A Forecast Rationality Test that Allows for Loss Function Asymmetries

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Abstract

In this paper, we propose a conditional moment type test for forecast rationality that allows for an asymmetric loss function, has power against generic (non)linear alternatives and does not assume any particular functional form for the forecaster's loss function. The construction of the test is based on the simple idea that asymmetric preferences imply an unconditional bias of the forecast error but not a conditional bias. The null hypothesis of forecast rationality under asymmetric loss (i.e. no conditional bias) is tested by constructing a Bierens type test. We compare our nonlinear forecast rationality test to the linear J-test of Elliott, Komunjer and Timmermann (2005, 2008; EKT hereafter) through an empirical application using data from the Survey of Professional Forecasters issued by the Federal Reserve Bank of Philadelphia. In a Monte Carlo exercise we show that: i) the J-test is loss function sensitive, meaning that it is not robust to different choices of loss function and could lead to incorrect inferences if the forecaster's underlying loss function does not belong to the class of loss defined in EKT; and ii) our test has higher power than the J-test in the presence of nonlinear dependencies between the forecast error and the information set used to generate the forecasts.

Keywords: forecast rationality, asymmetric loss function, predictive accuracy tests, generically comprehensive tests
J.E.L Classification : C12, C32, C53

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1 Introduction

In the forecast evaluation literature, rationality is tested under different assumptions regarding the forecaster's underlying loss function. Early works such as Theil (1958) [15], Mincer and Zarnowitz (1963) [12] based their forecast evaluation framework on the assumption of a symmetric loss, where positive and negative forecast errors are equally weighted by the forecaster. Under a squared loss function, testing for forecast efficiency means testing whether the forecast errors have zero mean and whether they are uncorrelated with the available information. Nevertheless, the assumption of quadratic loss can be criticized due to different economic reasons. At a macroeconomic level, central banks are averse to bad outcomes such as lower than expected real output growth and higher than expected inflation and hence they incorporate this loss aversion into their forecasts. At the firm level, the cost of under-predicting demand which results in loss of sales should not necessarily be the same as the cost of over-predicting demand which means additional storage costs.

Given that symmetric loss functions such as the mean squared error or mean absolute error may not be flexible enough to capture the loss structures that forecasters face, another line of the literature: Zellner (1986) [16], Christoffersen and Diebold (1997) [5], Elliott, Komunjer and Timmermann (2005, 2008) [9] [10], Patton and Timmermann (2007a) [13], Komunjer and Owyang (2007) [11] argues that an asymmetric loss function that weights differently positive and negative forecast errors, could be more representative for the forecaster's intentions. However, under an asymmetric loss, standard forecast rationality tests could be misleading, not being able to distinguish whether the forecaster uses inefficiently her information, or whether the underlying loss function is just asymmetric. Thus, rejections of rationality in the standard rationality evaluation literature may largely be caused by the assumption of a squared loss function.

EKT provide a GMM based forecast optimality testing framework based on a general class of loss functions that allows for a parametrization of the asymmetry in the loss function. They construct a forecast rationality test that allows for an asymmetric loss function. However, as we show in this paper, their methodology is loss function sensitive. Thus, if the forecaster's true loss function does not belong to the parametrization of EKT, their test could lead to incorrect inferences. In general, the forecast evaluator does not have much information about the true loss function of the forecast producer. Hence, wrongly assuming that the forecasts have been constructed by minimizing the loss function given in EKT could lead to misleading results. In addition, the EKT test is based on the assumption that the forecasts were generated using a linear model. In their context, failure to reject the null of rationality means an absence of linear correlation between the forecast error and the information set used to generate the forecasts. Thus, their test may not detect nonlinear dependencies.

Our paper suggests an alternative test for forecast rationality that allows for an asymmetric loss function. Our test is consistent against generic (non)linear alternatives and relaxes the assumption that the forecaster's loss belongs to the parametrization of EKT. In fact, our framework accounts for the possibility of asymmetry without restricting the forecaster's loss to any particular parametric form of the loss function. In the construction of our test statistic, we require neither the knowledge of the underlying loss function nor the knowledge of the forecasting model used by the forecaster. Our test is related to the conditional moment type tests of Bierens (1982, 1990) [2] [3], de Jong (1996) [8], Corradi and Swanson (2002) [7] and Corradi, Fernandez and Swanson (2009) [6].

The paper is structured as follows. Section 2 provides a brief review of the EKT (2005) framework. In Section 3 we outline our suggested forecast rationality test. In section 4 we compare the finite sample properties of the two tests by Monte Carlo simulations. In the same section we show that misspecified losses lead to biased loss function estimates and important size distortions for the J-test. Section 5 presents an empirical illustration using data from the Survey of Professional Forecasters (SPF). Concluding remarks are provided in Section 6. The main simulation and empirical findings are presented in Appendix A1. Appendix A2 contains additional empirical results that support our findings.

2 The EKT Framework

Consider the general class of loss functions introduced by EKT (2005, 2008) that includes the mean squared error (MSE) or mean absolute error (MAE) as special cases. This generalized loss function is defined as follows:

$$L_1\left(\epsilon_{t+h}; p, \alpha\right) = [\alpha + (1-2\alpha)\cdot 1(\epsilon_{t+h} < 0)]\cdot |\epsilon_{t+h}|^p \tag{1}$$

where $\epsilon_{t+h} = y_{t+h} - f_{t+h}$ is the forecast error at horizon h, $h \geq 1$. The forecast, f_{t+h}, is defined as $f_{t+h} = \theta' W_t$, where θ is an unknown d-vector of parameters and W_t is a d-vector of variables that are $\mathcal{F}_t$-measurable. The shape parameters of the loss function, L_1, are p and α, with $p \in \mathbb{N}^*$ and $\alpha \in (0,1)$. Special cases of L_1 include: the absolute deviation loss function, $L_1(\epsilon_{t+h}; 1, 1/2) = \frac{1}{2}|\epsilon_{t+h}|$, the squared loss function $L_1(\epsilon_{t+h}; 2, 1/2) = \frac{1}{2}\epsilon^2_{t+h}$ and their asymmetrical counterparts obtained when $\alpha \neq 1/2$: the lin-lin loss $L_1(\epsilon_{t+h}; 1, \alpha)$ and the quad-quad loss $L_1(\epsilon_{t+h}; 2, \alpha)$.

The shape parameter α describes the degree of asymmetry in the forecaster's loss function. Values for α less than one half suggest that the forecaster gives higher weights on negative forecast errors than on positive ones of the same magnitude, or in other words over-prediction is more costly than under-prediction. Values greater than one half indicates a higher cost associated with positive forecast errors, or that under-prediction is more costly than over-prediction. In the symmetric case, α equals one half. The costs associated with positive and negative forecasts errors are equally weighted. The relative cost of a forecast error can be estimated as $\alpha/1-\alpha$ (Capistran 2008). For example if $\alpha = 0.75$, positive forecast errors (obtained by under-forecasting) are three times more costly then negative ones (obtained by over-forecasting).

EKT (2005, 2008) construct a test for forecast rationality allowing for an asymmetric loss function as defined in (1). Their test is based on the following moment conditions:

$$E(W_t(1(\epsilon^*_{t+h} \leq 0) - \alpha_0)|\epsilon^*_{t+h}|^{p_0 - 1}| = 0 \tag{2}$$

where $\epsilon^*_{t+h} = y_{t+h} - f^*_{t+h}$ is the optimal forecast error which depends on the unknown true values p_0 and α_0.

It is shown that a sub-vector V_t, of W_t is sufficient to identify α_0. Consequently, the forecast user does not require the entire set of variables W_t used by the forecaster to back out α_0, but only a sub-vector of these variables. This result is relevant in practical applications as the forecast user might not have access to the full information set used to generate the forecasts. The moment conditions from (2) thus become:

$$E(V_t(1(\epsilon^*_{t+h} \leq 0) - \alpha_0)|\epsilon^*_{t+h}|^{p_0-1}| = 0 \tag{3}$$

Using the moment conditions in (3), they obtain a GMM estimator for α and then construct a test for the validity of the $d-1$ overidentifying restrictions. The approach is used to test a composite null that the loss belongs to a general family of loss functions and the forecasts are rational. Testing this composite null hypothesis is conducted through the following test statistic:

$$\begin{aligned} J = \frac{1}{T} &\left(\sum_{t=\tau}^{T+\tau-1} v_t[1(\hat{e}_{t+1} < 0) - \hat{\alpha}_T]|\hat{e}_{t+1}|^{p_0-1} \right)' \hat{S}^{-1} \\ &\left(\sum_{t=\tau}^{T+\tau-1} v_t[1(\hat{e}_{t+1} < 0) - \hat{\alpha}_T]|\hat{e}_{t+1}|^{p_0-1} \right) \sim \chi^2_{d-1} \end{aligned} \tag{4}$$

The test is asymptotically distributed as a χ^2 with $d-1$ degrees of freedom and rejects for large values. In (4), $\hat{e}_{t+1}$ is the observed forecast error obtained as: $\hat{e}_{t+1} = y_{t+1} - \hat{f}_{t+1}$, where $\hat{f}_{t+1}$ is the observed forecast reported by the forecast producer. The sample size is denoted by T, τ is the beginning of the estimation sample, v_t are the observations of the vector of instruments V_t, $\hat{\alpha}_T$ is a linear instrumental variable estimator of the true value α_0,

$$\hat{\alpha}_T \equiv \frac{[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} v_t|\hat{e}_{t+1}|^{p_0-1}]'\hat{S}^{-1}[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} v_t 1(\hat{e}_{t+1} < 0)|\hat{e}_{t+1}|^{p_0-1}]}{[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} v_t|\hat{e}_{t+1}|^{p_0-1}]'\hat{S}^{-1}[\frac{1}{T}\sum_{t=\tau}^{T+\tau-1} v_t|\hat{e}_{t+1}|^{p_0-1}]} \tag{5}$$

and $\hat{S}$ defined as

$$\hat{S}(\bar{\alpha}_T) \equiv \frac{1}{T} \sum_{t=\tau}^{T+\tau-1} v_t v_t' (1(\hat{e}_{t+1} < 0) - \bar{\alpha}_T)^2 |\hat{e}_{t+1}|^{2p_0 - 2}$$

is a consistent estimate of a positive definite weighting matrix S, which depends on $\bar{\alpha}_T$, a consistent initial estimate for α_0.

In practice, the computation of the estimator $\hat{\alpha}_T$ can be done iteratively. First, we choose $S = I_{dxd}$ and use (5) to compute the corresponding $\hat{\alpha}_{T,1}$. Using $\hat{\alpha}_{T,1}$ a new weight matrix $\hat{S}(\hat{\alpha}_{T,1})$ is obtained. This is more efficient than the previous one. The new weight matrix is plugged into (5) obtaining $\hat{\alpha}_{T,2}$. These two steps are repeated until $\hat{\alpha}_{T,j}$ equals its previous value $\hat{\alpha}_{T,j-1}$.

3 The Nonlinear Forecast Rationality Test

The forecast rationality test of EKT is based on the assumption that the forecasts were generated using a linear model of the type: $f_{t+1} = \theta' W_t$, and thus the observed forecast error is: $\hat{e}_{t+1} = y_{t+1} - \hat{\theta}' W_t$. In this framework, failure to reject the null hypothesis of rationality, means an absence of linear correlation between the information set of the forecaster and the forecast error. Hence, possible nonlinear dependencies are not necessarily detected. The forecast error could be uncorrelated with W_t but correlated with a nonlinear function of W_t. Even more, the error could be correlated with some variables not included in W_t. In this section, we outline a forecast rationality test which allows for an asymmetric loss function and it is consistent against generic (non)linear alternatives. This test is able to detect any form of dependence between the forecasting error and the available information set. Moreover, our forecast rationality test is not based on the assumption of a particular loss function. This constitutes an important advantage over the J-test, because as we show in Section 4.1, if the true loss function of the forecaster does not belong to the class of loss functions defined in EKT, the results given by the J-test could be misleading.

The idea of our test is that asymmetric preferences imply an *unconditional bias* of the forecast error but *not a conditional bias*. Under the assumption of asymmetric preferences, the forecast error is unconditionally biased because forecasters systematically over or under-predict. The forecasters get different losses from over and under-prediction and thus it is rational for them to produce biased forecasts. However, the conditional bias of the forecast error is zero if there is no issue of inefficient use of the available information.

We start by defining the null hypothesis for our forecast rationality test under asymmetric preferences as follows:

$$H_0 : E(\epsilon_{t+1}|W_t) = E(\epsilon_{t+1}) \tag{6}$$

against the alternative:

$$H_1 : E(\epsilon_{t+1}|W_t) \neq E(\epsilon_{t+1})$$

where W_t contains all publicly available information relevant to predict a variable y_{t+1} at time t. If H_0 is true, it means that the forecast error is independent of any function which is measurable in terms of the information set available at time t. The forecasts are rational even though they may be biased. We can rewrite the null given in (6) as follows: $H_0 : E(\epsilon_{t+1}|W_t) - E(\epsilon_{t+1}) = 0 \Leftrightarrow H_0 : E(\epsilon_{t+1}|W_t) - E[E(\epsilon_{t+1})|W_t] = 0 \Leftrightarrow H_0 : E[\epsilon_{t+1}|W_t - E(\epsilon_{t+1}|W_t)] \Leftrightarrow$

$$H_0 : E[(\epsilon_{t+1} - E(\epsilon_{t+1}))|W_t] = 0 \tag{7}$$

The alternative of the new form of our null hypothesis, as given in (7), is:

$$H_1 : Pr\ [E[(\epsilon_{t+1} - E(\epsilon_{t+1}))|W_t] = 0] < 1$$

If the new form of H_0 is rejected, the conditional bias of the forecast error is not zero and we have evidence for non-rationality because the available information has been used inefficiently when constructing the forecasts. We are able now to test the new form of our null hypothesis, by constructing a Bierens (1982, 1990) type test, with an infinite number of moment conditions. Conditional moment type tests are constructed based on the idea that for correctly specified models, the conditional mean of certain functions of data are almost surely equal to zero. This quantity is the product of the model's residuals (in our case the conditional bias of the forecast error) and a weighting function which depends on the conditioning variables.

In order to test the null given in (7), we apply to our context the test statistic suggested by de Jong (1996) which generalizes the conditional moment type test proposed by Bierens (1990), to the framework of time series. Thus, we define:

$$M_T = sup_{\gamma \in \Gamma} |m_T(\gamma)|$$

where:

$$m_T(\gamma) = \frac{1}{\sqrt{T}} \sum_{t=0}^{T-1} (\hat{e}_{t+1} - \overline{e})\, w \left(\sum_{j=0}^{t-1} \gamma_j' \Phi(W_{t-j}) \right) \tag{8}$$

and the convention that the sum from 0 to -1 is zero. In (8), $\hat{e}_{t+1}$ is the observed one step ahead forecast error obtained as the difference between the actual realization and the forecasted value from the forecast producer, $\hat{e}_{t+1} = y_{t+1} - \hat{f}_{t+1}$. The mean $\overline{e}$ is defined as $\overline{e} = \frac{1}{T} \sum_{t=0}^{T-1} \hat{e}_{t+1}$. Our discussion focuses on $\hat{e}_{t+1}$, but the results generalize to $\hat{e}_{t+h}$, where $h > 1$ is the forecast horizon.

The function Φ is a measurable one to one mapping from $\Re^d$ to a bounded subset of $\Re^d$ - it can be choosen the arctangent function, for example. The weights, γ_j, attached to observations decrease over time. The function $w(\gamma', W_t)$ is a generically comprehensive function, a nonlinear transformation of the conditioning variables. Suppose that W_t is a d-dimensional vector. The generically comprehensive function can be taken for example: $w(\gamma', W_t) = \exp(\sum_{i=1}^{d} \gamma_i \Phi(W_{i,t}))$, or $w(\gamma', W_t) = 1/(1 + \exp(c - \sum_{i=1}^{d} \gamma_i \Phi(W_{i,t})))$, with the constant $c \neq 0$. The choice of the exponential in this weight function is not crucial. Stinchcombe and White (1998) show that any function that admits an infinite series approximation on compact sets with non-zero series coefficients can be used to obtain a consistent test.

It can be shown, (see for example Corradi, Fernandez and Swanson (2008)) that our test statistic has a limiting distribution that is a functional of a Gaussian process:

(i)Under H_0, $M_T \xrightarrow{d} \sup_{\gamma \in \Gamma} |m_T(\gamma)|$, where $m(\gamma)$ is a zero mean Gaussian process.

(ii)Under H_1, there exist an $\epsilon > 0$, such that:

$$Pr\left(\frac{1}{\sqrt{T}} M_T > \epsilon\right) \to 1$$

The proof follows from the empirical process CLT of Andrews (1991), for heterogeneous near epoch dependent (i.e. functions of mixing processes) arrays. The limiting distribution of our statistic, M_T is the supremum over a Gaussian process and hence standard critical values are not available. Also, note that M_T is not pivotal because the limiting distribution depends on the nuisance parameter $\gamma \in \Gamma$. The test has power against generic nonlinear alternatives, but the critical values have to be computed by bootstrap.

In the Monte Carlo study (Section 4) and the empirical part of the paper (Section 5), we employ the block bootstrap to obtain the critical values for our statistic. In the block bootstrap $\hat{e}_{t+1}$ and W_t are jointly resampled, in order to preserve the correct temporal behavior and to mimic the original statistic.

4 Monte Carlo Evidence

Our Monte Carlo study consists of two parts. First, we illustrate the effect that a misspecified loss function can have in the forecast evaluation framework of EKT. Then, we compare the empirical power of our Bierens type test with that of the J-test, in the presence of nonlinear dependencies between the sequence of forecast errors and the information set available at the time the forecast is made.

4.1 The Effect of a Misspecified Loss Function in Forecast Evaluation

To show that the J-test is loss function sensitive, we construct a Monte Carlo exercise where the forecaster's true loss function belongs to a different family than that assumed by the J-test. Nevertheless, the evaluation is done under the particular loss function introduced by EKT. To highlight the effect of a misspecified loss function, we examine the behavior of the estimator $\hat{\alpha}$ and study the properties of the J-test.

We assume that the variable of interest is generated by a simple AR(1) process:

$$x_{t+1} = c + bx_t + \epsilon_t$$

where $\epsilon_t \sim N\ (0,\ 0.5)$ and the parameters are set to c = 0.9 and b = 0.7. We generate random samples of size $T = R+P-1$, after discarding the first 100 observations to remove any initial values effect. Using a rolling window of size R, the forecaster constructs P one period ahead forecasts by minimizing the expected value of her loss function, L_1, assumed to be of the form given by (1). The observed one period ahead forecast is $\hat{f}_{t+1} = \hat{c} + \hat{a}x_t$ where:

$$(\hat{c}, \hat{b}) = \arg\min\ R^{-1} \sum_{t=1}^{R} L_1(\alpha_0,\ x_{t+1} - c - bx_t)$$

and where α_0 is the true value of the forecasters loss function asymmetry parameter. The sequence of the observed forecast errors is then computed as:

$$\{\hat{e}_{t+1}\}_{t=R}^{T} = \{x_{t+1} - \hat{c} - \hat{b}x_t\}_{t=R}^{T}$$

We perform 1000 Monte Carlo simulations for different choices of R, P and α_0. Our instrument set includes a constant and the lagged forecast error.

Table 1 reports the average α_0 estimates for various sample sizes and various values of the true asymmetry parameter. The estimator performs overall well when the loss function is correctly specified, the estimated values being close to the true values. Table 2 reports the empirical rejections probabilities for the J-test. Size is well controlled overall. We notice that there are some small size distortions in cases when $R/P \leq 1$.

Now, we examine the implications of falsely assuming that the forecasters true loss function belongs to (1), the class of loss functions introduced in EKT. For this, we reconstruct the Monte Carlo exercise from above, assuming that the forecasters true loss function is the widely used Linex loss defined as:

$$L_2(\epsilon_{t+h}; a) = \exp(a \cdot \epsilon_{t+h}) - a \cdot \epsilon_{t+h} - 1 \tag{9}$$

but this time, the forecast evaluation is inaccurately done under the loss function given in (1). In this case, we estimate $\hat{c}$ and $\hat{b}$ as:

$$(\hat{c}, \hat{b}) = \arg\min\ R^{-1} \sum_{t=1}^{R} L_2(a_0,\ x_{t+1} - c - bx_t)$$

where a_0 is the true value of the Linex loss function's asymmetry parameter.

Table 3 reports the average GMM estimates of α for different sample sizes and different values of the Linex loss function's true asymmetry parameter. The average estimates present large variations across different values of the true loss function's asymmetric parameter. The results given in Table 4 clearly indicate the size distortions of the J-test obtained when the forecast evaluation is done under a misspecified loss function. The J-test over rejects the null of rationality, the size distortions being larger, the larger is the Linex loss function's asymmetric parameter (in absolute value).

4.2 Empirical Size and Power Comparison

In our second Monte Carlo setup, we consider the following data generating process:

$$DGP:\ Y_{t+1} = \theta' W_t + \delta g(\phi' W_t) + U_t$$

where $\theta' = (\theta_1, \theta_2)$, $\phi' = (\phi_1, \phi_2)$ and $W_t = (W_{1t}, W_{2t})'$. We set the following expressions for the nonlinear function g, $g(x) = x^2$, $g(x) = exp(x)$, $g(x) = arctan(x)$. We also set different parametrizations for δ as indicated in Table 5. The parameters governing the process, θ and ϕ, are fixed to $(\theta_1, \theta_2) = (0.5, 0.5)$, $(\phi_1, \phi_2) = (0.7, 0.8)$. Here, the instruments are set to a constant and Z_t (which is generated so that it is correlated with W_{1t}, W_{2t} but uncorrelated with U_t). In order to ensure this, we generate W_{1t}, W_{2t}, Z_t, U_t from a multivariate normal distribution as follows:

$$\begin{pmatrix} W_{1t} \\ W_{2t} \\ Z_t \\ U_t \end{pmatrix} \sim N(0, \Sigma)$$

where Σ is the variance-covariance matrix set to:

$$\Sigma = \begin{pmatrix} 2 & 0.1 & 0.8 & 0.2 \\ 0.1 & 2 & 1 & 0.1 \\ 0.8 & 1 & 2 & 0 \\ 0.2 & 0.1 & 0 & 0.8 \end{pmatrix}$$

We generate a sample of size $T = 500$ for Y_{t+1} according to our data generating process. We assume the forecaster uses the first $R = 0.6 * T$ observations to estimte the parameters of the linear forecasting model:

$$Y_{t+1} = \theta_1 W_{1t} + \theta_2 W_{2t} + \epsilon_t$$

The observed one-step ahead forecasts $\hat{Y}_{t+1}$ and the observed one-step ahead forecast errors $\hat{e}_{t+1}$ are obtained using a recursive scheme. Given that a linear forecasting model was used to generate the forecast errors, even though the true data was generated by a nonlinear process, we ensure that the forecast error is correlated with some nonlinear function of W_t. This means that whenever $\delta \neq 0$, we can study the empirical power of the tests, while when we set δ to 0, we obtain the empirical sizes. We perform 1000

Monte Carlo simulations for both statistic. In addition, for the M_T statistic, in order to obtain the critical values, for each Monte Carlo replications we perform 100 bootstrap simulations.

Table 5 reports the rejection frequencies at a 10% significance level, for different non-linear functions, $g(x)$, and for different parameterizations for δ. When we set $\delta = 0$, the forecast error is uncorrelated with the available information and both tests have an empirical size close to the nominal size of 10%. In all the other cases, characterized by a nonlinear relationship between the forecast error and the information set, our M_T test which has power against all possible deviations from the null outperforms the J-test.

5 Empirical Illustration

In this section, we perform an empirical comparison of the linear J-test and the nonlinear M_T test. The data set used in the paper is from the Survey of Professional Forecasters (SPF) maintained by the Federal Reserve Bank of Philadelphia, where survey participants provide point forecasts for macroeconomic variables in quarterly surveys. The SPF does not specify the objective of the forecasting exercise and thus the objective of the forecasters is unknown. It is not sure at all that the forecasters simply minimizes a quadratic loss function and reports the conditional mean. It is thus reasonable not to impose too much structure on their unknown loss function. Nevertheless, the forecasts should indeed reflect the underlying loss function.

For our empirical illustration we use the following series: quarterly growth rates for real GNP/GDP (1968:4-2012:4) (the SPF provides data on GNP before 1992 and on GDP after 1992), the price index for GNP/GDP (1968:4-2012:4), and the quarterly growth rates for consumption (Real Personal Consumption Expenditures) (1981:3-2012:4). The growth rates are calculated as the difference in natural logs. In our empirical analysis, we focus on the median responses (results in Appendix A1), however for robustness we perform our analysis for the mean and range responses, too (see Appendix A2).

In the computation of the two statistics, we considered the one-step ahead forecast error obtained as the difference between the actual and the one-step ahead forecasted value. The point forecast data set of the SPF provides data on the year, the quarter, the most recent value known to the forecasters, the value for the current quarter (which is usually forecasted) and then forecasts for the next four quarters. To compute the one step ahead forecasted growth rates, we used values corresponding to the current quarter and the most recent value known. For the actual values, the SPF provides a real-time data set. In order to compute the actual growth rates we used the first release.

For the instruments of the J-test and the information set used in the computation of our M_T test, we considered the following 6 cases - Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors, Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant, lagged errors and lagged change in actual values, Case 6: constant, lagged errors, lagged change in actual values and lagged change in forecasts.

In the construction of the M_T statistic we chose the exponential function for w and the arctangent function for Φ. Following the literature, we set $\gamma_j \equiv \gamma(j+1)^{-2}$, where $\gamma \in [0,3]$ for Case 1. When γ is multidimensional we have for example

$$\gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \in [0,3] \times [0,3] \times [0,3]$$

and the test statistic say in Case 5 is computed as the supremum of the absolute value of:

$$\begin{aligned} m_T(\gamma) = \frac{1}{\sqrt{T}} \sum_{t=0}^{T-1} (\hat{e}_{t+1} - \overline{e}) \exp \left[\sum_{j=0}^{t-1} (\gamma_1 (j+1)^{-2} \tan^{-1}(Z_{1,t-j}) \right. \\ \left. + \gamma_2 (j+1)^{-2} \tan^{-1}(Z_{2,t-j}) + \gamma_3 (j+1)^{-2} \tan^{-1}(Z_{3,t-j})) \right] \end{aligned} \tag{10}$$

where Z_1 is a vector of ones, Z_2 contains the lagged errors and Z_3 the lagged change in actual values.

The critical values for the nonlinear forecast rationality test, M_T, are computed using the block bootstrap with overlapping blocks of length 5 and an overlap length of 2. Given the small sample sizes, we derive our conclusions based on the 10% bootstrap critical values.

Table 6 reports the results for the J forecast rationality test based on the median forecasts. For the real GNP/GDP, the estimates of the asymmetry parameter take values slightly less than 0.5, suggesting that forecasters tend to give higher weights on negative forecast errors than on positive ones. For the price index, the estimates take slightly higher values than 0.5, under-prediction being more costly than over-prediction. However, performing a t-test that tests H_0: α =0.5, we cannot reject the null of symmetric preferences for GNP/GDP and for the price index. Interestingly, this null hypothesis is rejected for consumption, variable for which forecasters tend to under-predict. At the 10% level, for GDP/GNP and the price index, the J-test does not reject the composite null hypothesis that the loss belongs to the family of loss functions defined in EKT and that the forecasts are rational. However, it rejects for most instrument sets cases of consumption, (more precisely for cases 1, 3, 5 and 6).

Analyzing now Table 7, where our suggested nonlinear test is computed, we notice that for the real GNP/GDP, our test results are in conformity with the J- test's results, forecast rationality not being rejected for this variable. For the price index and consumption, we obtain contrasting results that reveal interesting insights. Unlike the J-test, our M_T test rejects forecast rationality for the price index, which suggests that in the case of inflation, the forecast error depends in a nonlinear fashion on the information set used to produce the forecasts and the J-test is not able to detect these nonlinear dependencies. For consumption, our test does not reject rationality, even though the J-test rejects the null. This could indicate that the true loss function used to generate the forecasts for consumption was from a different family of loss than the one the J-test is based on, and consequently the J-test rejects the null.

The empirical results on the mean and the range responses, confirm our previous findings. We include the results on these two series in Appendix A2.

6 Concluding Remarks

In this paper, we propose a test for forecast rationality that allows for asymetric preferences, is consistent against generic (non)linear alternatives and does not assume any particular functional form for the forecaster's loss function. The key idea in the construction of our test is that under the null of forecast rationality, asymmetric preferences will always imply an unconditional bias of the forecast error, however the conditional bias has to be zero if the available information was efficiently used in the computation of the forecasts. Our test is a conditional moment type test in the spirit of Bierens (1982,1990), de Jong (1996).

We show through a Monte Carlo exercise that the forecast rationality test of EKT (2005, 2008) is loss function sensitive. The consequence of this is that if the true loss function of the forecaster does not belong to the particular class of loss under which the J-test is constructed, the test may lead to wrong inferences. In addition, simulations show that our test has higher power than the J-test in the presence of nonlinear dependencies between the forecast error and the information set used in forecasting.

Our empirical study highlights some different results that we obtain when applying the two forecast rationality tests to data from the Survey of Professional Forecasters. The contradiction in the results suggests that our proposed test could be used in broader contexts.

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Appendix A1

Table 1: GMM estimates for α obtained under the true loss function

R, P	$\alpha_0 = \mathbf{0.2}$	$\alpha_0 = \mathbf{0.4}$	$\alpha_0 = \mathbf{0.5}$	$\alpha_0 = \mathbf{0.6}$	$\alpha_0 = \mathbf{0.8}$
R=250, P=150	0.1970	0.3975	0.4976	0.6003	0.8028
R=250, P=200	0.1985	0.3992	0.4986	0.6020	0.8011
R=250, P=250	0.1995	0.4005	0.5021	0.5988	0.7998
R=200, P=250	0.2006	0.3988	0.4995	0.5990	0.8003
R=300, P=200	0.1997	0.3968	0.5009	0.6025	0.8011

NOTE: The table reports the average estimates for the asymmetry parameter α across 1000 Monte Carlo simulations for different values of the true asymmetry parameter α_0. R is the size of the rolling window used to construct the forecasts and P is the size of the evaluation sample.

Table 2: Rejection frequencies for the J-test when the forecast evaluation is done under the true loss function

R, P	$\alpha_0 = \mathbf{0.2}$	$\alpha_0 = \mathbf{0.4}$	$\alpha_0 = \mathbf{0.5}$	$\alpha_0 = \mathbf{0.6}$	$\alpha_0 = \mathbf{0.8}$
R=250, P=150	0.0370	0.0380	0.0460	0.0420	0.0420
R=250, P=200	0.0400	0.0360	0.0400	0.0390	0.0410
R=250, P=250	0.0320	0.0390	0.0330	0.0280	0.0380
R=200, P=250	0.0300	0.0330	0.0370	0.0290	0.0300
R=300, P=200	0.0450	0.0350	0.0410	0.0400	0.0470

NOTE: The table reports the percentage of rejections of the null of rationality at the 5% nominal level for different values of the true asymmetry parameter α_0. The forecaster's true loss function belongs to $L_1(\epsilon_{t+1}; p, \alpha) = [\alpha + (1-2\alpha)\cdot 1(\epsilon_{t+1} \leq 0)] \cdot |\epsilon_{t+1}|^p$. R is the size of the rolling window used to construct the forecasts and P is the size of the evaluation sample.

Table 3: GMM estimates for α obtained under a misspecified loss function

R, P	a_0=**-2**	a_0=**-1**	a_0=**-0.5**	a_0=**0.5**	a_0= **1**	a_0= **2**
R=250, P=150	0.0189	0.0153	0.0782	0.9216	0.9734	0.5961
R=250, P=200	0.0181	0.0158	0.0813	0.9198	0.9711	0.6010
R=250, P=250	0.0163	0.0164	0.0816	0.9191	0.9787	0.6081
R=200, P=250	0.0130	0.0185	0.0841	0.9166	0.9757	0.6249
R=300, P=200	0.0219	0.0149	0.0791	0.9231	0.9565	0.5926

NOTE: The table reports the average estimates for the asymmetry parameter α across 1000 Monte Carlo simulations for different values of the Linex loss asymmetry parameter and for different sizes of the rolling window R and forecast evaluation sample P.

Table 4: Rejection frequencies for the J-test when the forecast evaluation is done under a misspecified loss function

R, P	a_0=**-2**	a_0=**-1**	a_0=**-0.5**	$a_0 4 =$ **0.5**	a_0= **1**	a_0= **2**
R=250, P=150	0.3290	0.0530	0.0740	0.0790	0.2390	0.4000
R=250, P=200	0.3620	0.0610	0.0740	0.0860	0.3400	0.4540
R=250, P=250	0.4640	0.0980	0.0660	0.0850	0.2440	0.4990
R=200, P=250	0.4250	0.0850	0.0680	0.0830	0.4620	0.4860
R=300, P=200	0.4570	0.0840	0.0890	0.0880	0.4960	0.4350

NOTE: The table reports the percentage of rejections of the null of rationality at the 5% nominal level for different values of the Linex loss asymmetry parameter and for different sizes of the rolling window R and forecast evaluation sample P. The forecaster's true loss function is the Linex loss: $L_2(\epsilon_{t+1}; a) = \exp(a \cdot \epsilon_{t+1}) - a \cdot \epsilon_{t+1} - 1$.

Table 5: Empirical Size and Power for the two tests

		J-Stat	M_T-Stat
$g(x) = x^2$			
	$\delta = 0.2$	0.186	0.436
	$\delta = 0.5$	0.244	0.662
	$\delta = 1$	0.242	0.592
	$\delta = 2$	0.310	0.800
$g(x) = arctan(x)$			
	$\delta = 0.2$	0.296	0.610
	$\delta = 0.5$	0.302	0.724
	$\delta = 1$	0.298	0.762
	$\delta = 2$	0.320	0.812
$g(x) = exp(x)$			
	$\delta = 0.2$	0.290	0.616
	$\delta = 0.5$	0.324	0.736
	$\delta = 1$	0.362	0.856
	$\delta = 2$	0.430	0.816
	$\delta = 0$	0.104	0.116

NOTE: The table reports test rejection frequencies at a 10% significance level using a sample size of $T = 500$. The number of Monte Carlo replications is $M = 1000$. For the M_T-stat the number of bootstrap replications for each Monte Carlo replication is $B = 100$.

Table 6: Linear Test for Rationality based on **Median** Forecasts

	Instrument	α	std.err.	t-Stat	J-Stat	CV at 10%	p-value
Real GNP/GDP							
	Case 1	0.4630	0.0488	-0.7586	1.2149	2.71	0.2704
	Case 2	0.4759	0.0484	-0.4966	0.5967	2.71	0.4398
	Case 3	0.4651	0.0483	-0.7216	0.2062	2.71	0.6498
	Case 4	0.4715	0.0483	-0.5890	0.0640	2.71	0.8003
	Case 5	0.4683	0.0483	-0.6570	1.6501	4.60	0.4382
	Case 6	0.4662	0.0482	-0.7010	2.5526	6.25	0.4659
Price Index GNP/GDP							
	Case 1	0.5541	0.0471	1.1480	0.4478	2.71	0.5034
	Case 2	0.5676	0.0463	1.4601	2.6596	2.71	0.1029
	Case 3	0.5626	0.0460	1.3622	0.9246	2.71	0.3363
	Case 4	0.5610	0.0459	1.3312	0.6171	2.71	0.4321
	Case 5	0.5621	0.0458	1.3549	0.9456	4.60	0.6233
	Case 6	0.5587	0.0457	1.2841	1.9772	6.25	0.5772
Consumption							
	Case 1	0.2760	0.0502	-4.4603	5.9011	2.71	0.0151
	Case 2	0.3075	0.0522	-3.6884	0.7563	2.71	0.3845
	Case 3	0.2815	0.0503	-4.3453	4.1666	2.71	0.0412
	Case 4	0.3057	0.0519	-3.7425	0.3205	2.71	0.5713
	Case 5	0.2732	0.0498	-4.5532	6.0723	4.60	0.0480
	Case 6	0.2651	0.0492	-4.7755	7.1208	6.25	0.0681

NOTE: The table reports the asymmetry parameter α estimates, corresponding standard errors, values of the t-statistic testing H_0: α=0.5, the values of the J statistic and its critical values and p-values for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts. The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption.

Table 7: Non-Linear Test for Rationality based on **Median** Forecasts

	Information Set	**Test Statistic**	**Boot. CV at** 5%	**Boot. CV at** 10%
Real GNP/GDP				
	Case 1	0.0229	0.0581	0.0466
	Case 2	0.0224	0.0439	0.0439
	Case 3	0.0200	0.0457	0.0391
	Case 4	0.0164	0.0554	0.0493
	Case 5	0.0235	0.0470	0.0370
	Case 6	0.0208	0.0577	0.0494
Price Index GNP/GDP				
	Case 1	0.0316	0.0340	0.0303
	Case 2	0.0321	0.0257	0.0219
	Case 3	0.0322	0.0419	0.0302
	Case 4	0.0336	0.0319	0.0281
	Case 5	0.0308	0.0315	0.0305
	Case 6	0.0313	0.0371	0.0303
Consumption				
	Case 1	0.0152	0.0378	0.0338
	Case 2	0.0114	0.0397	0.0347
	Case 3	0.0171	0.0512	0.0400
	Case 4	0.0123	0.0410	0.0377
	Case 5	0.0220	0.0540	0.0471
	Case 6	0.0239	0.0520	0.0464

NOTE: The table reports the values of our nonlinear test statistics and its Bootstrap Critical Values at 5% and at 10% for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts. The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption. The block length is 5. The blocks are overlapping with an overlap length of 2. The number of bootstrap replications is $B = 100$.

Appendix A2

Table 8: Linear Test for Rationality based on **Mean** Forecasts

	Instrument	α	**std.err.**	**t-Stat**	J**-Stat**	**CV at 10%**	p**-value**
Real GNP/GDP							
	Case 1	0.4556	0.0488	-0.9100	0.7695	2.71	0.3804
	Case 2	0.4688	0.0484	-0.6450	1.5282	2.71	0.2164
	Case 3	0.4550	0.0481	-0.9348	0.2026	2.71	0.6526
	Case 4	0.4603	0.0479	-0.8283	0.0144	2.71	0.9045
	Case 5	0.4591	0.0479	-0.8532	0.8986	4.60	0.6381
	Case 6	0.4567	0.0478	-0.9055	1.8447	6.25	0.6053
Price Index GNP/GDP							
	Case 1	0.5532	0.0472	1.1252	0.7229	2.71	0.3952
	Case 2	0.5543	0.0468	1.1586	0.3312	2.71	0.5650
	Case 3	0.5619	0.0461	1.3422	1.1376	2.71	0.2862
	Case 4	0.5602	0.0460	1.3082	0.7403	2.71	0.3896
	Case 5	0.5610	0.0460	1.3262	1.1972	4.60	0.5496
	Case 6	0.5578	0.0459	1.2594	2.2457	6.25	0.5230
Consumption							
	Case 1	0.3041	0.0523	-3.7489	5.4346	2.71	0.0197
	Case 2	0.3325	0.0538	-3.1113	0.3351	2.71	0.5627
	Case 3	0.3152	0.0527	-3.5047	2.8750	2.71	0.0900
	Case 4	0.3325	0.0538	-3.1158	0.0037	2.71	0.9515
	Case 5	0.3043	0.0522	-3.7486	5.4401	4.60	0.0659
	Case 6	0.2976	0.0518	-3.9110	6.4760	6.25	0.0906

NOTE: The table reports the asymmetry parameter α estimates, corresponding standard errors, values of the t-statistic testing H_0: α=0.5, the values of the J statistic and its critical values and p-values for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts. The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption.

Table 9: Linear Test for Rationality based on **Range** Forecasts

	Instrument	α	**std.err.**	**t-Stat**	J**-Stat**	**CV at 10%**	p**-value**
Real GNP/GDP							
	Case 1	0.4606	0.0487	-0.8091	1.2459	2.71	0.2643
	Case 2	0.4749	0.0483	-0.5192	0.7263	2.71	0.3941
	Case 3	0.4651	0.0482	-0.7732	0.2062	2.71	0.6498
	Case 4	0.4628	0.0482	-0.6253	0.2251	2.71	0.6352
	Case 5	0.4664	0.0481	-0.6995	1.6993	4.60	0.4276
	Case 6	0.4643	0.0480	-0.7430	2.6056	6.25	0.4565
Price Index GNP/GDP							
	Case 1	0.5454	0.0470	0.9656	1.1797	2.71	0.2774
	Case 2	0.5492	0.0466	1.0556	1.1358	2.71	0.2865
	Case 3	0.5503	0.0460	1.0922	0.8136	2.71	0.3671
	Case 4	0.5464	0.0460	1.0077	0.2856	2.71	0.5931
	Case 5	0.5486	0.0460	1.0575	1.2741	4.60	0.5289
	Case 6	0.5454	0.0459	0.9894	2.3338	6.25	0.5061
Consumption							
	Case 1	0.2662	0.0494	-4.7295	6.7588	2.71	0.0093
	Case 2	0.3042	0.0520	-3.7668	0.9609	2.71	0.3270
	Case 3	0.2826	0.0503	-4.3194	3.7277	2.71	0.0535
	Case 4	0.3069	0.0520	-3.7135	0.0219	2.71	0.8824
	Case 5	0.2665	0.0493	-4.7306	6.7658	4.60	0.0339
	Case 6	0.2584	0.0487	-4.9597	7.8038	6.25	0.0502

NOTE: The table reports the asymmetry parameter α estimates, corresponding standard errors, values of the t-statistic testing H_0: α=0.5, the values of the J statistic and its critical values and p-values for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts.The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption.

Table 10: Non-Linear Test for Rationality based on **Mean** Forecasts

	Information Set	**Test Statistic**	**Boot. CV at** 5%	**Boot. CV at** 10%
Real GNP/GDP				
	Case 1	0.0054	0.0460	0.0406
	Case 2	0.0078	0.0612	0.0445
	Case 3	0.0028	0.0578	0.0454
	Case 4	0.0010	0.0453	0.0397
	Case 5	0.0049	0.0600	0.0461
	Case 6	0.0023	0.0542	0.0512
Price Index GNP/GDP				
	Case 1	0.0298	0.0306	0.0228
	Case 2	0.0312	0.0380	0.0292
	Case 3	0.0297	0.0361	0.0266
	Case 4	0.0311	0.0326	0.0284
	Case 5	0.0283	0.0297	0.0260
	Case 6	0.0281	0.0309	0.0263
Consumption				
	Case 1	0.0130	0.0382	0.0350
	Case 2	0.0087	0.0429	0.0365
	Case 3	0.0137	0.0467	0.0387
	Case 4	0.0086	0.0485	0.0376
	Case 5	0.0188	0.0539	0.0436
	Case 6	0.0196	0.0547	0.0481

NOTE: The table reports the values of our nonlinear test statistics and its Bootstrap Critical Values at 5% and at 10% for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts. The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption. The block length is 5. The blocks are overlapping with an overlap length of 2. The number of bootstrap replications is $B = 100$.

Table 11: Non-Linear Test for Rationality based on **Range** Forecasts

	Information Set	**Test Statistic**	**Boot. CV at** 5%	**Boot. CV at** 10%
Real GNP/GDP				
	Case 1	0.0114	0.0601	0.0408
	Case 2	0.0113	0.0522	0.0414
	Case 3	0.0082	0.0477	0.0403
	Case 4	0.0047	0.0572	0.0487
	Case 5	0.0116	0.0601	0.0505
	Case 6	0.0084	0.0697	0.0538
Price Index GNP/GDP				
	Case 1	0.0367	0.0337	0.0272
	Case 2	0.0380	0.0377	0.0298
	Case 3	0.0382	0.0357	0.0298
	Case 4	0.0399	0.0416	0.0328
	Case 5	0.0365	0.0348	0.0287
	Case 6	0.0381	0.0417	0.0338
Consumption				
	Case 1	0.0116	0.0431	0.0361
	Case 2	0.0074	0.0360	0.0306
	Case 3	0.0124	0.0477	0.0376
	Case 4	0.0071	0.0418	0.0360
	Case 5	0.0178	0.0500	0.0432
	Case 6	0.0187	0.0530	0.0485

NOTE: The table reports the values of our nonlinear test statistics and its Bootstrap Critical Values at 5% and at 10% for Cases 1-6. Case 1: constant and lagged errors, Case 2: constant and absolute lagged errors Case 3: constant and lagged change in actual values, Case 4: constant and lagged change in forecasts, Case 5: constant and lagged errors plus lagged change in actual values, Case 6: constant and lagged errors and lagged change in actual values and lagged change in forecasts. The sample size is $T = 177$ for Output and Prices and $T = 126$ for Consumption. The block length is 5. The blocks are overlapping with an overlap length of 2. The number of bootstrap replications is $B = 100$.