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Liquidity, Assets and Business Cycles

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Abstract

I construct a tractable model to evaluate the liquidity shock hypothesis that exogenous shocks to equity market liquidity are an important cause of the business cycle. After calibrating the model, I find that a large and persistent negative liquidity shock can generate large drops in investment, employment and output. Contrary to the hypothesis, however, a negative liquidity shock generates an equity price boom. This counterfactual response of equity price is robust, provided that a negative liquidity shock tightens firms' financing constraint on investment. Also, I demonstrate that the same counterfactual response of equity price arises when there is a financial shock to a firm's collateral constraint on borrowing. For equity price to fall as it typically does in a recession, the negative liquidity/financial shock must be accompanied or caused by other changes that relax firms' financing constraint on investment. I discuss some candidates of these concurrent changes.

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1. Introduction

The financial crisis in 2008 in the United States has brought asset market liquidity to the forefront of policy debate and academic research. The severe shortage of liquid assets during the height of the crisis prompted the US government to inject a massive amount of liquidity into the asset market, in various forms of bailouts and quantitative easing. There is little doubt that the liquidity shortage in that crisis was caused by changes in economic fundamentals. Specifically, the realization that many asset-backed securities had much lower quality and much higher default risks than previously thought triggered a flight of funds from those securities to safer and more liquid assets. Despite this critical role of the fundamentals, the crisis has raised a more general question about the role of asset market liquidity: Can exogenous shocks to such liquidity be an important cause of the business cycle?

An affirmative answer to this question is the basis of the following hypothesis, which I will refer to as the *liquidity shock hypothesis*. A sudden drop in asset market liquidity, which may not necessarily be related to changes in economic fundamentals, causes equity price to fall. In a world where firms face financing constraints on investment, this fall in equity price reduces the funds for investment that a firm can raise by issuing equity and/or using equity as collateral in borrowing. Thus, investment falls, output falls and an economic recession starts. The objective of this paper is to reformulate this hypothesis and evaluate it quantitatively.

The liquidity shock hypothesis has become popular in macroeconomic models that emphasize financial frictions (e.g., Kiyotaki and Moore, 2012, Jermann and Quadrini, 2010). The intuitive appeal of the hypothesis comes partly from the link between investment and asset prices, which accords well with recent business cycles. Figure 1 depicts the time series of a broad stock price index and non-residential investment in the US from 1999 to 2011. The series are percentage deviations of the quarterly data from the trend, as signified by "dev" in the labels.[1] It is clear that investment and the stock price move closely together. More importantly, the stock price leads investment by one to two quarters in the business cycle. This lead-lag structure suggests that shocks might affect investment through asset prices.

[1] The stock price index is the Wilshire 5000 price full cap index (Wilshire Associates Incorporated, also available at the Federal Reserve Data Center). This is an index of the market value of all stocks actively traded in the US, weighted by market capitalization. The designation "full cap" signifies a float adjusted market capitalization that includes shares of stocks not considered available to ordinary investors. The data is available on the daily basis, but the series used here is the price of the last trading day in each quarter. Investment is private nonresidential fixed investment, which is available at the US Department of Commerce: Bureau of Economic Analysis. The variables in Figure 1 are quarterly data deflated with the GDP deflator, with the first quarter of year 2005 as the base period. They are filtered through the Hodrick-Prescott filter with a parameter 1600. I have multiplied the deviation of investment from its trend by 2.

Besides its intuitive appeal, the liquidity shock hypothesis has immediate policy implications. If fluctuations in asset liquidity are a cause of the business cycle, then a government can attenuate the business cycle by making the supply of liquid assets counter-cyclical. In particular, by injecting liquidity to support asset prices in a recession, a government can prevent business investment from deteriorating precipitously, thereby stabilizing the economy. Such interventions are warranted when exogenous shocks to asset liquidity are the source of fluctuations.

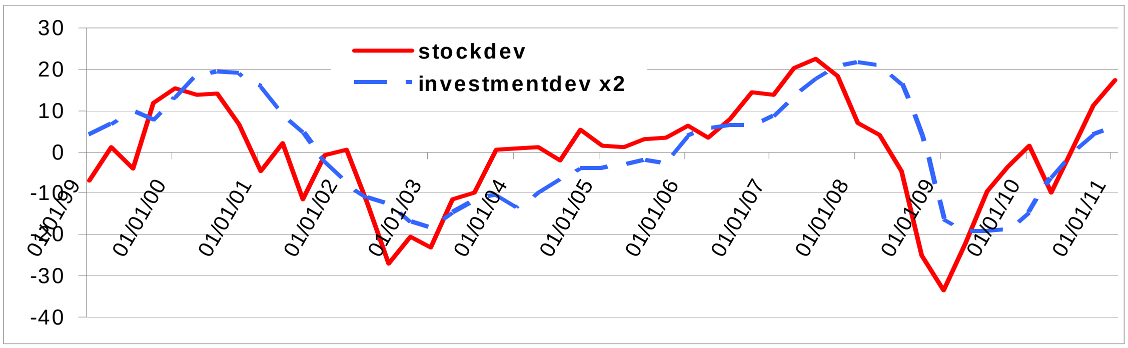

Figure 1. Deviations of stock price and investment from trend (%)

Given the intuitive appeal and the immediate policy implication of the liquidity shock hypothesis, it is important to evaluate the hypothesis formally and clearly. For concreteness, I focus on the version of the hypothesis modeled by Kiyotaki and Moore (2012, *KM*, henceforth). Later I will show that a main result of this model also holds in many other models that emphasize the financing constraint on investment. KM place two equity-market frictions at the center. One is the difficulty to issue new equity: a firm can issue new equity on at most a fraction $\theta \in (0, 1)$ of investment. Another friction is the lack of resaleability of equity; that is, only a fraction $\phi \in (0, 1)$ of existing equity can be resold in any given period. KM model a liquidity shock as an exogenous and unexpected change in equity resaleability, ϕ.

I reformulate the KM model by assuming that each household consists of many members who perform different tasks in the market. While retaining the two equity market frictions in KM, this large-household construct simplifies the analysis significantly in two ways. First, it allows the use of a representative household which leads to straightforward aggregation of households' decisions. In contrast, aggregation in KM is tractable only with logarithmic preferences. Second, the reformulation enables me to formulate an individual household's decision as a dynamic programming problem, rather than the sequence problem in KM. Dynamic programming leads to a construction of a recursive competitive equilibrium, which facilitates the analysis of how the liquidity shock works in a stochastic and dynamic environment.

Then I calibrate the model to evaluate the liquidity shock hypothesis quantitatively. The calibrated model shows that a strong and persistent negative liquidity shock generates large and persistent reductions in aggregate investment, employment and output. However, contrary to the liquidity shock hypothesis, the negative liquidity shock generates an asset price boom. Such a boom is rarely observed in economic recessions. This finding casts doubt on the liquidity shock hypothesis because, according to the hypothesis, a fall in equity price is the primer of the propagation of a negative liquidity shock.

The counterfactual response of equity price is not unique to the KM model or to the particular form of the liquidity shock. Rather, it is a general feature of many models where equity is important for financing investment. To demonstrate this generality, I introduce debt finance into the KM model and capture the indirect role that existing equity can help financing new investment by relaxing a firm's collateral constraint. Specifically, the amount that a firm can borrow is proportional to the value of the firm's holdings of resaleable assets at the end of a period. Popularized by Kiyotaki and Moore (1997) and Jermann and Quadrini (2010), such a collateral constraint allows one to examine "financial shocks" that affect the ratio of a firm's borrowing capacity to the value of collateral. In this extension, a negative liquidity shock reduces both the amount of resaleable equity and the borrowing capacity. I show that a negative liquidity shock still increases equity price. Moreover, a negative financial shock alone also increases equity price, even when liquidity is fixed. In fact, all shocks that reduce firms' ability to finance investment tend to increase equity price.

The puzzling response of equity price to liquidity/financial shocks has a simple explanation. Suppose that a negative shock of this type tightens the financing constraint on firm investment. Then, the demand for, and the price of, liquid assets will rise, as long as investment projects are still attractive. Because the portion of equity that remains resaleable is liquid, its price will rise as well. In section 5, I will discuss some resolutions to the puzzle, all of which rely on direct or induced changes in effective productivity to accompany the liquidity shock.

Other authors have independently discovered the puzzling response of equity price to liquidity shocks. Nezafat and Slavik (2010) show that a negative shock to θ increases equity price, and Ajello (2010) shows that a negative shock to ϕ increases equity price. However, these authors do not focus on the puzzling response of equity price. Instead, Nezafat and Slavik (2010) focus on the importance of shocks to θ in explaining the volatility of asset prices, and Ajello (2010) on the importance of shocks to the intermediation cost in explaining the volatility of investment and output. Another closely related paper is Del Negro et al. (2011), who quantitatively evaluate the non-standard monetary policy intervention in the 2008. Both Ajello (2010) and Del Negro et al. (2011) incorporate a range of elements into KM, such as wage/price rigidity, adjustment costs in investment and habit persistence in consumption. These elements are intended to be realistic for addressing the issues in the two papers, but they cloud the picture of how liquidity shocks affect equity price. I simplify the KM model rather than complicate it. The simplified model enables me to clearly illustrate the counterfactual response of equity price to liquidity/financial shocks. In section 3.4, I will explain why adding the aforementioned elements to the model does not overturn the counterfactual response of equity price. The tractable formulation in my model should also be useful broadly for studying the role of the asset market in macro.[2]

More generally, financial frictions have been the focus of business cycle research for quite some time. The literature is too large to be surveyed here (see Bernanke et al., 1999, for a partial survey). One approach emphasizes the role of financial intermediaries in economizing on the cost of lending to and monitoring entrepreneurs who have private information on their projects' outcome (see Townsend, 1979). Williamson (1987) seems the first to use this approach to study the business cycle, and Bernanke and Gertler (1989, 1990) construct popular models along this line. The main mechanism in this approach is that net worth of entrepreneurs and/or financial intermediaries is pro-cyclical, which generates the financial multiplier. A related approach emphasizes a borrower's assets as collateral in securing debt when there is limited enforceability on debt repayment (see Kiyotaki and Moore, 1997, Jermann and Quadrini, 2010, and Liu et al., 2011). I will incorporate such a collateral constraint in section 4. A further comparison with the literature will appear in section 5.

2. A Macro Model with Asset Market Frictions

2.1. The model environment

Consider an infinite-horizon economy with discrete time. The economy is populated by a continuum of households, with measure one. Each household has a unit measure of members. At the beginning of each period, all members of a household are identical and share the household's assets. During the period, the members are separated from each other, and each member receives a shock that determines the role of the member in the period. A member will be an entrepreneur with probability $\pi \in (0, 1)$ and a worker with probability $1 - \pi$. These shocks are *iid* among the members and across time. An entrepreneur has an investment project and no labor endowment, while a worker has one unit of labor endowment and no investment project. The members'

[2] After I completed the first draft of this paper and communicated with Del Negro et al., they revised their paper to adopt the construct of large households to simplify aggregation. Also, some parts of the current paper are summarized in Shi (2011), where the focus is on the steady state.

preferences are aggregated and represented by the following utility function of the household:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi \; u\left(c_t^e\right) + (1-\pi) \left[U(c_t^w) - h(\ell_t) \right] \right\}, \quad \beta \in (0,1).$$

Here, the expectation is taken over aggregate shocks to (A, ϕ) which will be described below. The variable c_t^e is an entrepreneur's consumption, c_t^w a worker's consumption, and ℓ_t a worker's labor supply. The functions u, U and h are assumed to have standard properties. The household maximizes the above utility function by choosing the actions for the members, and the members implement these choices. In the presence of ex post heterogeneity among the individuals, this large household structure facilitates aggregation.[3]

Let me describe the technologies in the economy together with the timing of events in an arbitrary period t. The time subscript t is suppressed and the variables in period $t \pm j$ are given the subscript $\pm j$. A period is divided into four stages: households' decisions, production, investment, and consumption. In the stage of households' decisions, all members of a household are together to pool their assets. Aggregate shocks to (A, ϕ) are realized.[4] The household holds (physical) capital k, equity claims s, and liquid assets b. Capital resides in the household and will be rented to firms in the second stage to produce consumption goods. On every unit of capital there is a claim which is either sold to the outsiders or retained by the household. Thus, a household holds a diversified portfolio of equity claims on the capital stock in the economy.[5] Liquid assets are government bonds. Because all members of the household are identical in this stage, the household evenly divides the assets among the members. The household also gives each member the instructions on the choices in the period contingent on whether the member will be an entrepreneur or a worker in the second stage. For an entrepreneur, the household instructs him to consume an amount c^e, invest i, and hold a portfolio of equity and liquid assets (s_{+1}^e, b_{+1}^e) at the end of the period. For a worker, the household instructs him to consume an amount c^w, supply labor ℓ, and hold a portfolio (s_{+1}^w, b_{+1}^w) at the end of the period. After receiving these instructions, the members go to the market and will remain separated from each other until the beginning of the next period.

At the beginning of the production stage, each member receives the shock whose realization determines whether the individual is an entrepreneur or a worker. Competitive firms rent capital from the households and hire labor from workers to produce consumption goods according to

[3] A similar household structure has been used in monetary theory by Shi (1997). In a related environment, Lucas (1990) uses a two-member household structure to facilitate aggregation.

[4] This timing of aggregate shocks simplifies the analysis. If A and ϕ are realized in the second stage, instead, there may be precautionary holdings of assets.

[5] As in KM, I simplify the analysis by assuming that the claims on the household's own capital and other households' capital have the same liquidity, and so they have the same price.

$y = A\,F(k^D, \ell^D)$, where the superscript D indicates the demand. The function F has diminishing marginal productivity of each factor and constant returns to scale. Total factor productivity A follows a Markov process. After production, a worker receives wage income, and an individual who holds equity claims receives the rental income of capital.[6] Then, a fraction $(1-\sigma)$ of existing capital depreciates, where $\sigma \in (0,1)$, and every existing equity claim is rescaled by a factor σ.

The third stage in the period is the investment stage where entrepreneurs seek finance and undertake investment projects. To simplify, I assume that all investment projects are identical and each project can transform any amount $i \geq 0$ units of consumption goods into i units of new capital that will be added to next period's capital stock. In this stage, the asset market and the goods market are open. Individuals trade assets to finance new investments and to achieve the portfolio of asset holdings instructed earlier by their households.

In the final stage of the period, a worker consumes c^w and an entrepreneur consumes c^e. Then, individuals return to their households, arriving at the beginning of the next period.

There are two frictions in the equity market, as emphasized by KM. The first is that an entrepreneur can issue equity in the market on at most a fraction $\theta \in (0,1)$ of investment. The rest of the equity on new investment is retained temporarily by the entrepreneur's household. The second friction is that an individual can sell at most a fraction $\phi \in (0,1)$ of existing equity in a period. One may be able to explicitly specify the impediments in the asset market to generate these bounds endogenously.[7] As a first pass, however, I take θ and ϕ as exogenous, as KM did. Also following KM, I focus on equity resaleability ϕ by assuming that ϕ follows a Markov process while θ is fixed. Shocks to ϕ are interpreted as shocks to equity liquidity.

The asset market frictions amount to putting a lower bound on an entrepreneur's equity holdings at the end of a period. Because of the bound on new equity issues, an entrepreneur must retain $(1-\theta)i$ claims on the new capital formed by his investment. In addition, after capital depreciates, the entrepreneur has σs claims on existing capital, of which the entrepreneur must hold onto at least the amount $(1-\phi)\sigma s$. Thus, the entrepreneur's equity holdings at the end of the period, s^e_{+1}, must satisfy the following equity liquidity constraint:

$$s^e_{+1} \geq (1-\theta)i + (1-\phi)\sigma s. \tag{2.1}$$

[6] I will discuss this assumption on timing at the end of subsection 3.3.

[7] For example, if new investment differs in quality which is the entrepreneur's private information, then the entrepreneur may not be able to finance the investment entirely with equity. Also, if investment requires an enterpreneur's (non-contractible) labor input as well as the input of goods, then moral hazard on labor input may put an upper bound on θ (see Hart and Moore, 1994). The difficulty in re-selling equity, as captured by $\phi < 1$, may be caused by the lemons problem in the asset market that induces asset prices to fall sharply as the quantity sold increases. Instead of modelling this difficulty with a smoothly decreasing function, I use the two-step function in KM to simplify the analysis.

For (2.1) to be binding, an entrepreneur must face a tight borrowing limit. I set this limit as zero for now, as in KM, and will introduce debt finance in section 4. Note that the borrowing constraint is enforced by temporary separation of the members from each other in a period. This separation ensures that a household cannot shift funds from its workers to its entrepreneurs in the investment stage to circumvent entrepreneurs' liquidity constraint. This role of temporary separation is similar to that in the literature of limited participation, e.g., Lucas (1990).

Government policies are kept simple. In each period, the government spends g per household, redeems all matured bonds, and issues an amount B of new real bonds per household, where g and B are positive constants. The government collects lump-sum taxes τ per household to balance the budget in each period. (If $\tau < 0$, they are transfers to the households.) Let p_b be the price of bonds. Then, the government budget constraint is

$$g = \tau + (p_b - 1)B. \tag{2.2}$$

2.2. A household's decisions

In a period, a household chooses $(i, c^e, s^e_{+1}, b^e_{+1})$ for each entrepreneur and $(\ell, c^w, s^w_{+1}, b^w_{+1})$ for each worker. In addition to the liquidity constraint, the household faces a resource constraint on each member. On an entrepreneur, the resource constraint is:

$$rs + (b - p_b b^e_{+1}) + q(i + \sigma s - s^e_{+1}) \geq i + c^e + \tau, \tag{2.3}$$

where r is the rental rate of capital and q the price of an equity claim, measured in consumption goods. This constraint is explained as follows. An entrepreneur has three items of expenditure: consumption c^e, investment i, and the tax liability τ. The entrepreneur has three sources of funds to finance these expenditures. The first is the rental income of capital, rs. The second is the net receipt from trading liquid assets, $(b - p_b b^e_{+1})$, which is the amount obtained from redeeming matured bonds minus the amount spent on new bonds. The third is the net receipts from trading equity. After capital depreciates in the period, the entrepreneur holds σs claims on existing equity. The entrepreneur's investment creates i units of new capital. There is one claim on each unit of new capital, which is either sold to other households or retained by the entrepreneur for the household. Thus, the entrepreneur's total holdings of equity claims are $(i + \sigma s)$. Because the entrepreneur has to hold onto s^e_{+1} claims at the end of the period, the rest is sold to the market. Thus, the entrepreneur's net receipt from trading equity claims is $q(i + \sigma s - s^e_{+1})$.[8]

[8]Note that the liquidity constraint (2.1) ensures that the receipt from trading equity is strictly positive, which prevents the entrepreneur from going short on equity.

I focus on the economy where the liquidity constraint (2.1) binds.[9] In this case, since an entrepreneur is constrained in the ability to finance investment, the entrepreneur will optimally push equity holdings at the end of the period to the minimum allowed by the liquidity constraint, and liquid asset holdings to zero. That is, s^e_{+1} satisfies (2.1) with equality, and $b^e_{+1} = 0$. Substituting these quantities of (s^e_{+1}, b^e_{+1}) into the entrepreneur's resource constraint, (2.3), I consolidate the entrepreneur's *financing constraint* as follows:

$$(r + \phi\sigma q)s + b - \tau \geq c^e + (1 - \theta q)i. \tag{2.4}$$

This financing constraint reveals two features. First, the resaleability of equity increases an entrepreneur's ability to finance investment. Second, an entrepreneur's "downpayment" on each unit of investment is $1 - \theta q$, because the entrepreneur can raise an amount θq by issuing equity in the market. Note that an entrepreneur's resource constraint (2.3) holds with equality, provided that the entrepreneur's marginal utility of consumption is strictly positive. Thus, an entrepreneur's liquidity constraint (2.3) is binding if and only if the consolidated constraint (2.4) is binding.

A worker faces a resource constraint similar to (2.3), except that a worker has labor income and no investment project. Let w be the real wage rate. This constraint is:

$$rs + w\ell + q(\sigma s - s^w_{+1}) + (b - p_b b^w_{+1}) - \tau \geq c^w. \tag{2.5}$$

A worker's equity holdings at the end of the period should also satisfy the constraint: $s^w_{+1} \geq (1 - \phi)\sigma s$. However, this constraint is not binding because, in the equilibrium, workers are the buyers of the new and existing equity sold by entrepreneurs.

Denote average consumption per member in the household as c and the average holdings of the portfolio per member at the end of the period as (s_{+1}, b_{+1}). Then,

$$x = \pi x^e + (1 - \pi)x^w, \text{ for } x \in \{c, s, b, s_{+1}, b_{+1}\}. \tag{2.6}$$

Multiply (2.3) by π and (2.5) by $1 - \pi$. Adding up yields the household's resource constraint:

$$(r + \sigma q)s - qs_{+1} + (1 - \pi)w\ell + (q - 1)\pi i + (b - p_b b_{+1}) - \tau \geq c. \tag{2.7}$$

Now I can formulate a household's decisions with dynamic programming. The aggregate state of the economy at the beginning of a period is (K, Z), where K is the stock of capital per household and $Z = (A, \phi)$ is the realizations of the exogenous shocks to total factor productivity and equity resaleability. I omit the amount of equity per household and the supply of liquid

[9] As shown later, the necessary and sufficient condition for these constraints to bind is $1 < q < 1/\theta$, which is satisfied in the steady state in the calibrated economy.

assets from the list of aggregate state variables because the former is equal to K and the latter is a constant $B \geq 0$. Let $q(K,Z)$ be the price of equity, $p_b(K,Z)$ the price of liquid assets, $r(K,Z)$ the rental rate of capital, and $w(K,Z)$ the wage rate. All prices are expressed in terms of the consumption good, which is the numeraire.[10]

A household's state variables consist of equity claims, s, and liquid assets, b, in addition to the aggregate state. Denote the household's value function as $v(s,b;K,Z)$. The household's choices in a period are $(i, c^e, s^e_{+1}, b^e_{+1})$ for each entrepreneur, ℓ for each worker, and (c, s_{+1}, b_{+1}) for the average quantities per member. Note that in this list, I use the quantities per member instead of the choices for a worker, $(c^w, s^w_{+1}, b^w_{+1})$. Similarly, I can use the household's resource constraint (2.7) in lieu of a worker's resource constraint (2.5). When the financing constraint (2.4) binds, the optimal choices of (s^e_{+1}, b^e_{+1}) are $s^e_{+1} = (1-\theta)i + (1-\phi)\sigma s$ and $b^e_{+1} = 0$, respectively. The other choices, $(i, c^e, \ell, c, s_{+1}, b_{+1})$, solve:

$$v(s,b;K,Z) = \max \ \{\pi \ u(c^e) + (1-\pi)\left[U(c^w) - h(\ell)\right] + \beta \ \mathbb{E}v(s_{+1}, b_{+1}; K_{+1}, Z_{+1})\} \tag{2.8}$$

subject to (2.4), (2.7), and the following constraints:

$$i \geq 0, \ c^e \geq 0, \ c^w \geq 0, \ s^w_{+1} \geq 0, \ b^w_{+1} \geq 0, \tag{2.9}$$

where $(c^w, s^w_{+1}, b^w_{+1})$ are functions of (c, s_{+1}, b_{+1}) and $(c^e, s^e_{+1}, b^e_{+1})$ defined through (2.6). The expectation in the objective function is taken over next period's aggregate state (K_{+1}, Z_{+1}), and I have suppressed the arguments of price functions r, w, q and p_b in the constraints.

Let $\lambda^e \pi U'(c^w)$ be the Lagrangian multiplier of the financing constraint, (2.4), where the rescaling by $\pi U'(c^w)$ simplifies various expressions below. The multiplier λ^e is liquidity services provided by cash flows, measured in consumption units. As explained above, the liquidity constraint (2.1) binds if and only if $\lambda^e > 0$. Moreover, the optimal choices of (ℓ, c^e, i) yield:

$$\frac{h'(\ell)}{U'(c^w)} = w, \tag{2.10}$$

$$u'(c^e) = U'(c^w)\left(1+\lambda^e\right), \tag{2.11}$$

$$q - 1 \leq (1-\theta q)\lambda^e \text{ and } i \geq 0, \tag{2.12}$$

where the two inequalities in (2.12) hold with complementary slackness.[11] Condition (2.10) is the standard condition for optimal labor supply. Condition (2.11) captures the fact that a marginal unit of the resource is more valuable to an entrepreneur than to a worker if an entrepreneur's

[10] As is standard, the price of equity is the so-called post-dividend price; i.e., it is measured after the rental income of capital is distributed to shareholders.

[11] The constraints $c^e \geq 0$, $c^w \geq 0$, $b^w_{+1} \geq 0$ and $s^w_{+1} \geq 0$ do not bind.

financing constraint is binding, in which case the additional value to an entrepreneur is captured by $\lambda^e U'(c^w)$. The conditions in (2.12) characterize the optimal choice of investment. As explained above, the downpayment on each unit of investment in terms of goods is $1-\theta q$, the cost of which in terms of utility is $(1-\theta q)\lambda^e U'(c^w)$. For the household, a unit of investment increases the resource by $(q-1)$, the benefit of which in terms of utility is $(q-1)U'(c^w)$. Investment is zero if the cost exceeds the benefit, and positive if the cost is equal to the benefit.

It is clear from (2.12) that the financing constraint is binding (i.e., $\lambda^e > 0$) if and only if $1 < q < 1/\theta$. Note that the direct cost of replacing a unit of capital is one. Thus, when the financing constraint binds, equity price exceeds the replacement cost of capital, despite the absence of adjustment costs in investment. Intuitively, a binding constraint in financing investment creates an implicit cost that drives a wedge between equity price and the replacement cost of capital.

Finally, the optimality conditions on asset holdings at the end of the period and the envelope conditions on asset holdings together give rise to the asset-pricing equations below:

$$q = \beta\,\mathbb{E}\left\{\frac{U'(c^w_{+1})}{U'(c^w)}\left[r_{+1} + \sigma q_{+1} + \pi\lambda^e_{+1}\left(r_{+1} + \phi_{+1}\sigma q_{+1}\right)\right]\right\}, \tag{2.13}$$

$$p_b = \beta\,\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1+\pi\lambda^e_{+1}\right)\right]. \tag{2.14}$$

These asset-pricing equations incorporate liquidity services provided by the assets as implicit returns. The shadow price λ^e_{+1} enters the right-hand sides of both pricing equations because existing equity and liquid assets can both be sold to raise funds for new investment, thereby relaxing the financing constraint on an entrepreneur. However, only a fraction ϕ_{+1} of existing equity can be sold next period while all liquid assets can be sold. Thus, ϕ_{+1} appears in the pricing equation for equity but not in that for liquid assets.

2.3. Definition of a recursive equilibrium

The formulation thus far suggests a straightforward definition of an equilibrium. Let $\mathcal{K} \subset \mathbb{R}_+$ be a compact set which contains all possible values of K and $\mathcal{Z} \subset \mathbb{R}_+\times[0,1]$ a compact set which contains all possible values of Z. Let $\mathcal{C}_1$ be the set containing all continuous functions that map $\mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$, $\mathcal{C}_2$ the set containing all continuous functions that map $\mathcal{K} \times [0, B] \times \mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$ and $\mathcal{C}_3$ the set containing all continuous functions that map $\mathcal{K} \times [0, B] \times \mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}$. A *recursive competitive equilibrium* consists of asset and factor price functions (q, p_b, r, w) belonging in $\mathcal{C}_1$, a household's policy functions $(i, c^e, s^e_{+1}, b^e_{+1}, \ell, c, s_{+1}, b_{+1})$ belonging in $\mathcal{C}_2$, the value function $v \in \mathcal{C}_3$, the demand for factors by final-goods producers, (k^D, ℓ^D), and the law of motion of the aggregate capital stock that meet the following requirements:

(i) Given price functions and the aggregate state, a household's value and policy functions solve a household's optimization problem in (2.8);
(ii) Given price functions and the aggregate state, factor demands satisfy $r = AF_1'(k^D, \ell^D)$ and $w = AF_2'(k^D, \ell^D)$, where the subscripts of F indicate partial derivatives;
(iii) Given the law of motion of the aggregate state, prices clear the markets:

$$\text{goods:} \quad c(s, b; K, Z) + \pi i(s, b; K, Z) + g = AF(k^D, \ell^D), \tag{2.15}$$

$$\text{labor:} \quad \ell^D = (1 - \pi)\ell(s, b; K, Z), \tag{2.16}$$

$$\text{capital:} \quad k^D = K = s, \tag{2.17}$$

$$\text{liquid assets:} \quad b_{+1}(s, b; K, Z) = b \equiv B, \tag{2.18}$$

$$\text{equity:} \quad s_{+1}(s, b; K, Z) = \sigma s + \pi \ i(s, b; K, Z); \tag{2.19}$$

(iv) The law of motion of the aggregate capital stock is consistent with the aggregation of individual households' choices:

$$K_{+1} = \sigma K + \pi \ i(K, B; K, Z). \tag{2.20}$$

Since the explanations for the requirements (i)-(iv) are straightforward, I only add the following clarifications. In the capital market clearing condition, the equality $K = s$ states the fact that there are claims on all capital. In the equity market clearing condition, new equity claims are equal to new investment, πi, because s is defined to include not only equity claims sold in the market but also claims retained by the household. Condition (iv) is explicitly imposed here because it is needed for the households to compute the expectations in (2.8). However, because $K = s$, the law of motion of the capital stock duplicates the equity market clearing condition – a reflection of the Walras' law.

Determining an equilibrium amounts to solving for asset price functions $q(K, Z)$ and $p_b(K, Z)$. Once these functions are determined, other equilibrium functions can be recovered from a household's first-order conditions, the Bellman equation in (2.8), the market clearing conditions and factor demand conditions. To solve for asset price functions, I can use the right-hand sides of the asset pricing equations, (2.13) and (2.14), to construct a mapping T that maps a pair of functions in $\mathcal{C}_1$ back into $\mathcal{C}_1$ (see Appendix A). The pair of functions (q, p_b) in an equilibrium is a fixed point of T. I will implement this procedure numerically in subsection 3.1.

I relegate the discussion on the value of liquidity and the equity premium to Appendix B and the steady state to Appendix C. For comparative statics of the model, see Shi (2011).

3. Equilibrium Response to Shocks

I calibrate the model and examine how the equilibrium responds to equity liquidity shocks.

3.1. Calibration and computation

For the utility and production functions, I choose the following standard forms:

$$U(c^w) = \frac{(c^w)^{1-\rho} - 1}{1-\rho}, \quad u(c^e) = u_0 U(c^e),$$

$$h(\ell) = h_0 \ell^\eta, \quad F(K, (1-\pi)\ell) = K^\alpha \left[(1-\pi)\ell\right]^{1-\alpha}.$$

For the exogenous state of the economy (A, ϕ), I assume:

$$\log A_{t+1} = (1-\delta_A)\log A^* + \delta_A \log A_t + \varepsilon_{A,t+1}, \tag{3.1}$$

$$-\log(\frac{1}{\phi_{t+1}} - 1) = -(1-\delta_\phi)\log\left(\frac{1}{\phi^*} - 1\right) - \delta_\phi \log\left(\frac{1}{\phi_t} - 1\right) + \varepsilon_{\phi,t+1}. \tag{3.2}$$

The superscript $*$ indicates the non-stochastic steady state. These processes ensure $A \geq 0$ and $\phi \in [0,1]$. The quantitative analysis below will take ε_A and ε_ϕ as one-time shocks.

I choose the length of a period to be one quarter and calibrate the non-stochastic steady state to the US data. The steady state and the calibration are described is Appendix C. The value of the discount factor β and the relative risk aversion are standard; so are the following targets and parameter values. The elasticity of labor supply is equal to two, and aggregate hours of work in the steady state are 0.25. The share of labor income in output is $1-\alpha = 0.64$, the ratio of annual investment to capital in the steady state is $4(1-\sigma) = 0.076$, and the ratio of capital to annual output is 3.32. The steady state value of productivity is normalized to $A^* = 1$ and the persistence of productivity is $\delta_A = 0.95$. Government spending g is set to be 18% of the steady state level of output. Note that the parameter u_0 is identified by the ratio of capital to output because u_0 affects entrepreneur's consumption which in turn affects the capital stock.

Let me discuss the remaining identification restrictions. First, the parameter π can be interpreted as the fraction of firms that adjust their capital in a period. The estimate of this fraction ranges from 0.20 (Doms and Dunne, 1998) to 0.40 (Cooper et al., 1999) annually. I choose a value 0.24 in this range, which leads to $\pi = 0.06$ quarterly. Second, θ is set to be equal to ϕ^* as a benchmark. Third, liquidity shocks must be persistent in order to generate persistent effects. Thus, I set $\delta_\phi = 0.9$ in the baseline calibration. Fourth, the rate of return to liquid assets and the fraction of liquid assets in the total value of assets come from the evidence in Del Negro et al. (2011). These authors report that the annualized net rate of return to the US government liabilities is 1.72% for one-year maturities and 2.57% for ten-year maturities. I choose a value in this interval, 0.02.[12] Finally, Del Negro et al. (2011) use the US Flow of Funds between 1952 and 2008 to compute the share of liquid assets in asset holdings. Their measure of liquid assets consists of all liabilities of the Federal Government, that is, Treasury securities net of holdings by the monetary authority and the budget agency plus reserves, vault cash and currency net of remittances to the Federal Government. The sample average of the share of liquidity assets is close to 0.12, which I target in the calibration.

Table 1. Parameters and calibration targets

parameter	value	calibration target
β: discount factor	0.992	exogenously chosen
ρ: relative risk aversion	2	exogenously chosen
π: fraction of entrepreneurs	0.06	annual fraction of investing firms = 0.24
u_0: constant in entrep. utility	42.803	capital stock/annual output = 3.32
h_0: constant in labor disutility	16.780	hours of work = 0.25
η: curvature in labor disutility	1.5	labor supply elasticity $1/(\eta-1)=2$
α: capital share	0.36	labor income share $(1-\alpha)=0.64$
σ: survival rate of capital	0.981	annual investment/capital = 0.076
A^*: steady-state TFP	1	normalization
δ_A: persistence in TFP	0.95	persistence in TFP = 0.95
B: stock of liquid assets	2.021	fraction of liquid assets in portfolio = 0.12
ϕ^*: steady-state resaleability	0.273	annual return to liquid assets = 0.02
δ_ϕ: persistence in resaleability	0.9	exogenously chosen
θ: fraction of new equity	0.273	set to equal to ϕ^*
g: government spending	0.193	government spending/GDP = 0.18

Some of the identified values are worth mentioning. First, equity resaleability in the steady state is $\phi^* = 0.273$.[13] Because this is significantly less than one, the resale market for equity is far from being liquid. Notice that ϕ^* is identified by the target that the annual yield on liquid assets is 0.02. If all assets were liquid, then the yield on liquid assets would be equal to the discount rate, $\beta^{-1/4} - 1 = 0.0327$. The difference between the yield on liquid assets and the discount rate is accounted for by the liquidity service performed by liquid assets. Second, the price of equity in the steady state is $q^* = 1.0367$. Note that this value of q^* satisfies $1 < q^* < 1/\theta$, and so the financing constraint binds. Third, in the steady state, the rental rate of capital is $r^* = 0.0271$ and the price of liquid assets is $p_b^* = 0.9951$. So, the annualized equity premium in the steady state is

[12] Note that the pricing equation for liquid assets in the steady state imposes the constraint $p_b^* \geq \beta$. Thus, given the value of β, the upper bound on the annual rate of return to liquid assets is $\beta^{-4} - 1 = 0.0327$. Thus, the value chosen for the rate of return to liquid assets is in this feasible region.

[13] Nezafat and Slavik (2010) use the US Flow of Funds data for non-financial firms to estimate the stochastic process of θ. Interpreting θ as the ratio of funds raised in the market to fixed investment, they find that the mean of θ is 0.284. This is close to the value $\theta = 0.273$ that I use here.

$4(r^*/q^* + \sigma - 1/p_b^*) = 0.0087$. This premium is significant, considering that it is associated with the steady state where no risk is present.

Suppose that the error terms in the processes of A and ϕ are zero except possibly for $t = 1$. That is, the paths of A and ϕ are all realized at the beginning of $t = 1$, which is the case with one-time shocks. I follow the procedure in Appendix A to compute equilibrium asset price functions $(q, p_b)(K, Z)$, where $Z = (A, \phi)$. Then, I recover an individual household's policy functions $x(s, b; K, Z)$, where x is any element in the list $(c, i, c^e, s^e_{+1}, \ell, c^w, s_{+1}, b_{+1})$. Since the equilibrium has $s = K$ and $b = B$, where B is a constant, I shorten the notation $x(K, B; K, Z)$ as $x(K, Z)$.

Most of the policy functions have predictable properties. For example, consumption, investment and output are increasing functions of the capital stock, K. An exception might be the dependence of asset prices on the capital stock. For most values of the capital stock, equity price and the price of liquid assets are decreasing functions of the capital stock. On equity price, a plausible explanation is that as the capital stock increases, the rental rate of capital falls which reduces equity price. On the price of liquid assets, a plausible explanation is that as the capital stock increases, the need for further investment falls, which reduces the demand for liquid assets.

3.2. Equilibrium response to an asset liquidity shock

Suppose that the economy is in the non-stochastic steady state at time $t = 0$. At the beginning of $t = 1$, there is an unanticipated drop in liquidity to 0.221, a 19% drop from ϕ^*. After this shock, ϕ follows the process in (3.2), with $\varepsilon_{\phi,t} = 0$ for all $t \geq 2$. To focus on this shock, let me assume for the moment that the fraction of new equity issuance, θ, and total factor productivity, A, are fixed at their steady state levels. The dynamics are computed as in Appendix C.[14]

Figure 2.1 graphs aggregate investment ($I = \pi i$) and equity resaleability, where the vertical axis is percentage deviations of the variables from their steady-state levels and the horizontal axis is the number of quarters after the shock. In period 1, the negative liquidity shock reduces investment by 17%. Although the negative liquidity shock is large by construction, the size of the reduction in investment may still be surprising in the following sense. Because θ does not fall with the liquidity shock in this experiment, entrepreneurs can still issue new equity to finance new investment. The large fall in investment indicates that a majority of new investment is financed by selling existing equity and other cash flows rather than issuing new equity. Figure 2.1 also shows that investment closely follows the dynamics of equity liquidity. Because the liquidity shock is assumed to be persistent, the shock has persistent effects on investment. Three years

[14] Note that the dynamics are computed using the policy functions and asset pricing functions instead of linearizing the equilibrium system. This approach has the advantage of being able to deal with large shocks.

after the shock, investment is still about 7% below the steady state.

Figure 2.2 exhibits the percentage deviations of aggregate employment ($L = (1-\pi)\ell$) and output (Y) from the steady state. Both variables fall by significant amounts when the negative liquidity shock hits. Employment falls by 5.3% and output by 3.4% in period 1. Note that the reduction in output in period 1 comes entirely from the reduction in employment, because the capital stock in period 1 is predetermined and total factor productivity is fixed in this experiment. After period 1, however, the capital stock (not depicted) also falls below the steady state due to lower investment, which keeps output low. The responses of these aggregate variables are persistent. Three years after the shock, employment and output are still about 2% below the steady state. These responses of aggregate quantities seem to suggest that liquidity shocks to the equity market can potentially be an important cause of aggregate fluctuations.

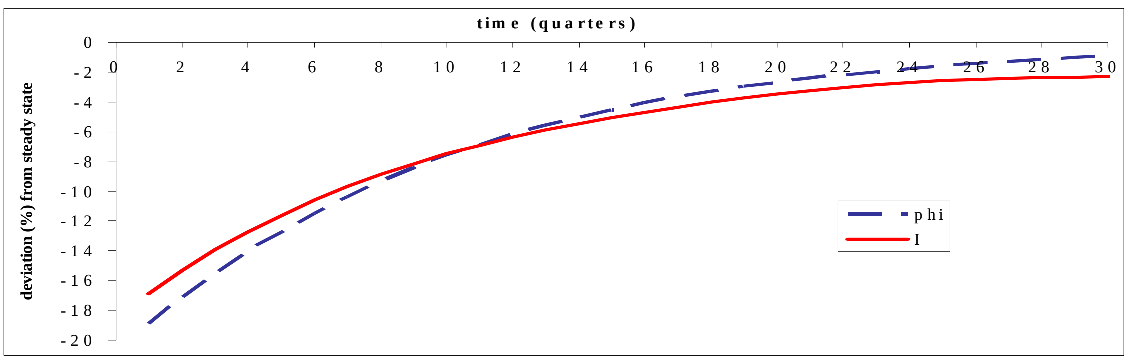

Figure 2.1. Investment and liquidity after a negative liquidity shock

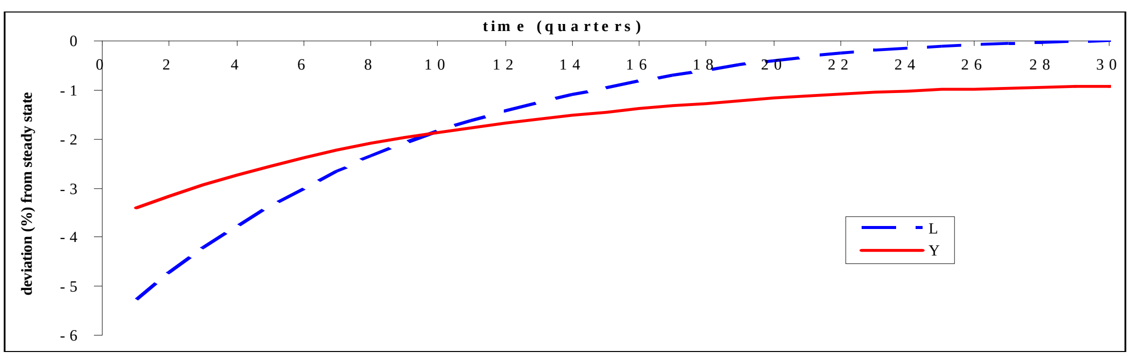

Figure 2.2. Employment and output after a negative liquidity shock

One can question whether shocks to equity liquidity are as large and persistent as I assumed. Instead of getting into this debate, let me check how asset prices respond to the liquidity shock. As Figure 2.3 shows, a negative shock to equity liquidity generates an asset price boom! Immediately after the shock, equity price increases by 4.5% and the price of liquid assets increases by 0.5%.

Asset prices stay above the steady state for quite a long time. Three years after the shock, equity price is still 2% above the steady state. Thus, the negative liquidity shock to the asset market can generate large and persistent fluctuations in equity price, but the direction of this response is opposite to what is conjectured by KM and opposite to what is observed in the data.

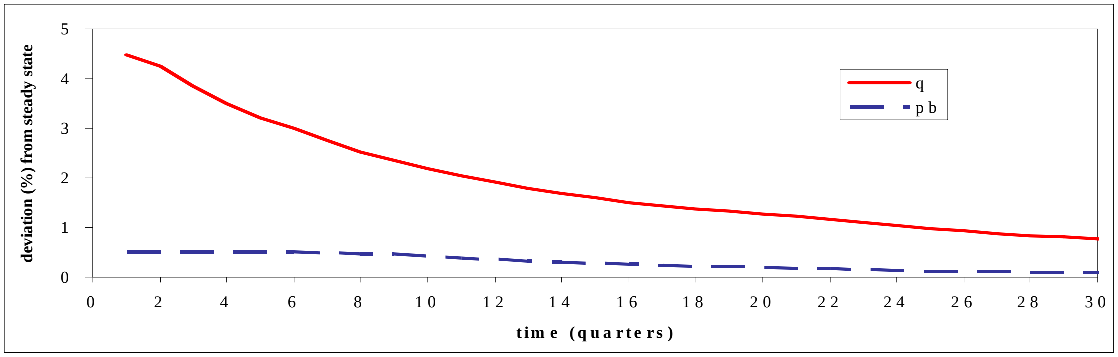

Figure 2.3. Asset prices after a negative liquidity shock

3.3. What is the cause of this problem?

One suspect is the fixed θ, the fraction of investment that can be financed by issuing new equity. I will investigate the likely scenario that θ falls with ϕ. Another suspect is that the model is too simplistic. By abstracting from many realistic elements, this model might have forced some variables to respond to the liquidity shock in the wrong magnitude or direction, in which case equity price might respond to the liquidity shock in the wrong direction in order to make up for the unrealistic responses in other variables. The following is a partial list of omitted elements:
(i) wage rigidity: the absence of it in my model may imply that output does not fall enough after the negative liquidity shock;
(ii) habit persistence in consumption: the absence of this ingredient may imply that consumption responds to the liquidity shock by too much or in the wrong direction;
(iii) adjustment costs of investment: the absence of these costs may imply that investment may fall by too much immediately after the negative liquidity shock.
These items are related. If the fall in output is sufficiently deficient and the fall in investment is sufficiently large, then consumption must increase after the negative liquidity shock in order to clear the goods market. In fact, after the negative liquidity shock described above, an entrepreneur's consumption falls, a worker's consumption increases, and aggregate consumption increases (not depicted here). Given this apparent defect of the model, one may be tempted to put items (i)-(iii) above into the model.[15]

[15]Subsection 3.4 will show that large adjustment costs in investment can make aggregate consumption fall

Such an effort will be futile in overturning the positive response of equity price to a negative liquidity shock. So will be the effort of allowing θ to fall together with the shock. To explain, let me examine the condition of optimal investment, (2.12). When investment is positive, this condition becomes:

$$q - 1 = (1 - \theta q)\lambda^e. \quad (3.3)$$

Because this equation is central to the argument, let me repeat the meanings of the terms in it. For each unit of capital formed by investment, the price is q and the direct marginal cost is one. So, the benefit of investment is $(q-1)$. If there were no frictions in the equity market, investment could be positive and finite in the equilibrium if and only if $q = 1$. Since there are frictions in issuing new equity, as modeled by $\theta < 1$, the funds raised by issuing new equity are θq. The remainder of the funds for the investment, $1-\theta q$, must come from other sources. The cost of this downpayment on investment depends on the implicit cost of the financing constraint (2.4), λ^e. Thus, $(1 - \theta q)\lambda^e$ is the implicit marginal cost of a unit of investment. Condition (3.3) requires the marginal benefit of investment to be equal to the marginal cost.

Condition (3.3) provides a simple explanation for why asset prices increase after a negative liquidity shock. The condition contains only two variables, equity price q and the shadow price of the financing constraint, λ^e. For any given λ^e, the marginal benefit of investment is a strictly increasing function of q and the downpayment on investment a strictly decreasing function of q. As long as a negative liquidity shock reduces an entrepreneur's ability to finance the downpayment of investment, the shock tightens the financing constraint (2.4) and increases its shadow price λ^e. The higher λ^e raises the implicit marginal cost of investment for any given equity price. To restore the balance between the marginal benefit and cost of investment, equity price must increase.

This explanation is general and can be phrased as a *rule of thumb*: Whenever a shock tightens the entrepreneur's financing constraint, all assets that can help raising funds for investment experience price gains because they become more valuable to the entrepreneur. The resaleable portion of equity is one such asset, and liquid assets are another. At the risk of over-simplification, let me phrase the result in terms of the demand for and the supply of equity. A reduction in equity resaleability reduces the supply of equity. In contrast, the demand for equity is not affected so much by the reduction, because there is no change to the quality of investment projects. As a result, the price of equity must increase to clear the equity market. With this generality, the argument can survive a wide range of variations/extensions of the model and the liquidity shock.

together with investment, employment and output in response to a negative liquidity shock. In the absence of adjustment costs, combined shocks to liquidity and productivity can also generate this positive comovement (see section 5). Ajello (2010) emphasizes the importance of nominal wage rigidity in producing this positive comovement in response to financial shocks when nominal prices are rigid.

I discuss some of these variations in the remainder of this section and the next section.

Consider first the plausible scenario that θ falls with ϕ. This concurrent fall in θ exacerbates the problem in the response of equity price to ϕ. Because the reduction in θ further tightens an entrepreneur's financing constraint, it makes the resaleable portion of equity even more valuable than if θ is fixed. This effect is clear from (3.3). For any given equity price and given ϕ, a fall in θ increases the downpayment needed for each unit of investment, which increases the implicit marginal cost of investment. To restore the balance between the marginal benefit and cost of investment, equity price must rise even further after a negative liquidity shock.[16]

Next, consider the large household construct used in this model. With this construct, entrepreneurs and workers in a household pool their assets at the beginning of each period, and so heterogeneity in asset holdings among individuals created by trade during a period lasts only for one period. Because this pooling allows entrepreneurs in the next period to use the assets accumulated by other members in the current period, it reduces the persistence of the negative liquidity shock on an entrepreneur's financing constraint. When such pooling is not allowed, as in KM, the financing constraint will be tighter, which will require equity price to increase by even more in response to a negative liquidity shock.

Another assumption in this model and in KM is that an entrepreneur has an immediate access to capital income in the period. That is, the income rs is available for financing current investment, as can be seen from (2.3). One may consider the alternative timing according to which the income rs is available only for financing consumption at the end of the period but not for financing investment in the current period. In this case, a fall in equity resaleability will tighten the financing constraint to a greater extent than it does in the current model and, hence, will increase equity price by even more.

3.4. Robustness to adding the usual elements

The usual elements considered in this subsection are wage rigidity, habit persistence in consumption, and adjustment costs in investment.

Wage rigidity and habit persistence do not directly affect the marginal benefit and cost of investment, as it is clear from (3.3). Their indirect effects may tighten the financing constraint, (2.4), even further and exacerbate the problem in the response of equity price to a liquidity shock. To see this, consider wage rigidity first. When there is a fall in equity liquidity, labor

[16]Note that this explanation suggests that a negative shock to θ by itself increases equity price even if ϕ is fixed. This explains a result in Nezafat and Slavik (2010). Setting $\phi = 1$ and focusing on the volatility of asset prices, they find that a negative shock to θ increases equity price.

demand and, hence, output is likely to fall by more with rigid wages than with flexible wages. As a result, the rental income of capital will fall by more when wages are rigid. Because capital income is part of the resource that an entrepreneur uses to finance investment, the financing constraint (2.4) becomes tighter, and so equity price rises by more with rigid wages than with flexible wages. Next consider habit persistence in consumption. When an entrepreneur cannot adjust consumption quickly because of habit persistence, the entrepreneur needs resource not only to finance investment but also to support persistently high consumption. Again, a negative liquidity shock will tighten the financing constraint (2.4) by more in this case than when there is no habit persistence, which requires equity price to increase by more.

In contrast to wage rigidity and habit persistence, adjustment costs in investment directly affect the condition of optimal investment. For concreteness, let me adopt the conventional assumption that entrepreneurs purchase newly installed capital goods from capital-goods producers who are perfectly competitive. Producing and installing I units of new capital costs $[I + I^*\Psi(I/I^*)]$ units of consumption goods, where I^* is steady-state investment and Ψ satisfies $\Psi(1) = 0$, $\Psi'(1) = 0$, and $\Psi'' > 0$.[17] Let p_I denote the price of newly installed capital. A capital-goods producer maximizes profit, $p_I I - I - I^*\Psi(I/I^*)$, and the optimal choice of I satisfies $1 + \Psi'(I/I^*) = p_I$. Profit of such a firm is zero in the steady state. Outside the steady state, profit can be non-zero, which is rebated to the household in a lump sum and hence added to the resource side of (2.4), (2.5) and (2.7). Because a unit of investment costs an entrepreneur p_I units of consumption goods, the term i on the right-hand side of an entrepreneur's resource constraint, (2.3), is replaced with $p_I i$; the term $\pi(q-1)i$ in the household's resource constraint (2.7) is replaced with $\pi(q - p_I)i$; and the term $(1 - \theta q)i$ in the consolidated financing constraint (2.4) is replaced with $(p_I - \theta q)i$. A household's optimal investment satisfies:

$$q - (1 + \Psi') = (1 + \Psi' - \theta q)\lambda^e. \tag{3.4}$$

where I have substituted the result $p_I = 1 + \Psi'(I/I^*)$.

Adjustment costs add two effects on optimal investment. One is the effect on λ^e through the total downpayment, $(p_I - \theta q)i$, and this effect is ambiguous analytically. On the one hand, adjustment costs prevent investment from falling by as much as in the baseline model. This has a positive effect on the total downpayment on investment. One the other hand, the presence of adjustment costs allows the replacement cost of capital (p_I) to fall, which reduces the total downpayment. After a negative liquidity shock, the shadow price λ^e increases by more with adjustment costs than in the baseline model if the total downpayment falls by less than in the

[17] Del Negro et al. (2010) use a similar specification.

baseline model. The second effect of adjustment costs is that the marginal cost of adjustment directly enters (3.4). For any given (q, λ^e), the marginal benefit of investment is a decreasing function of i and the marginal cost an increasing function of i. When investment falls after a negative liquidity shock, the marginal adjustment cost falls, which reduces the replacement cost. Such savings at the margin increase the net marginal benefit of investment for any given (q, λ^e) and mitigates the upward pressure on equity price caused by the increase in λ^e.

Although the overall effect of adjustment costs on (3.4) is analytically ambiguous, the effect is unlikely to overturn the positive response of equity price to a negative liquidity shock. For the marginal savings from reduced investment to be significant, the marginal cost of adjustment, Ψ', must be sufficiently steep, which implies that the reduction in investment must be sufficiently small. This implication may be inconsistent with the observed large reduction in investment at the beginning of a recession (see Figure 1). Moreover, for equity price to fall with a negative liquidity shock, the savings from adjustment costs have to be so large that they wipe out the direct effect of the shock that tightens the financing constraint. This does not seem plausible.

For a concrete illustration, I set $\Psi(\frac{I}{I^*}) = \frac{1}{\psi}\left|\frac{I}{I^*} - 1\right|^{\psi}$ and $\psi = 2$. Figures 3.1 - 3.3 depict the responses of some variables to the negative liquidity shock examined in subsection 3.2. The adjustment cost is sizable in this example. The replacement cost of capital falls by 6.5% on impact of the shock, and investment falls by less than 40% of that in Figure 2.1.[18] Despite such large savings from a lower cost of capital, the negative liquidity shock still tightens the financing constraint substantially, as depicted by the large increase in λ^e in Figure 3.2, where the percentage change in λ^e is rescaled by a factor 1/100. As a result, equity price and the bond price increase after the shock, as in the baseline model.

Figure 3.3 shows that employment and output fall by about a half of the amounts in the baseline model in response to the negative liquidity shock, because investment falls by a smaller amount when there are adjustment costs. However, the reductions in employment and output are still sizable. Notice that aggregate consumption falls after the negative liquidity shock, in contrast to the rise in the baseline model. The reason is that total expenditure on investment, which is equal to $p_I I$, falls by 13% instead of 17% as in the baseline model. With this smaller reduction in investment expenditure, consumption also needs to fall to help accounting for the reduction in GDP. Thus, large adjustment costs are useful for producing the positive comovement between aggregate consumption and other major macro quantities.

[18]With the particular function Ψ and the value $\psi = 2$, the percentage deviation of p_I from the steady state is equal to that of I. This is why the two lines in Figure 3.1 coincide.

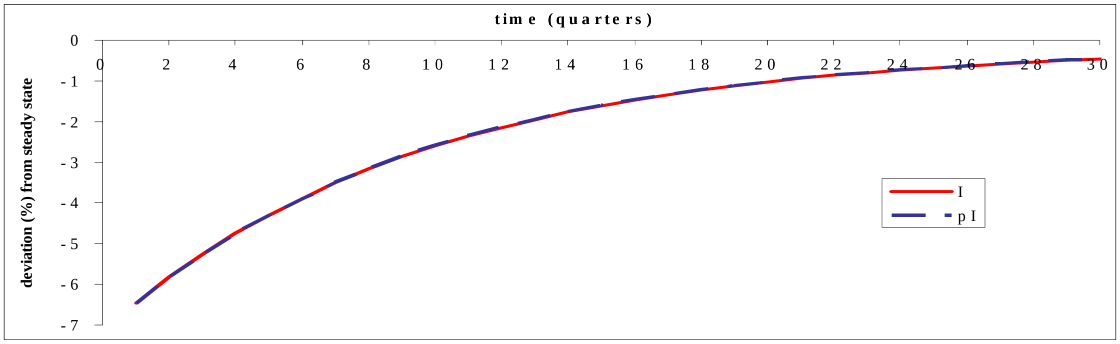

Figure 3.1. Investment and the cost of capital after a negative liquidity shock

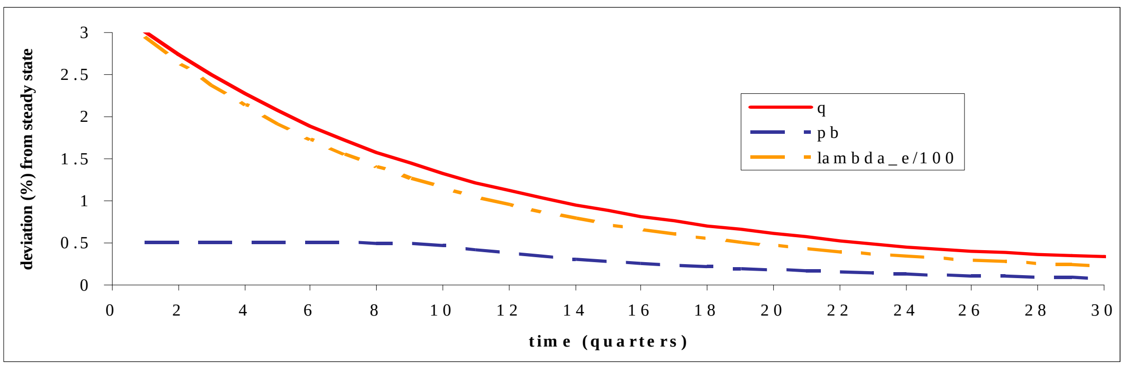

Figure 3.2. Asset prices and λ^e after a negative liquidity shock

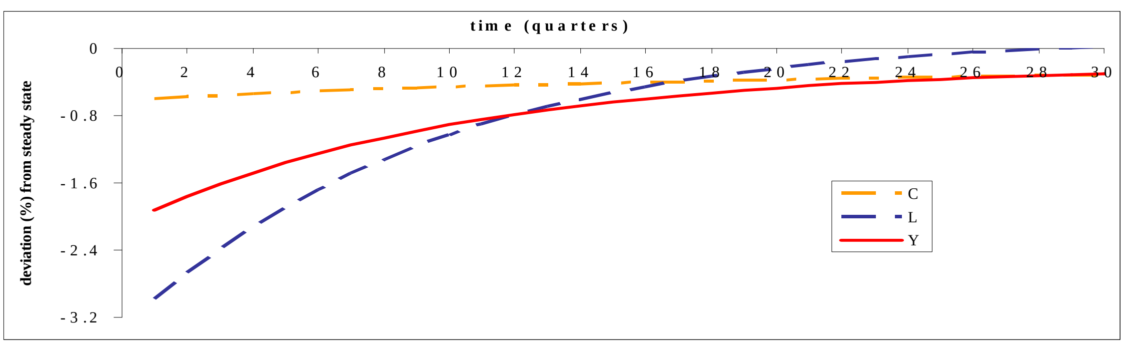

Figure 3.3. Aggregate C, L and Y after a negative liquidity shock

4. Debt Finance and Collateral

So far I have abstracted from debt finance. Debt finance can be important for the business cycle, as shown in the literature discussed at the end of the introduction. In particular, Kiyotaki and Moore (1997) have shown that cyclical fluctuations in the asset value can amplify and propagate the business cycle by affecting the value of collateral on borrowing. In this section, I introduce debt finance with collateral for two purposes. One is to demonstrate that introducing debt finance does not overturn the counterfactual response of equity price to liquidity shocks. The other is to show that equity price responds in such a counterfactual way to a broad set of financial shocks. In particular, a negative shock that reduces the borrowing capacity also increases equity price, even when equity liquidity is fixed.

Suppose that individuals can borrow and lend through a perfectly competitive intermediary. In a period, let d^e_{+1} be the amount borrowed by an entrepreneur and d^w_{+1} the amount borrowed by a worker. Define $d_{+1} = \pi d^e_{+1} + (1-\pi)d^w_{+1}$ as the amount of borrowing per member in the household. Such borrowing among the households should be distinguished from the borrowing between the government and the households, the latter of which is still denoted b. Assume that borrowing is in the form of one-period debt. At the beginning of each period, the household pools all members' outstanding debts and divide them evenly among the members before they go to the market. During the period, each member repays the outstanding debt allocated to him. The amount of outstanding debt per member in the household at the beginning of the period is d. Let R be the gross interest rate on the outstanding debt and R_{+1} the rate on new debt.[19] For an entrepreneur, the net receipt from new borrowing minus the repayment on the outstanding debt is $(d^e_{+1} - Rd)$, which is added to the resource side of the entrepreneur's resource constraint, (2.3). Similarly, the term $(d^w_{+1} - Rd)$ is added to the resource side of a worker's resource constraint, (2.5), and the term $(d_{+1} - Rd)$ to the resource side of a household's resource constraint, (2.7). The liquidity constraint on an entrepreneur, (2.1), still applies.

The borrowing limit can depend on the asset value, as emphasized by Kiyotaki and Moore (1997). Specifically, because a borrower can renege on the repayment, a lender asks the borrower to put up collateral and is only willing to lend up to a fraction of the value of the collateral. For an entrepreneur, the collateral is equity holdings at the end of the period, whose value is qs^e_{+1}. Let $m(\phi)$ be the fraction of this value that can be collateralized, which will be referred to as the collateral multiplier. Then, an entrepreneur's borrowing limit is:

$$m(\phi) q s^e_{+1} \geq d^e_{+1}, \quad \text{where } m(\phi) \in [0,1) \text{ and } m'(\phi) \geq 0. \tag{4.1}$$

The case $m = 0$ is the baseline model. The assumption $m < 1$ captures the fact that a borrower cannot borrow against the value of the entire asset holdings, partly because some of those assets are not immediately liquid. The assumption $m'(\phi) \geq 0$ captures the feature that a lender is

[19] In the equilibrium, workers are the lenders. Because a worker is indifferent between lending to the intermediary and lending to the government, the gross interest rate of lending to the intermediary must be equal to $1/p_b$. Moreover, because there is perfect competition in intermediation, the borrowing rate must be equal to the lending rate. Thus, $R_{+1} = 1/p_b$ in the equilibrium.

willing to lend more if the borrower's collateral is more liquid, because the lender can sell the collateral in the market more easily in this case if the borrower defaults on the debt.[20] Although the collateral constraint is exogenously imposed here, I will remark later that endogenizing the constraint does not change the qualitative response of equity price to liquidity shocks.

Again, focus on the relevant case where the liquidity constraint, (2.1), is binding. Since $m < 1$, it can be verified that the collateral constraint, (4.1), also binds in this case. These binding constraints solve for (s^e_{+1}, d^e_{+1}). Substituting these quantities into an entrepreneur's resource constraint, I obtain the following consolidated financing constraint on an entrepreneur:

$$[r + \phi\sigma q + (1-\phi)m(\phi)\sigma q]\, s + b - Rd - \tau \geq c^e + [1 - \theta q - (1-\theta)m(\phi)q]\, i. \tag{4.2}$$

This constraint modifies (2.4) as follows. First, there is repayment on the outstanding debt, Rd. Second, the effective downpayment on each unit of investment is reduced from $(1-\theta q)$ to $[1 - \theta q - (1-\theta)m(\phi)q]$. The additional reduction, $(1-\theta)m(\phi)q$, comes from the role of collateral served by the entrepreneur's holdings of equity at the end of the period. That is, although a fraction $(1-\theta)$ of investment cannot be financed by issuing new equity, the capital created by this fraction of investment can be used as collateral to secure the amount of borrowing $(1-\theta)m(\phi)q$, which reduces the cash flow needed for investment. Third, the funds from each claim on existing equity are increased from $(r + \phi\sigma q)$ to $[r + \phi\sigma q + (1-\phi)m(\phi)\sigma q]$. Again, the additional amount, $(1-\phi)m(\phi)\sigma q$, comes from the role of assets as collateral in borrowing. That is, the fraction $(1-\phi)$ of each existing equity claim that cannot be immediately sold in the market can be used as collateral to secure the amount of borrowing, $(1-\phi)m(\phi)\sigma q$.

The outstanding debt in each period is a state variable for a household, and so the household's value function is modified as $v(s, b, d; K, Z)$. I can reformulate the household's decision problem by adding d_{+1} to the list of choices, where the constraints are (2.9), (4.2), and the modified resource constraint on the household. In the optimality conditions, (2.12) and (2.13), the term θq is replaced with $[\theta + (1-\theta)m(\phi)]q$, and the term ϕq is replaced with $[\phi + (1-\phi)m(\phi)]q$, for the reasons explained above. In the equilibrium definition, the loan market clearing condition, $d_{+1} = d = 0$, is added, which comes from the fact that all households are identical.

To check the quantitative response of the equilibrium to a negative liquidity shock, I specify the collateral multiplier as

$$m(\phi) = \max\{\phi + \mu(1-\phi), 0\}, \tag{4.3}$$

where $\mu < 1$ is a constant. The assumption $\mu < 1$ is imposed to guarantee $m < 1$. However, I do

[20] Note that a worker faces a similar constraint, $m(\phi)qs^w_{+1} \geq d^w_{+1}$, but this constraint is not binding, because a worker is a lender in the equilibrium.

not assume $\mu \geq 0$. If $\mu < 0$, the debt limit is lower than the value of the part of the collateral that can be immediately sold. If $\mu > 0$, a borrower can borrow more than the value of this immediately liquid fraction of the collateral, possibly because a lender can sell the remaining fraction in the future. To identify μ, I use the micro evidence in Covas and den Haan (2011) who report the ratio of debt issuance to assets and the ratio of equity sales to assets in each period for US firms. Dividing these two ratios yields the ratio of debt issuance to equity sales, denoted as DE. This ratio has large variations across firms and typically increases in firm size. I target the value for the bottom 50% firms, 1.287, in order for the collateral constraint to be sufficiently binding. I identify μ by equating DE to the ratio of debt issuance to equity sales produced by the steady state of the model.[21] Although all of the calibration targets in Table 1 are kept, incorporating the collateral role of asset holdings changes some of the equilibrium conditions and, hence, some of the parameter values. The changes in the parameter values are:

$$u_0 = 40.484, \quad h_0 = 16.556, \quad \phi^* = \theta = 0.118, \quad \mu = 0.062.$$

The implied value of the collateral multiplier is $m(\phi^*) = 0.173$.

Suppose that the economy is in the steady state at $t = 0$ and there is a shock to liquidity at the beginning of $t = 1$ that reduces ϕ from $\phi^* = 0.118$ to $\phi_1 = 0.094$. Figure 4.1 depicts the dynamics of equity liquidity ϕ and the collateral multiplier m. The negative liquidity shock has large and persistent effects on the collateral multiplier. On impact of the shock, the collateral multiplier falls by 13%. Three years after the shock, the collateral multiplier is still 4.8% below the steady state. Figure 4.2 depicts the responses of the prices of equity and liquid assets. As in the baseline model, a negative liquidity shock increases both prices. Also similar to the baseline model, investment and output fall (not depicted).

The positive response of equity price to a negative liquidity shock conforms with the rule of thumb described in subsection 3.3. That is, by tightening an entrepreneur's financing constraint, the shock makes the resaleable portion of equity more valuable. To see why this rule of thumb still applies in the presence of debt finance, let us examine the condition for optimal investment, which can be written as follows:

$$q - 1 = \left[1 - \theta q - (1-\theta) m(\phi) q\right] \lambda^e. \tag{4.4}$$

[21] In the steady state, debt issuance by an entrepreneur in a period is equal to $m(\phi^*)q^* s^{e*}$, where The amount of equity sold by an entrepreneur in a period is $\theta q^* i^* + \phi^* \sigma s^* q^*$. Thus,

$$DE = \frac{m(\phi^*) s^{e*}}{\theta i^* + \phi^* \sigma s^*} = m(\phi^*) \frac{(1-\theta) i^*/K^* + (1-\phi^*)\sigma}{\theta i^*/K^* + \phi^* \sigma}.$$

As in the baseline model, the negative liquidity shock increases the implicit cost of investment through λ^e by tightening an entrepreneur's financing constraint. In addition, the negative liquidity shock reduces the collateral multiplier $m(\phi)$, which reduces the amount of borrowing by $(1-\theta)mq$. As the downpayment on investment increases, an entrepreneur's financing constraint is tightened further. To keep investment optimal in this case, equity price must rise to increase the benefit of investment, $q-1$, and to mitigate the increase in the downpayment.[22]

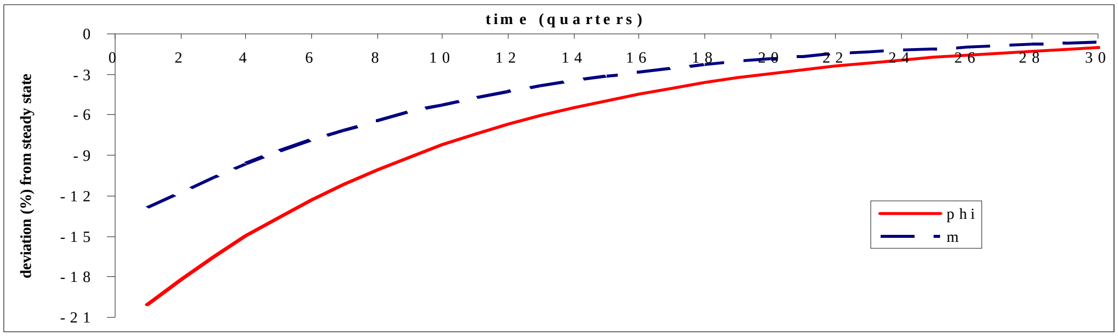

Figure 4.1. Equity liquidity and the collateral multiplier

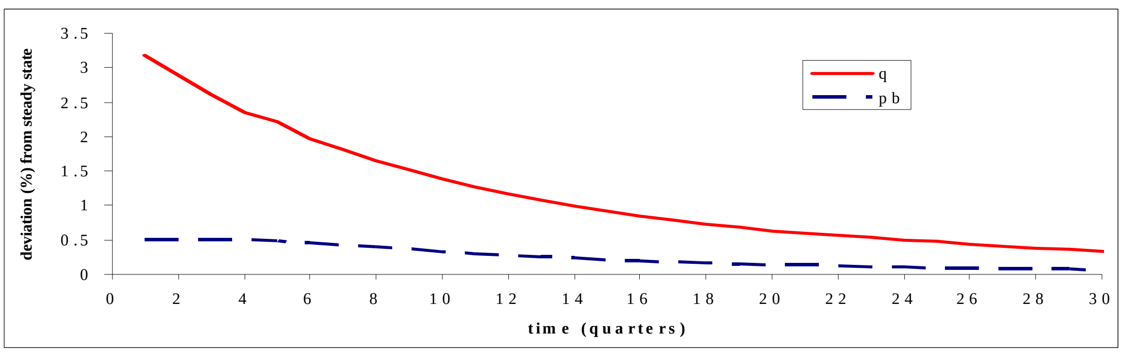

Figure 4.2. Responses of asset prices to a negative liquidity shock

A notable special case of the above model is $\theta = 0$. In this case, an entrepreneur must finance investment entirely with debt and the receipts from selling existing assets, rather than issuing new equity to the market. Since all new equity is retained by the entrepreneur for the household for the period in this case, it helps financing investment only as collateral to secure borrowing. Even in this case, a negative shock to liquidity generates an equity price boom.

Let me remark on the assumption $m'(\phi) \geq 0$ and the outcome of endogenizing the debt limit. First, the assumption $m'(\phi) \geq 0$ captures the intuitive feature that a reduction in equity liquidity does not increase an entrepreneur's borrowing capacity. This intuitive feature is supported by

[22]Note the size of the liquidity shock in Figure 4.1 is much smaller than that in the economy without debt finance. However, the responses in asset prices in the two economies are comparable in size. This is because the shock in the economy without debt finance affects both the ability to sell equity and the ability to borrow.

the evidence documented by Covas and den Haan (2011). Using micro data of US firms, they find that the ratio of debt finance to firm output is procyclical for an overwhelming majority of firms in the US. This suggests $m'(\phi) \geq 0$ if asset liquidity is procyclical. Second, because equity price increases in response to a negative liquidity shock when $m'(\phi) = 0$, continuity implies that this positive response also arises even when $m'(\phi)$ is negative and sufficiently small. Third, endogenizing the debt limit will not overturn the positive response of equity price to a negative liquidity shock. Specifically, as long as the endogenous collateral limit has the realistic feature $m'(\phi) \geq 0$, the same mechanism as the above induces equity price to increase after a negative shock to equity liquidity.

The collateral constraint, (4.1), enables me to examine other financial shocks in addition to the liquidity shock. In particular, there can be shocks to the borrowing capacity that are unrelated to equity liquidity. For example, financial development and regulations can change the amount that an entrepreneur is able to leverage against the collateral even when the liquidity of the collateral remains unchanged. Such shocks can be modeled as shocks to μ in (4.3). By examining how equity price responds to such financial shocks, I hope to illustrate the generality of this response. To this end, let me assume that μ obeys the following process:

$$-\log(\frac{1-\underline{\mu}}{\mu_{t+1}-\underline{\mu}}-1) = -(1-\delta_\mu)\log(\frac{1-\underline{\mu}}{\mu^*-\underline{\mu}}-1) - \delta_\mu \log(\frac{1-\underline{\mu}}{\mu_t-\underline{\mu}}-1) + \varepsilon_{\mu,t+1}, \tag{4.5}$$

where μ^* is the steady state of μ and $\underline{\mu} < 1$ is the lower bound of μ. This process ensures $\mu \in [\underline{\mu}, 1)$. The identification procedure above for μ implies $\mu^* = 0.062$. I set $\underline{\mu} = -0.1$.

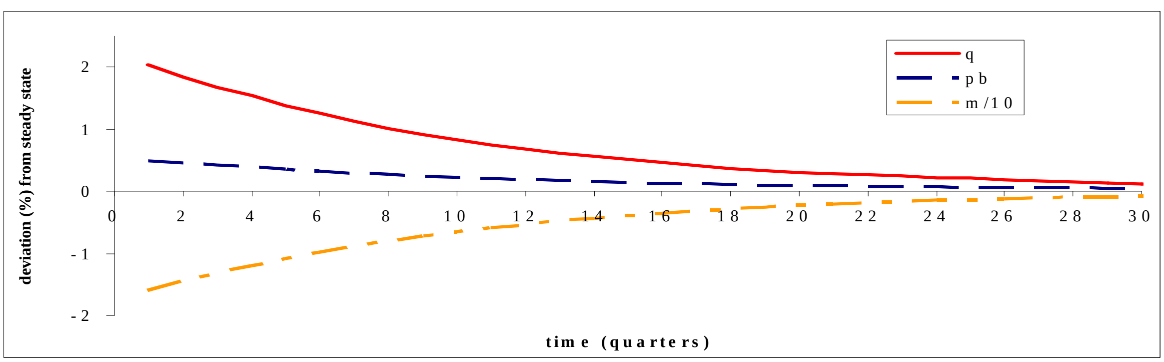

Figure 5. Responses to a negative financial shock to the borrowing capacity

Consider a shock at the beginning of $t = 1$ that reduces μ from $\mu^* = 0.062$ to $\mu_1 = 0.03$, while (A, ϕ) remain at their steady-state values. Figure 5 depicts the responses of the collateral multiplier and asset prices, where the percentage deviation of the collateral multiplier from the steady state is divided by 10. Although the negative shock to μ does not affect equity liquidity, it reduces an entrepreneur's ability to borrow. Immediately after the shock, the collateral multiplier falls by 16%. As the collateral multiplier falls, the prices of equity and liquid assets increase. Again, this increase in equity price in response to the negative shock to μ can be explained with (4.4). By reducing the collateral multiplier, the negative shock to μ increases the downpayment on investment and, hence, increases the implicit cost of investment. In addition, by tightening the financing constraint on an entrepreneur, the negative shock increases λ^e, which further increases the implicit cost of investment. To make investment optimal, equity price must rise to increase the benefit of investment. This positive response of equity price to a negative shock to μ provides another illustration of the simple rule of thumb described earlier.

5. Some Solutions to the Problem

I have shown that equity price increases in response to negative shocks to an entrepreneur's financing constraint, regardless of whether the shock is to the liquidity of assets or to the borrowing capacity. For equity price to fall after such a negative financing shock, as it often does during recessions, the financing constraint must become less tight. To generate this paradoxical outcome, there must be other changes concurrent with the financing shock that sufficiently reduce the need for investment. In this section I discuss some candidates of these concurrent changes and relate the analysis to other attempts in the literature.

One candidate is a perceived fall in the quality of capital, which can be caused by worsening adverse selection in the asset market (e.g., Kurlat, 2010). If market participants perceive the quality of capital to deteriorate quickly in a recession, they will move resources from equity to relative safe and liquid assets. This will depress equity price and drive up the price of liquid assets. One way to model this shock is to assume that the effective capital stock is κK, where κ is the quality of capital. The effective capital stock, instead of the raw capital stock, appears in the production function. Because total factor productivity is $A\kappa^{\alpha}$, a negative shock to the quality of capital is similar to a negative shock to productivity A, which I examine below.[23]

Consider a negative shock to total factor productivity, A. This shock reduces investment by reducing the marginal productivity of capital. If the shock is sufficiently persistent, then a household will also scale down consumption. These reductions in investment and consumption reduce an entrepreneur's expenditure, given by the right-hand side of the financing constraint (2.4). However, the negative shock to productivity may also reduce the rental income of capital, rs, which appears on the left-hand side of (2.4). If the reductions in investment and consumption dominate the reduction in the rental income, then the financing constraint becomes less tight

[23] One can also consider a negative shock to π – the fraction of individuals who have investment projects in a period. Again, a reasonable cause of a reduction in π is a negative shock to productivity.

and, by (3.3), equity price falls.

As an illustration of this possibility, I return to the baseline model and compute the response of the equilibrium to simultaneous shocks to ϕ and A. Suppose that the economy is in the steady state before $t = 1$ and, at the beginning of $t = 1$, there are unanticipated reductions in ϕ about 12% and in A about 5%. After these shocks, A and ϕ follow the deterministic dynamics of the processes in (3.1) and (3.2). Figure 6 depicts the responses of asset prices and the Lagrangian multiplier of the financing constraint, where the percentage change in λ^e is rescaled by a factor 1/20. After the shocks, equity price falls and approaches the steady state from below. Supporting the above intuitive explanation, equity price falls because the negative productivity shock relaxes the financing constraint, as captured by the fall in λ^e.

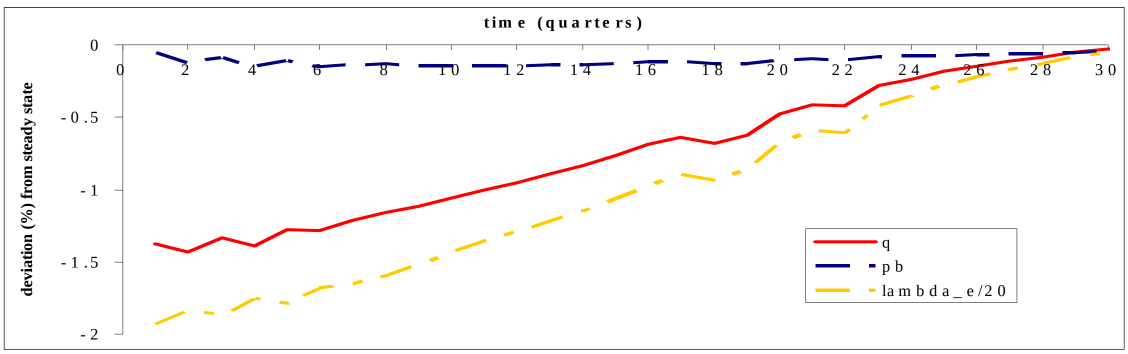

Figure 6. Asset prices and λ^e after negative shocks to (A, ϕ)

Let me relate this analysis to some papers that generate a negative response of equity price to negative liquidity shocks. The KM paper proposes two ways to resolve the problematic response of equity price to the liquidity shock. One is to introduce a storage technology to allow entrepreneurs to shift resources between investment and storage. The analyses in sections 3 and 4 suggest that the role of storage will be limited. Since the condition of optimal investment, (3.3), must hold regardless of whether or not a storage technology exists, equity price will still increase after a negative liquidity shock as long as the shock tightens the financing constraint on investment. For storage to change the qualitative response of equity price, it must relax the financing constraint after a negative liquidity shock. This seems implausible. The other proposal is to assume that the government sells liquid assets to buy private equity in the event of a negative liquidity shock. If the injection of liquidity is sufficiently large that relaxes an entrepreneur's financing constraint, then it is not surprising from (3.3) that equity price will fall. This outcome illustrates an interesting role of government intervention in the asset market, but it does not negate the finding that a negative liquidity shock will push up equity price if it tightens the financing constraint.

Del Negro et al. (2011) introduce into KM an interest-rate policy rule and non-standard policy interventions as well as the elements discussed in subsection 3.4. They suggest that a persistent negative liquidity shock may reduce equity price. Although it is not clear how this result can occur in their model, they seem to attribute it to an intricate interaction between nominal rigidities and the expectation of the zero lower bound on the nominal interest rate under the described policy rule. As in KM, this analysis may be useful for understanding the role of government policy, but it does not contradict the finding that a pure negative liquidity shock increases equity price. In addition, as shown by Ajello (2010) (discussed below), nominal rigidities alone do not generate a negative response of equity price to a negative liquidity shock.

Ajello (2010) incorporates into KM nominal price/wage rigidities and the elements discussed in subsection 3.4. In addition, he makes investment projects heterogeneous in quality among entrepreneurs and the intermediation process costly to channel funds to investment. Despite all these elements, he still finds that a negative shock to ϕ increases equity price, which is consistent with the analysis in subsection 3.4. He then illustrates that a shock that increases the cost of intermediation can reduce equity price and other aggregate quantities. By increasing the spread between the rates of return to an entrepreneur and the intermediary, the shock to intermediation affects the amount of funds that can be effectively channeled to investment and the quality distribution of investments that are undertaken in the equilibrium. As such, the intermediation shock acts as a negative shock to effective productivity, as well as to liquidity. It is then conceivable that such a negative shock can reduce equity price.

Jermann and Quadrini (2010) use a model that differs from KM in at least the following dimensions: (i) investment is undertaken by the same firms that produce consumption goods, rather than by a separate group of individuals called entrepreneurs; (ii) a firm needs working capital for all expenditures in a period, including wage payments, instead of just investment and consumption; and (iii) a firm faces a borrowing limit similar to the one in (4.1) where the financial shock is to μ instead of ϕ. Despite all these differences, Jermann and Quadrini (2010) find that a negative shock to μ increases equity price under the baseline calibration. They then illustrate that introducing sizable adjustment costs to investment can make equity price fall after a negative shock to μ. This result is surprising at first glance, given the analysis in section 4. However, a close inspection reveals that Jermann and Quadrini (2010) have estimated a vector autoregression (VAR) between μ and total factor productivity. Because the off-diagonal elements in this VAR are non-zero, a negative shock to μ acts as combined shocks to current liquidity and future productivity. Figure 6 illustrates that such combined shocks can reduce equity price. Jermann and Quadrini's result is instructive in revealing the importance of the inter-dependence between asset liquidity and productivity. However, this inter-dependence itself should be explained rather than assumed in order to understand the role of liquidity shocks in the business cycle.

6. Conclusion

In this paper, I have constructed a tractable model to evaluate the liquidity shock hypothesis that exogenous shocks to equity market liquidity are an important cause of the business cycle. After calibrating the model to the US data and computing the dynamic equilibrium, I have found that a large and persistent negative liquidity shock can generate large drops in investment, employment and output. Contrary to the hypothesis, however, a negative liquidity shock generates an equity price boom. This counterfactual response of equity price is robust, provided that a negative liquidity shock tightens firms' financing constraints on investment. Also, I have demonstrated that the same counterfactual response of equity price arises when there is a financial shock to a collateral constraint that changes firms' borrowing capacity. For equity price to fall as it typically does in a recession, the negative liquidity/financial shock must be accompanied or caused by other shocks that reduce the need for investment sufficiently and relax firms' financing constraints on investment. I have discussed some candidates of these concurrent shocks, all of which can be traced to changes in factor productivity.

The main message of this analysis is not that asset liquidity is not important in general for the macro economy. Rather, the message is that *exogenous* shocks to asset liquidity alone fail to account for the cyclical movement of equity price. Despite the negative tone, this main result of the paper should be interpreted as being constructive, because it provides specific guidance to future research. If one wants to explain why changes in asset liquidity or the financing capacity play an important role in the business cycle, as they did in 2008, one should not follow the majority of macro models in this field to treat such changes as exogenous shocks that have no consequence on productivity. Instead, effort should be devoted to endogenizing a channel through which liquidity/financial shocks affect investment quality and effective productivity, as indicated by the VAR evidence in Jermann and Quadrini (2010). In this regard, it may be particularly fruitful to incorporate information frictions, such as those in Chang (2010), Kurlat (2010), and Guerrieri and Shimer (2011), into dynamic stochastic general equilibrium models.

Appendix

A. The Mapping T on Asset Price Functions

To construct the mapping T on asset price functions, start with arbitrary functions $q, p_b \in \mathcal{C}_1$. The following procedure constructs the updated asset price functions Tq and Tp_b:

(i) Substitute the factor market clearing conditions (2.16) and (2.17) into requirement (ii) of the equilibrium definition. This step generates r and w as functions of (ℓ, K, Z).

(ii) Substitute the functions for (r, w) in step (i) and the conditions (2.16) - (2.19) into the optimality conditions (2.4) - (2.12). Then, solve $(i, c^e, c^w, c, \lambda^e)$ as functions of (ℓ, K, Z).

(iii) Substitute the resulting functions in steps (i) and (ii) into (2.15) to solve ℓ as a function of (K, Z). With this function $\ell(K, Z)$, the functions solved in steps (i) and (ii) express (r, w) and $(i, c^e, c^w, c, \lambda^e)$ as functions of (K, Z). Similarly, (r_{+1}, w_{+1}) and $(\ell, i, c^e, c^w, c, \lambda^e)_{+1}$ can be expressed as functions of (K_{+1}, Z_{+1}).

(iv) Substitute the functions obtained in step (iii) and the condition (2.20) into the right-hand sides of the asset pricing equations (2.13) and (2.14). The result is a pair of functions of (K, Z), provided that Z follows a Markov process. These are the updated asset price functions, denoted as $T(q, p_b)(K, Z)$.

It can be verified that for any $q, p_b \in \mathcal{C}_1$, Tq and Tp_b are continuous functions of (K, Z) and their values lie in $\mathbb{R}_+$. That is, T maps a pair of elements in $\mathcal{C}_1$ back into $\mathcal{C}_1$, where $\mathcal{C}_1$ is the set containing all continuous functions from $\mathcal{K} \times \mathcal{Z}$ into $\mathbb{R}_+$. A fixed point of T is a pair of asset price functions in the equilibrium. After obtaining these asset price functions, I can retrieve the policy functions of individuals' decisions and other variables.

B. Value of Liquidity and the Equity Premium

Liquidity has a positive value in this model only if the financing constraint, (2.4), is binding. Precisely, the *value of liquidity* in the aggregate state (K, Z) can be measured as $\pi\lambda^e(K, Z)$ units of consumption, where $\lambda^e \pi U'(c^w)$ is the shadow price of (2.4). Note that the price of liquid assets in a standard model without the financing constraint is $\beta\mathbb{E}\frac{U'(c^w_{+1})}{U'(c^w)}$. According to (2.14), liquid assets have a higher price in the current model than the standard one if and only if liquidity is expected to have a positive value in the next period.

While a liquid asset provides full liquidity, equity provides only partial liquidity. By assumption, the return to existing equity can be used to finance new investment. So can a fraction ϕ of the equity itself. Thus, an intuitive measure of the value of liquidity provided by a unit of existing equity is $(\pi\lambda^e)\frac{r+\phi\sigma q}{r+\sigma q}$. The value of the additional liquidity generated by liquid assets relative to existing equity is:

$$\Delta \equiv \pi\lambda^e \left[1 - \frac{r+\phi\sigma q}{r+\sigma q}\right] = \pi\lambda^e \frac{(1-\phi)\sigma q}{r+\sigma q}. \tag{B.1}$$

Intuitively, this additional value is strictly positive if and only if liquidity has a positive value (i.e., if $\lambda^e > 0$) and if existing equity is not fully liquid (i.e., if $\phi < 1$).

The measure Δ is in terms of consumption units. Another measure is the *equity premium*, i.e., the additional rate of return to equity that is needed to compensate for the lower liquidity of equity. The gross rate of return to equity is $(r_{+1} + \sigma q_{+1})/q$, and to liquid assets is $1/p_b$. The realized equity premium in the next period will be $(\frac{r_{+1}+\sigma q_{+1}}{q} - \frac{1}{p_b})$. The equity premium is closely related to the measure Δ. The asset-pricing equations (2.13) and (2.14) imply that the equity premium is strictly positive on average if and only if the value of the additional liquidity provided by liquid assets relative to existing equity is positive on average. Precisely, the following two inequalities are equivalent:

$$\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(1+\pi\lambda^e_{+1}\right)\left(\frac{r_{+1}+\sigma q_{+1}}{q} - \frac{1}{p_b}\right)\right] > 0$$

$$\mathbb{E}\left[\frac{U'(c^w_{+1})}{U'(c^w)}\left(\frac{r_{+1}+\sigma q_{+1}}{q}\right)\Delta_{+1}\right] > 0.$$

C. Steady State, Calibration, and Computing Dynamics

I characterize the non-stochastic steady state first, which is indicated with the superscript $*$. In the non-stochastic steady state, the exogenous state is constant at $Z = Z^* = (A^*, \phi^*)$, and all endogenous variables are constant over time. In the steady state, investment is strictly positive because it is equal to $i^* = (1-\sigma)K^*/\pi > 0$. By (2.12), the shadow price of the financing constraint in the steady state is

$$\lambda^{e*} = \frac{q^* - 1}{1 - \theta q^*}. \tag{C.1}$$

The asset-pricing equations, (2.13) and (2.14), yield the following steady-state relations:

$$\lambda^{e*} = \frac{(\beta^{-1} - \sigma)q^* - r^*}{\pi(r^* + \phi^*\sigma q^*)}, \tag{C.2}$$

$$p_b^* = \beta(1 + \pi\lambda^{e*}). \tag{C.3}$$

The equations, (C.1) and (C.2), determine (λ^{e*}, q^*). Substituting these solutions into (C.3) yields p_b^*. Other steady-state conditions are:

$$u'(c^{e*}) = U'(c^{w*})\frac{(1-\theta)q^*}{1-\theta q^*}, \tag{C.4}$$

$$c^* = AF(K^*, (1-\pi)\ell^*) - (1-\sigma)K^* - g, \tag{C.5}$$

$$p_b^* B = g + c^{e*} - \left[r^* + \phi^* \sigma q^* - \frac{1-\sigma}{\pi}(1-\theta q^*)\right] K^*. \tag{C.6}$$

Equation (C.4) is the steady-state version of the first-order condition of c^e, (2.11), with λ^{e*} being substituted. Equation (C.5) is the steady-state version of the goods-market clearing condition, and (C.6) is the steady-state version of the financing constraint, (2.4). Together with $r^* = AF_1$ and $h'(\ell^*) = U'(c^{w*})AF_2$, (C.2) - (C.6) solve for $(q^*, p_b^*, c^{e*}, c^*, K^*, r^*, \ell^*)$.

Next, I identify the parameters. The values of β, ρ, A^*, δ_A, δ_ϕ and π are exogenously set and the explanations for these values are given in the text. The parameter η is calculated from the elasticity of labor supply, $\frac{1}{\eta-1} = 1$, and the capital share in output is $\alpha = 0.36$. Since aggregate investment in the steady state is $\pi i^* = (1-\sigma)K^*$, the ratio of annual investment to capital is $4\pi i^*/K^* = 4(1-\sigma)$. Equating this to the target, 0.076, yields σ.

Setting total hours of work to the target yields $(1-\pi)\ell^* = 0.25$. Given π, this solves ℓ^*. Setting the ratio of capital to annual output in the steady state to the target yields $K^*/(4A^*F^*) = 3.32$. With the value of α identified above, this condition solves r^*. Since $r^* = A^*F_1'$, I can solve K^* and recover $i^* = (1-\sigma)K^*/\pi$. Also, $w^* = A^*F_2'$. These values of $(\ell^*, K^*, i^*, r^*, w^*)$ will be used to identify some parameters below. Since the ratio of government spending to output is $g/(A^*F^*) = 0.18$, I can solve g. Setting the annualized net rate of return to liquid assets to the target, I have $1/(p_b^*)^4 - 1 = 0.02$. This solves for p_b^*. Substituting p_b^* into the asset pricing equations in the steady state, (C.2) and (C.3), using the value of r^* identified above, and using $\theta = \phi^*$, I can solve ϕ^*, θ and q^*. Using the target on the share of liquid assets, I have $p_b^*B/(p_b^*B + q^*K^*) = 0.12$. Because p_b^*, q^* and K^* are all solved by now, this condition solves B.

Two parameters are still to be solved, (u_0, h_0). To solve them, I substitute the values of $(r^*, q^*, \phi^*, K^*, i^*, B, g)$ into the steady-state version of (2.4) to solve c^{e*}. The goods-market clearing condition yields $c^* = A^*F^* - \pi i^* - g$. Then, the definition of c yields $c^{w*} = (c^* - \pi c^{e*})/(1-\pi)$. Substituting the values of (c^{w*}, ℓ^*, w^*) into the steady-state version of (2.10), I obtain the value of h_0. Note that $i^* > 0$. Combining the steady-state versions of (2.11) and (2.12) to eliminate λ^{e*}, I obtain an equation in which u_0 is the only item still to be solved. Solve this equation for u_0.

Finally, I describe how to compute the dynamics of the equilibrium after some shocks. Suppose that there are shocks to A or ϕ or both. Let me focus on various cases where the new paths of A and ϕ are completely known after the shocks are realized at the beginning of time $t = 1$, because only such cases are examined in the text. An example is a one-time shock to A or ϕ or both that occurs at $t = 1$, after which A and ϕ follow (3.1) and (3.2) with the error terms being zero for all $t \geq 2$. Another example is a change to the path of A or ϕ or both that becomes known at $t = 1$. With such shocks, the entire path of $Z = (A, \phi)$ becomes known immediately after the beginning of $t = 1$. Given this path of Z, I can use the equilibrium asset price functions and the policy functions to compute the paths of equilibrium variables. Specifically, at $t = 1$, asset prices are $q_1 = q(K_1, Z_1)$ and $p_{b,1} = p_b(K_1, Z_1)$, where K_1 is predetermined and Z_1 is known. An individual household's optimal decisions in period 1 are given by $x_1 = x(K_1, B; K_1, Z_1)$, where x is the policy function for any variable in the list $(c, i, c^e, s^e_{+1}, \ell, c^w, s_{+1}, b_{+1})$. Similarly, I can compute factor prices (w_1, r_1) and aggregate variables in period 1. Using (2.20), I can obtain K_2. With (K_2, Z_2), I repeat the process to obtain all equilibrium variables in period 2. Continuing this process yields the dynamic path of the equilibrium after the shocks.

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