late 2008 onwards. Similarly, Row (c) shows the estimates under a specification where the Fed Funds rate is replaced with the interest rate on a 10-year Treasury note. This allows us to estimate the model using the entire sample without worrying about the zero lower bound. The resulting estimate of the probability of commitment is 0.8.

Table 6: Estimates under various specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Posterior mode</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_y$</td>
<td>$w_r$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>(a) Benchmark</td>
<td>0.015</td>
<td>1.824</td>
<td>0.811</td>
</tr>
<tr>
<td>(b) Pre-fin crisis sample</td>
<td>0.031</td>
<td>0.874</td>
<td>0.781</td>
</tr>
<tr>
<td>(c) 10-year interest rate</td>
<td>0.019</td>
<td>1.244</td>
<td>0.803</td>
</tr>
<tr>
<td>(d) Beta(0.5,0.5) prior for $\gamma$</td>
<td>0.015</td>
<td>1.829</td>
<td>0.812</td>
</tr>
<tr>
<td>(e) Gamma(2,4) prior for $w_r$ and $w_y$</td>
<td>0.016</td>
<td>1.950</td>
<td>0.812</td>
</tr>
<tr>
<td>(f) AR(1) price-markup shocks</td>
<td>0.012</td>
<td>2.009</td>
<td>0.729</td>
</tr>
<tr>
<td>(g) $\sigma_I = 0.5$</td>
<td>0.022</td>
<td>0.879</td>
<td>0.801</td>
</tr>
<tr>
<td>(h) $\sigma_I = 1.92$</td>
<td>0.014</td>
<td>1.804</td>
<td>0.812</td>
</tr>
<tr>
<td>(i) $\sigma_I = 3$</td>
<td>0.022</td>
<td>0.879</td>
<td>0.801</td>
</tr>
</tbody>
</table>

Note: The table shows the posterior mode estimates of the loss function weight on output $w_y$, the loss function weight on interest rate smoothing $w_r$, and the probability of commitment $\gamma$ for various specifications.
We also consider different prior specifications for the monetary policy parameters. Row (d) considers a Beta prior for the probability of commitment $\gamma$, with both shape parameters set to 0.5. This prior distribution gives roughly the same weight to values in the $[0.2,0.8]$ interval while putting more weight on values near the end points 0 and 1. Such specification confirms that even with a higher prior probability weight on discretion (0) and commitment (1), the data chooses a value of $\gamma$ close to 0.8. Row (e) considers Gamma distribution for $w_y$ and $w_r$ with a higher variance. Specifically we use the Gamma distribution with mean 2 and variance 4, but the resulting estimates are very similar to the benchmark case.

We then estimate the model allowing for a different specification of the price-markup process. In the benchmark estimation the price-markup disturbance is modeled as an ARMA(1,1), $\varepsilon_p^t = \rho_p \varepsilon_p^{t-1} + \eta_{p}^t - \mu_p \eta_p^{t-1}$. The estimates from Table 2 show that we have an issue of near-cancelation of the roots, and potential weak identification. A similar problem is reported in Justiniano and Primiceri (2008), who end up using an i.i.d. specification. To ensure the our estimate of the probability of commitment is not affected by this issue, we consider an additional specification of the price-markup process, where $\varepsilon_p^t$ is AR(1). The results are presented in row (f) of table 6. In this specification the estimate of $\rho_p$ is very close to zero, making this very similar to the i.i.d. case of Justiniano and Primiceri (2008). Importantly, the estimates of $\gamma, w_y$ and $w_r$ are not affected much by the specification of the price-markup process.

Finally, as opposed to the original SW model, in our benchmark estimation we fix the elasticity of labor supply with respect to the real wage, $\sigma_l = 1$. That parameter displayed very large confidence bands in the SW estimates, and fixing it improves the convergence properties of our estimation algorithm. We then consider three alternative values of $\sigma_l$: a value of 0.5, that is close to what is typically estimated in micro studies; a value of 1.92 corresponding to the posterior mode SW; and a value of 3 that is close to the estimates in Bianchi (2012) (who estimates a regime-switching model similar to ours). The estimates of $w_y$ and $\gamma$ (shown in the last 3 rows) are very similar to the benchmark case, while the estimate of $w_r$ does change somewhat (see section 5.1 for a more in-depth discussion of the sensitivity of this parameter.

In conclusion, the estimate of the probability of commitment $\gamma$ is stable for all the different specifications considered.
A-2 Comparison of fit of the model

Our model nests both the cases of commitment and discretion, thus the estimated results imply that the data gives a higher weight to the posterior at $\gamma = .81$ as compared to $\gamma = 0$ or $\gamma = 1$. However a potentially different way to estimate the model at the end points, $\gamma = 0$ or $\gamma = 1$, is to use an estimation strategy that does not involve regime-switching in the coefficient matrices. What we are worried about is the following. Even though the posterior (and also the likelihood given our diffuse priors) puts a higher weight on $\gamma = .81$, once we take into account the estimation uncertainty involved with the regime-switching, is it possible that the fit of the discretion or commitment model can be better than the loose commitment model? To explore this, we separately estimate the commitment and discretion versions of the model using an estimation strategy that does not involve regime-switching in the coefficients. There is still regime-switching in the variance of the shocks.

Table 7 shows the marginal likelihoods. These are based on the harmonic mean method of Gelfand and Dey (1994) and specific one we use is the modified harmonic mean estimator of Geweke (1999). It is reassuring to see that the fit of the loose commitment model is significantly better than the discretion case. With commitment, we had difficulty getting the Bayesian MCMC algorithm to converge. But the posterior mode for commitment is much lower and we are confident that discretion and loose commitment fit the data much better than for commitment. See Givens (2012) among others for supporting evidence that commitment tends to fit the data much worse than discretion.

Finally, we compare our model with the original SW model to compare the fit. For this exercise the models are estimated stopping the sample before the start of the financial crisis. We use the original priors used in the SW model, and the baseline priors for the loose commitment model. The marginal likelihood for our model is higher but not by much. These marginal likelihood calculations do depend to a small extent on the priors and so we conclude that our model fits the data at least as well as the SW model, which is the workhorse model in the literature.
### Table 7: Comparison of Marginal Likelihood

<table>
<thead>
<tr>
<th>Model</th>
<th>Data Sample</th>
<th>Marginal Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion</td>
<td>Full</td>
<td>-1127.69</td>
</tr>
<tr>
<td>Benchmark</td>
<td>Full</td>
<td>-1020.36</td>
</tr>
<tr>
<td>Smets &amp; Wouters (2007)</td>
<td>Pre-fin crisis</td>
<td>-880.25</td>
</tr>
<tr>
<td>Benchmark</td>
<td>Pre-fin crisis</td>
<td>-874.45</td>
</tr>
</tbody>
</table>

### A-3 Details of the Estimation Algorithm

#### A-3.1 Evaluating the Likelihood Function

Consider the state-space form that can be obtained combining eqs. (2) and (5)

\[
x^{\text{obs}}_t = A + H\beta_t
\]

\[
\beta_t = F_s \beta_{t-1} + G v_t
\]

where \( \beta_t \equiv [x_t, \lambda_t] \). Also, let’s denote with \( M \) the number of possible states the the Markov-switching process can take. In our case we have 2 states for the policy regimes, and 2 states for the volatility regimes. Thus, the combined Markov-switching process \( S_t \equiv [s_t, s_{t}^{\text{var}}] \) can take \( M = 4 \) possible values. The likelihood function is evaluated in four steps.

**Step 1:** Perform the Kalman Filter for \( i = 1, \ldots, M, j = 1, \ldots, M \)

\[
\beta_{i,j|t-1}^{i,j} = F_{j}^{i} \beta_{i|t-1}^{j}
\]

\[
P_{i,j|t-1}^{i,j} = F_{j}^{i} P_{i-1|t-1}^{j} F_{j}^{i} + G Q G'
\]

\[
\eta_{i,j|t-1}^{i,j} = x_{t}^{\text{obs}} - H \beta_{i,j|t-1}^{i,j} - A z_t
\]

\[
f_{i,j|t-1}^{i,j} = H P_{i,j|t-1}^{i,j} H'
\]

\[
\beta_{i|t}^{i,j} = \beta_{i|t-1}^{i,j} + P_{i|t-1}^{i,j} H'[f_{i|t-1}^{j}]^{-1} \eta_{i,j|t-1}^{i,j}
\]

\[
P_{i|t}^{i,j} = (I - P_{i|t-1}^{i,j} H'[f_{i|t-1}^{j}]^{-1} H) P_{i|t-1}^{i,j}
\]

42
Step 2: Perform the Hamilton Filter

\[ P(S_t, S_{t-1}) = P(S_t|S_{t-1})P(S_{t-1}|\psi_{t-1}) \]
\[ f(x_t^{obs}|\psi_{t-1}) = \sum_{S_t} \sum_{S_{t-1}} f(x_t^{obs}|S_t, S_{t-1}, \psi_{t-1})P(S_t, S_{t-1}|\psi_{t-1}) \]
\[ P(S_t|\psi_t) = \sum_{S_{t-1}} P(S_t, S_{t-1}|\psi_t) \]

Note that the conditional density is normal and given by

\[ f(x_t^{obs}|S_{t-1} = i, S_{t-1} = j, \psi_{t-1}) = (2\pi)^{-\frac{N}{2}} \left| f_{i,j}^{i,j} \right|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \eta_{i,j}^{i,j} (f_{i,j}^{i,j} - \beta_{i,j}^{i,j})\right\} \]

Step 3: Perform the Kim & Nelson approximations to collapse the \( M^2 \) unobservable \( \beta_{i,j}^{i,j} \) into \( M \) ones. For each \( j \) calculate the following

\[ \beta_{i,j}^{i,j} = \frac{\sum_{i=1}^{M} P(S_t = i, S_t = j|\psi_t) \beta_{i,j}^{i,j}}{P(S_t = j|\psi_t)} \]
\[ P_{i,j}^{i,j} = \sum_{i=1}^{M} P(S_t = j, S_{t-1} = i|\psi_t) \left[ P_{i,j}^{i,j} + (\beta_{i,j}^{i,j} - \beta_{i,j}^{i,j}) (\beta_{i,j}^{i,j} - \beta_{i,j}^{i,j})' \right] \]
\[ P(S_t = j|\psi_t) \]

Step 4: After performing steps 1-3 \( \forall t \) we can evaluate the log likelihood function

\[ l(\theta) = \sum_{t=1}^{T} \ln(f(x_t^{obs}|\psi_{t-1})) \]

A-3.2 Metropolis-Hastings Algorithm

This section explains the Bayesian estimation procedure that uses the Metropolis-Hastings algorithm and will be used to estimate all the parameters of the model jointly. Our estimation method follows the Metropolis-Hastings algorithm used in SW. The main difference is that the evaluation of the likelihood has to be modified to deal with the addition of regime-switching. The estimation follows a two step procedure. In the first step we numerically maximize the log posterior distribution to get an estimate of the posterior mode. In the second step, using the posterior mode calculated in the first step as a starting value, we use the Metropolis-Hastings algorithm to completely characterize the posterior distribution.

Let \( \theta \) be the parameters to be estimated. The M-H algorithm involves generating a draw from a candidate generating density, \( q(.) \). Given a draw \( \theta^{(g)} \), let the candidate draw be called
Then this new draw is accepted with the following probability.

\[
\alpha(\theta^{(g+1)}, \theta^{(g)}) = \min \left( \frac{p(\theta^{(g+1)}|Y)q(\theta^{(g)})}{p(\theta^{(g)}|Y)q(\theta^{(g+1)})}, 1 \right)
\]

Following SW(2007) we use the inverse of the Hessian at the posterior mode (that comes out of the numerical optimization procedure) in the candidate generating density which is centered around the current draw \(\delta^{(g)}\).

\[
\delta^{(g+1)} = \delta^{(g)} + c\tilde{H}^{-1}
\]

where \(c\) is a scale factor and \(\tilde{H}\) is the Hessian at the posterior mode. This is known as a random walk Metropolis-Hastings step. We then tune the parameter \(c\) to get an acceptance rate of between 25% and 35% as recommended by Gamerman and Lopes (2006). The full parameter vector \(\theta\) is sampled in one block. We have also tried blocking by splitting the parameter vector \(\theta\) into 2 or more blocks but found that the Metropolis-Hastings algorithm ran most efficiently with one block and had good convergence properties as discussed below.

### A-3.3 Convergence Diagnostics

The Metropolis-Hastings algorithm is run for 500,000 draws where the first 25,000 draws are discarded. Note since the chain is initialized at the posterior mode, a large number of burn-in draws is not required. From the remaining 475,000 draws, one of out every ten draws is saved resulting in an effective sample of 47,500. Figure 9 shows the trace plots of the draws from the Metropolis-Hastings algorithm for the three structural parameters that are new to our setting. As can be seen the draws are mixing well. To get a better idea of the correlation of the draws the top panel of figure 10 plots the 20th order autocorrelation for each of the estimated parameters. These autocorrelations are all below 0.6 and most them lower than 0.3, suggesting that the dependence in the draws diminishes fairly rapidly. The lower panel shows the inefficiency factors, this is the inverse of the relative numerical efficiency of Geweke (1992). These numbers are mostly below 40 while some of them are a bit higher. Note these are much lower as compared to the SW M-H algorithm as reported by Chib and Ramamurthy (2010). Thus, overall the convergence diagnostics are satisfactory.
Figure 9: Trace Plots

Note: This figure shows the MCMC draws for loss function weight on output gap: $w_y$, the weight on interest rate smoothing: $w_r$ and the probability of commitment: $\gamma$. 
Figure 10: 20th Order Autocorrelations

Note: The top panel of this figure shows the 20th order autocorrelations (y-axis) of the MCMC draws for all the estimated parameters (x-axis) while the bottom panel shows inefficiency factors.